Government Financing of R&D:
A Mechanism Design Approach[1]

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August 4, 2017

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Abstract

We study the design of a government loan program for risky R&D projects generating externalities, undertaken by entrepreneurs in a competitive capital market. With adverse selection, the optimal contract requires a high interest rate but nearly zero co-financing by the entrepreneur. This contrasts sharply with observed policies, typified by low interest rates and high co-financing. When we add moral hazard, the optimal policy consists of a menu of at most two contracts, one with high interest/zero self-financing and a second with lower interest but also a co-financing requirement. Calibrated simulations compare welfare from the optimal policy and observed program designs.

1 Introduction

It is widely recognized that innovation, and the knowledge spillovers that underpin it, are the primary source of economic growth. They lie at the heart of modern macroeconomic growth models and are central to policy debates over how best to promote sustained growth and competitiveness (Grossman and Helpman, 1991; Aghion and Howitt, 1992; Acemoglu and Akcigit, 2012). Governments have adopted a variety of policy instruments to support innovation. Some countries rely mainly on indirect fiscal incentives such as R&D tax credits; others focus on direct instruments such as grants, loans and equity finance.[1] In this paper we study the optimal design of government loan policies for R&D projects.

There are two basic justifications for government support of R&D: knowledge spillovers and capital market failure. An extensive empirical literature documents substantial and pervasive spillovers, which have the consequence that the market generates underinvestment in R&D – i.e., the social rate of return to R&D is much larger than the private return.[2] However, the mere fact that R&D generates positive externalities is not itself a justification for government support. Some of those R&D projects might be financed by private capital markets in any event, so that government support would be 'redundant.' Since public funds are costly, government support that is redundant would be welfare-reducing. Conversely, even if capital markets were perfect, there could be R&D projects that cannot get market financing based on their private returns but which generate sufficient social returns (including knowledge spillovers) to justify being undertaken. In this context, government finance can be thought of as a way of purchasing the (expected) social returns from projects that would otherwise not be realized.

The second justification for government support is imperfect capital markets. Economic theory points to information asymmetry – adverse selection and moral hazard – as the source of capital market failure, and there is evidence that access to finance constrains innovation activity, especially for small, young firms (Hall and Lerner, 2010). Government support can

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[1] Countries that rely heavily on indirect support include Australia, Belgium, Canada, France, Ireland, Japan and the Netherlands. Those that focus on direct funding include Finland, Germany, Israel, Italy, New Zealand, Poland, Russia, Spain, Sweden and the United States. For details see OECD (2013).

[2] Bloom, Schankerman and Van Reenen (2013) show that R&D also creates a negative externality from product market rivalry, but their empirical results confirm that the positive externality dominate – the average social rate of return is more than twice the private rate of return.

potentially mitigate this problem, but this depends critically on whether government has better information than private capital markets or can elicit it by suitable mechanisms not available to market agents, or is better at diversifying risk. Be that as it may, we show below that under certain conditions the optimal loan scheme imposes almost no self-financing requirements on firms, which implies that capital market imperfections need not pose a serious problem.

There have been both survey-based and econometric studies that assess the 'additionality' of existing R&D subsidy and loan schemes – i.e., the extent to which financing increases the number and/or performance of projects relative to the counter-factual of no support program. Most studies find some degree of additionality (i.e., not fully crowding out), and also reveal considerable variation across programs in the degree of additionality.[3] However, to our knowledge there is no research on how program design features affect their comparative performance in terms of additionality. More generally, we are not aware of any theoretical research on how loan programs should be optimally designed to maximize welfare.

Government support programs vary along three main dimensions: (1) whether grants or loans are used; (2) the interest rate charged in the case of loans; and (3) the co-financing requirements from the applicant for both grants and loans. Many programs are pure grant schemes, requiring no repayment of principal or interest. Others are loan programs that require repayment, sometimes only when the project is deemed successful (generating revenues). In all loan schemes we examined for OECD countries, there is either a zero or nominal interest rate imposed. Finally, many of the grant and loan schemes require the applicant to co-finance (from her own or other private sources) the cost of the R&D project. Private co-financing rates vary from about 20 to 80 percent.[4]

This paper applies the techniques of mechanism design to analyse the optimal structure of R&D loan schemes. We develop a model in which projects generate positive externalities and the government has limited information about the projects. In order to focus on externalities, we abstract from capital market imperfections (but we do incorporate a constraint

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[3]Studies include Busom (2000), Klette, Moen and Griliches (2000), Wallsten (2000), Lach (2002), Gonzalez, Jaumandreu and Pazo (2005). For general discussion of additionality, see OECD (2006). For a structural model of the welfare effects of R&D subsidies, see Takalo, Tanayama and Toivanen (2013a, 2013b and 2017).

[4]For more information about R&D support programs, see http://erawatch.jrc.ec.europa.eu/erawatch/opencms/information/country_pages/.

on the ability of the entrepreneur to self-finance). In our model, entrepreneurs have R&D projects characterized by three features: a probability of success, private returns (assumed, for simplicity, to be common to all projects; this restriction is relaxed later) and an externality. Both the success probability and social returns vary across projects. We assume that the success probability is known to the entrepreneur and the competitive capital market, but not to the government. Since the competitive market interest rate depends inversely on the project probability of success (or entrepreneur type), the private cost of funds varies across projects but is not known to the government. We assume that the government knows (or has a signal on) the social returns and has two instruments at its disposal: the interest rate on the loan and a co-financing (matching funds) requirement.

We first derive the welfare maximizing policy under the assumption that the government does not know the success probabilities of projects but there is no moral hazard (i.e., exogenous success probabilities). In this case, the optimal policy achieves 'first-best' efficiency and involves selecting exactly those projects that are socially profitable but will not be financed by the capital market. The optimal policy is to set the interest rate at close as possible to the ex post rate of return of the most successful project that would still not be supported by the private market together with a co-financing rate that approximates zero.

The basic trade-off in the model is between making government funding 'additional' and inducing the implementation of all projects that generate positive expected social returns (netting out the social cost of public funds). Projects with high success probability (low interest rates in the private market) need to be screened out to ensure additionality, but projects with very low success probability also need to be excluded because their expected social returns will not justify undertaking them. In this sense, the optimal policy needs to 'target the middle'. Using a high interest rate – in the limit, the *ex post* rate of return – ensures additionality, and a low co-financing requirement maximizes the set of projects with sufficiently high social returns from which the government can select. This result differs sharply from the typical R&D loan schemes observed in the real world which have significant co-financing requirements but zero or negative interest rates. The optimal policy is also very different from the commonly observed pure grant schemes (equivalent to a loan scheme with an interest rate of minus 100 percent).

When we introduce moral hazard – allowing the entrepreneur to exert costly effort to

improve the project's success probability – we find that the optimal policy involves a lower interest rate, in order to provide incentives to the entrepreneur to undertake effort, together with a higher co-financing requirement. The optimal interest rate is decreasing, and the co-financing requirement is increasing, in the degree of moral hazard. Thus, as moral hazard concerns become more serious, the optimal policy moves in the direction of policies that are observed in practice.

We use simulation analysis to illustrate how the optimal policy varies with the shadow price of government funds, and to assess how alternative policies, commonly observed in practice, perform relative to the optimal design in terms of additionality and welfare. We show that there are substantial welfare gains from moving to the optimal policy for a wide range of parameters, notably the size of the project externality and the social cost of public funds.

From a theoretical perspective, the optimal policy is notable for two reasons. First, it describes a mechanism design problem for which the optimal solution is 'simple' in the sense that the optimal menu consists of at most two alternatives (at most one for each level of induced effort, two in our model). This is very unusual in mechanism design. Moreover, the contracts we consider induce payoffs that are linear in the entrepreneur's type.[5]

The feature that generates this simplified mechanism is that the optimal policy involves 'targeting the middle' – the high types (high probability of success) will be funded by the private market and low types do not justify public financing because the expected social gains are negative. We show that if a given type will take the government loan, so will a higher type. Because of the incentive compatibility constraint, offering another contract to the higher type involves leaving more rent to the entrepreneur and there is no social payoff to doing that. Moreover, the simulations show that for reasonable parameter values the optimal solution consists of just one alternative.

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[5]In our model the optimal mechanism is a single (linear) contract for each effort level. Laffont and Tirole (1986) provide an early example of a mechanism design with adverse selection and moral hazard that generates a menu of linear incentive contracts.

For discussion of 'simple' mechanisms and how to achieve them, see Hurwicz (1973), Wilson (1985), Dasgupta and Maskin (2000), and Bergemann and Morris (2005). For other examples of simple mechanisms, see Myerson (1981), Samuelson (1984), Fuchs and Skrypacz (2005), Brusco and Hopehayn (2007), and Bergemann, Bonatti and Smolin (2017). In related work, McAfee (2002) shows that, under strong assumptions, a simple menu comprised of a small number of contracts can extract close to the full surplus even when the optimal menu consists of a continuum of contracts.

Second, our finding that the optimal solution consists of at most two contracts is robust to the introduction of two-dimensional uncertainty, where there is both asymmetric information about the project probability of success and the private return when successful. This is also very unusual, and we are not aware of any other problem in the mechanism design literature for which this is the case.

The paper is organized as follows. Section 2 describes the set up of the model. In Section 3 we derive the optimal policy when there is adverse selection but no moral hazard. Section 4 introduces moral hazard – i.e. allows the entrepreneur to make costly effort to affect the project's probability of success and shows that this materially changes the structure of the optimal policy. In Section 5 we present simulations, calibrated on observed data, to assess the welfare performance of different policies against the benchmark of the optimal policy with moral hazard. We conclude with a brief summary and implications for policy. All proofs are relegated to an appendix.

2 Model

2.1 Definitions and Assumptions

We consider a model where a government faces a large number of entrepreneurs. Each entrepreneur has a project that generates both private and social benefits, and the government has to determine whether and how to support these projects.

An entrepreneur or project is characterized by a pair (p, s) where p is the project's probability of success provided the entrepreneur exerts effort, and s is the (non-negative) externality it generates. A successful project generates a commonly known (private) return $R > 1$.[6] If the project fails, then the private return and social contribution are both zero. The cost of the project is commonly known to be $c_I + c_P \equiv 1$. We interpret c_I and c_P as the costs of 'inspiration' and 'perspiration,' respectively. Incurring the 'inspiration cost' c_I is necessary in order to come up with the idea for the project. Incurring the 'perspiration cost' c_P enhances the project's probability of success but is not necessary for the project to succeed.

Let probability $p \in [0, 1]$ be the project's success probability provided the entrepreneur

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[6]The assumption that R is commonly known is relaxed in Section 4.2.1. It simplifies the analysis but does not affect our main result.

exerts the effort $c_I + c_P \equiv 1$. We assume that p is known by the entrepreneur and observable to private lenders (the 'private market'), but not to the government.[7] If the entrepreneur only exerts the effort c_I then the project's probability of success is kp for some $k \in [0, 1)$. For simplicity, in the analysis (and simulations) that follow, we assume that the success probability reflects constant returns to scale in effort, which implies $k = c_I$.[8] This assumption does not affect the qualitative nature of our results. We assume that the entrepreneur's effort c_P is unobservable to the market and the government. Below we distinguish between the case with and without moral hazard. If $k = 1$ (and so $c_P = 0$) then there is no moral hazard. The severity of the moral hazard problem facing the entrepreneur is decreasing in k.

We assume that the government observes a signal of the externality from a project, $\sigma \in [0, \infty)$, which we normalize to be such that $\sigma \equiv E[s|\sigma]$. Because σ provides the best estimate of the unobserved s and the government is risk neutral, no loss of generality is involved by simply replacing s by σ below. Thus we assume that a project is characterized by a pair (p, σ) instead of (p, s), and that the government believes that p and σ are drawn from a commonly known joint distribution $F(p, \sigma)$. In particular, in the model we do not make any assumptions about the correlation between p, R and σ, so the correlation (if any) between expected private and total social returns, pR and $p(R + \sigma)$, is unrestricted.

2.2 The Capital Market

Entrepreneurs have funds $\bar{b} \le 1$ of their own. They are able to finance the first (inspiration) stage of the project on their own, i.e., $c_I \le \bar{b}$. In addition, they have access to a perfectly competitive capital market in which they can borrow additional funds to help finance the second stage of their projects at an interest rate that reflects the project's success probability.

The entrepreneur approaches a bank and asks for a loan of size $1 - \bar{b}$ that would allow it to increase the project's success probability from kp to p. The bank assesses the success probability, which depends on whether the entrepreneur would indeed exert full effort $c_I + c_P \equiv$

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[7] The probability of success p may be replaced by anything else that is positively associated with the quality of the project such as the terms and conditions associated with a market loan, that are commonly known between the entrepreneur and the market, but are unobservable or unverifiable by the government.

[8] The (arc) elasticity of the success probability with respect to cost is $\eta = \frac{(1-k)p}{kp} / \frac{1-c_I}{c_I}$. With constant returns, $\eta = 1$ which implies $k = c_I$.

1 or not, and offers the entrepreneur a loan that ensures that the bank breaks even on its loan in expectation. Then the entrepreneur decides whether to incur the additional cost $c_P = 1 - c_I$. We assume that the bank is the senior claimant on the return of the project. If the project succeeds, then the payoff to the entrepreneur is equal to whatever remains after the entrepreneur returns the loan plus interest to the bank. If the project fails the bank gets nothing in return.

We normalize the risk-free interest rate in this capital market to zero and ignore discounting. The interest rate at which private lenders break even on a loan made to an entrepreneur of type p who is induced to exert the effort $c_I + c_P \equiv 1$ is

$$r_P(p) = \frac{1}{p} - 1.$$

An entrepreneur does not need a loan if it only intends to exert partial effort and, as shown below, there are no incentives for taking a loan if the project is not going to be fully developed. Therefore, we focus on the case in which entrepreneurs borrow and lenders lend, if at all, only the amount $1 - \bar{b}$, provided this loan induces the entrepreneur to exert the full effort $c_I + c_P \equiv 1$. This yields the following *moral hazard constraint*:

$$p\left(R - \left(1 - \bar{b}\right)\left(1 + r_P(p)\right)\right) + \left(1 - \bar{b}\right) - 1 \geq kp\left(R - \left(1 - \bar{b}\right)\left(1 + r_P(p)\right)\right) + \left(1 - \bar{b}\right) - c_I.$$

The left hand side is the expected benefits from exerting full effort: a project is successful with probability p and generates a return R minus the amount the entrepreneur has to pay back to the lender. From this we subtract the entrepreneur's own funds invested in the project. The right hand side is similar except for the fact that with partial effort, the probability of success decreases to kp and the cost of effort decreases to c_I. This constraint ensures that the loan $\left(1 - \bar{b}\right)$ induces the entrepreneur who gets it to exert full effort. Under constant returns to scale $(k = c_I)$ the moral hazard constraint simplifies to

$$p \geq \frac{2 - \bar{b}}{R}.$$

This moral hazard constraint is also the lenders' participation constraint if the entrepreneur is induced to exert full effort.[9]

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[9] Another relevant constraint is that the entrepreneur has sufficient funds to return the loan when the project succeeds, or $\left(1 + r_P(p)\right)\left(1 - \bar{b}\right) \leq R$, but this constraint simplifies to $p \geq \frac{1 - \bar{b}}{R}$, which is weaker than the moral hazard constraint.

An entrepreneur of type p such that

$$p\left(R - \left(1 - \bar{b}\right)\left(1 + r_P(p)\right)\right) + \left(1 - \bar{b}\right) - 1 \geq kpR - c_I,$$

prefers a loan of size $1 - \bar{b}$ at the interest rate $r_P(p)$ in which it exerts full effort to not taking a loan and exerting only partial effort. Under constant returns this inequality simplifies to

$$p \geq \frac{c_P}{(1 - k)\,R} = \frac{1}{R}$$

Thus, any entrepreneur that satisfies the moral hazard constraint, and would thus be offered a loan of size $1 - \bar{b}$ at interest rate $r_P(p)$, prefers to accept such a loan over rejecting it and only exerting partial effort on his project.

If $kpR - c_I \geq 0$ the project generates enough expected returns for the entrepreneur to gain also on a partial investment c_I. This inequality induces a participation constraint for an entrepreneur who does not intend to exert full effort, or

$$p \geq \frac{c_I}{kR} = \frac{1}{R}.$$

Notice that if a loan of size L with competitive interest rate $\frac{1}{kp} - 1$ were offered to an entrepreneur intending to exert only partial effort, it would generate an expected return $kp\left(R - L\left(1/kp\right)\right) - \left(c_I - L\right)$, which is identical to the expected return of partially developing the project without the bank loan. Thus, given the assumption that own funds are sufficient for the first stage, the private market will not fund partially-developed projects. Notice also that entrepreneurs of type $p < \frac{1}{R}$ will not accept a bank loan to develop the first stage of the project at the competitive interest rate $\frac{1}{kp} - 1$, and therefore such projects will not be developed.

The discussion above describes what loans would be offered and what effort they would induce if accepted by entrepreneurs. What would entrepreneurs actually do? An entrepreneur of type p will accept a loan of size $1 - \bar{b}$ that induces full effort at the interest rate $r_P(p)$ only if

$$p\left(R - \left(1 - \bar{b}\right)\left(1 + r_P(p)\right)\right) + \left(1 - \bar{b}\right) - 1 \geq \max\left\{kpR - c_I, 0\right\}$$

which, under constant returns, reduces to

$$p \geq \frac{1}{R}.$$

This last inequality yields the participation constraint for an entrepreneur induced to exert full effort.

The following proposition characterizes the set of projects funded by the private market without government intervention.

Proposition 1 *Entrepreneurs of type $p \in \left[0, \frac{1}{R}\right]$ abandon their projects, entrepreneurs of type $p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right]$ develop their projects but only exert partial effort without a bank loan, and entrepreneurs of type $p \in \left(\frac{2-\bar{b}}{R}, 1\right]$ receive a bank loan of size $1 - \bar{b}$ at the interest rate $r_P(p)$, exert full effort and develop their projects.*

Denote the payoff to the entrepreneur from developing its project on its own or borrowing in the private market by $U_P(p)$. Proposition 1 implies that U_P is given by

$$U_P(p) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{1}{R}\right] \\ k(pR - 1) & \text{if } p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right] \\ pR - 1 & \text{if } p \in \left(\frac{2-\bar{b}}{R}, 1\right] \end{cases}$$

This analysis shows that, absent government intervention, entrepreneurs with $p < \frac{2-\bar{b}}{R}$ will not be able to obtain financing for their projects from the private market and these projects may either be abandoned or only partially implemented. Entrepreneurs with $p < \frac{1}{R}$ would not want to develop their projects on their own even with partial effort, so these projects will be abandoned. However, to the extent that some of these projects may increase social welfare, the government may be interested in helping to fund them. In addition, some of the projects with $p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right]$, which are only partially implemented, may also increase social welfare and warrant government support to induce full implementation. We turn to the design of government funding in the next section.

2.3 Government Funding

We assume that the government supports projects in the following way. When an entrepreneur applies for a loan, the government obtains a signal σ about the externality of the project, and offers a menu of loans to the entrepreneur of $1 - b_\sigma$ at interest rate r_σ. Each loan enables the entrepreneur to implement the project, and she chooses (at most) one of the loan contracts offered. An entrepreneur who selects a loan of size $1 - b_\sigma$ needs to raise an amount b_σ from her

own or borrowed funds. We restrict attention to cases where $b_\sigma \leq \bar{b}$. Matching-loan schemes are used by many countries, but our specification has the additional feature that the co-payment requirement b_σ is allowed to depend on the externality generated by the project, σ. As we show below, the optimal menu consists of at most one pair (b_σ, r_σ) for each level of induced effort. For notational simplicity, in what follows we omit the subscript σ.

The shadow price of government funds is $1 + \lambda$ where $\lambda \geq 0$. The expected social welfare generated by a project (p, σ) that receives government support in the form of a loan of size $L = 1 - b$ at interest rate $r \leq \frac{R}{L} - 1$ (this inequality ensures the entrepreneur can pay back the loan to the government if the project is successful), and where the entrepreneur exerts full effort, is

$$W(p, \sigma, b, r) = p(R + \sigma) - 1 - \lambda(1 - b)(1 - p(1 + r)) \tag{1}$$

A project is successful with probability p and in this case generates a total return of $R + \sigma$, the cost of full effort is one, and the cost of public funds λ is incurred on all funds lent by the government that it is unable to collect in the event of project failure, i.e., $(1 - b)(1 - p(1 + r))$. If the entrepreneur exerts only partial effort, then the expression for social welfare is similar, except that the probability of success is kp instead of p and the cost of effort is c_I instead of 1.

The expected social welfare of a project in which the entrepreneur exerts full effort but does not receive government support is

$$W(p, \sigma) = p(R + \sigma) - 1 \tag{2}$$

As before, if the entrepreneur exerts only partial effort, the probability of success is kp and the cost of effort is c_I.

An entrepreneur who is offered a government loan with an interest rate that is lower than she can obtain in the private market may accept the government loan. When this happens, the government loan does *not* generate additional innovation since, absent the loan, the entrepreneur would have financed the project in the private market. In this case, government support of these projects is 'redundant' because the set of projects being implemented by entrepreneurs is not changed by the support program.

This suggests that in order to maximize expected social welfare the government should try to fund only those projects that will not be financed by the private market. However, as

we will show, this is only part of the story because avoiding redundancy (not funding projects with high p's that can be financed in the private market) can restrict the set of projects being implemented, but some of those projects may have large externalities.

3 The Case without Moral Hazard

In this section we solve for the optimal policy in the case where $k = c_I = 1$. The analysis of this case is intuitive and simple, and interesting in its own right. We first describe the first-best solution in this case, that is the optimal solution if the government can observe projects' probabilities of success p. In many cases, the first-best is only a theoretical benchmark that cannot be implemented in practice, but here the first-best is attainable if the entrepreneur faces no moral hazard problem.

3.1 First-Best

If the government can observe p, it should support a project if, and only if, the project would not otherwise be funded by the private market, that is[10]

$$p < \frac{1}{R}$$

and the project generates a positive expected social welfare:

$$p\left(R + \sigma\right) - 1 - \lambda\left(1 - b\right)\left(1 - p\left(1 + r\right)\right) \geq 0. \tag{3}$$

An entrepreneur will accept a government loan at interest rate r with self-financing requirement b if and only if it cannot obtain a loan with a lower interest rate in the private market,[11] and if it makes a nonnegative payoff

$$p\left(R - \left(1 - b\right)\left(1 + r\right)\right) + \left(1 - b\right) - 1 \geq 0. \tag{4}$$

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[10]Recall that without moral hazard, partial effort is equivalent to full effort.

[11]Observe that the size of the loan does not affect the entrepreneur's consideration. Namely, an entrepreneur prefers a smaller loan at a lower interest rate to a larger loan at the market interest rate, provided of course that the smaller loan still permits completion of the project.

Suppose that $r < r_P(p)$. The former loan generates an expected payoff of $p(R - L(1 + r)) + L - 1 = pR - 1 + L(1 - p(1 + r))$ and the latter generates an expected payoff of $p(R - L(1 + r_P(p))) + L - 1 = pR - 1$. The former is larger than the latter if and only if $L(1 - p(1 + r)) > 0$, which is equivalent to $r < \frac{1}{p} - 1 = r_P(p)$, irrespective of the size of the two loans.

If the government offers the entrepreneur a loan with an interest rate $r > R - 1$, then the expected payoff to an entrepreneur who borrows L from the government is $p(R - L(1 + r)) + L - 1 \leq p(R - LR) + L - 1 = (1 - L)(pR - 1)$. This is negative for entrepreneurs with $p < \frac{1}{R}$, which the government targets. It follows that we may assume that the interest rate that is charged by the government on any loan it makes is smaller than or equal to $R - 1$.

We solve for the smallest probability of success p that should be supported with any government loan. Inequality (3) implies

$$b \geq 1 - \frac{p(R + \sigma) - 1}{\lambda(1 - p(1 + r))} \tag{5}$$

which is decreasing in p and inequality (4) implies

$$b \leq \frac{p(R - (1 + r))}{1 - p(1 + r)}$$

which is increasing in p. It follows that the smallest p that should be supported by the government solves $1 - \frac{p(R+\sigma)-1}{\lambda(1-p(1+r))} = \frac{p(R-(1+r))}{1-p(1+r)}$ and is given by

$$p = \frac{1 + \lambda}{R(1 + \lambda) + \sigma}$$

Because private rents are subsidized by costly public funds, they should be minimized as much as possible. Note that the social welfare generated by a government contract (b, r) is equal to $p(R + \sigma) - 1 - \lambda + \lambda[b + p(1 - b)(1 + r)]$, which is increasing in $b + p(1 - b)(1 + r)$, while the expected payoff to entrepreneur p from contract (b, r) is equal to $pR - 1 - [b + p(1 - b)(1 + r)]$ which is decreasing in $b + p(1 - b)(1 + r)$. This implies that a contract (b, r) maximizes expected social welfare if and only if it minimizes entrepreneur p's payoff. Therefore, maximization of expected social welfare implies that inequality (4) should be binding. That is, the self-financing and interest rate b and r should satisfy the equation

$$1 - b = \frac{1 - pR}{1 - p(1 + r)}, \tag{6}$$

which leaves no rents to the entrepreneur.

Observe in (6) that in general b depends on r and p, but as $1 + r$ approaches R, b tends to zero independently of the value of p. This suggests that the government may be able to set b and r optimally even without being able to observe p. We show this in the next section.

3.2 Optimal Policy without Moral Hazard

Suppose that the government cannot observe p. Given a policy (b, r), denote the marginal type that accepts funding by the government by $p_{min}(b, r)$. From (4), it follows that

$$p_{min}(b, r) = \frac{b}{R - (1 - b)(1 + r)} \leq \frac{1}{R}$$

which is positive for all values of $b > 0$.

Consider the following family of mechanisms $\{b_\varepsilon, r_\varepsilon\}_{\varepsilon > 0}$ defined by

$$r_\varepsilon = R - 1 - \varepsilon; \quad b_\varepsilon = \frac{\varepsilon(1 + \lambda)}{\sigma + \varepsilon(1 + \lambda)}.$$

Observe that for each one of the mechanisms in this family

$$p_{min}(b_\varepsilon, r_\varepsilon) = \frac{1 + \lambda}{R(1 + \lambda) + \sigma}$$

as in the first-best. So the set of entrepreneurs seeking financing under the mechanism are all those with types p such that $p \geq p_{min}(b, r)$ and $r_\varepsilon \leq r_P(p)$, or

$$\frac{1 + \lambda}{R(1 + \lambda) + \sigma} \leq p < \frac{1}{R - \varepsilon}$$

It follows that by choosing a mechanism $(b_\varepsilon, r_\varepsilon)$ with a small $\varepsilon > 0$, the government can minimize the set of 'redundant' projects $p \in \left[\frac{1}{R}, \frac{1}{R - \varepsilon}\right]$ that would have obtained private funding. Moreover, because as ε nears 0, r_ε approaches $R - 1$, the government can extract approximately the entire rent from each participating entrepreneur. It follows that a mechanism $(b_\varepsilon, r_\varepsilon)$ with a small $\varepsilon > 0$ allows the government to approximate the first-best solution.

Proposition 2 *It is possible to approximate the first-best solution with a mechanism $(b_\varepsilon, r_\varepsilon)$ with a small $\varepsilon > 0$ that tends to $(b, r) = (0, R - 1)$ as ε tends to zero.*

The geometric intuition for the result is as follows. For a mechanism $(b_\varepsilon, r_\varepsilon)$, the expected payoff of entrepreneurs from accepting the government's loan is linear in p, and equal to zero at the point where $p = p_{min}(b_\varepsilon, r_\varepsilon) = \frac{1 + \lambda}{R(1 + \lambda) + \sigma}$. As ε tends to zero, the payoff to the entrepreneur pivots downward on the axis point p_{min}, which remains fixed. This allows the government to target the first-best set of projects and to decrease their rents toward zero. However, the government cannot attain the first-best by setting $\varepsilon = 0$, because in this case $p_{min}(b, r)$ jumps

discontinuously to zero, which prevents the exclusion of projects that generate a negative expected social welfare. In other words, when $\varepsilon = 0$ we have $r_\varepsilon = R - 1$ and $b_\varepsilon = 0$ so that expected payoffs are zero for any project that accepts a government loan. This implies that entrepreneurs with projects with $p < p_{\min}$, which generate negative expected welfare, are indifferent between accepting a government loan or not.[12]

The economic intuition is that the optimal policy charges a high interest rate to induce projects with $p \geq \frac{1}{R-\varepsilon}$ to prefer funding by the private market: the higher the interest rate (smaller ε) the smaller the set of subsidized projects that could have been financed by the market. But increasing the interest rate r also reduces the set of socially desirable projects that cannot be privately financed. To induce such entrepreneurs to seek a government loan, the government increases the size of the loan so as to make it just profitable for them to implement their projects. Notice also that, somewhat paradoxically, the optimal policy calls for (almost) fully funding the supported projects (b is approximately equal to zero) even though public funds are more expensive than private funds.

The mechanism described here has one unattractive property: it leaves almost no rent for the entrepreneur and thus gives no incentive to exert greater effort. This is not a problem if the project's success probability is exogenous, but if the entrepreneur's effort affects it we need the optimal mechanism to incorporate this moral hazard. We address this issue in the next section.

4 The Case with Moral Hazard

4.1 Optimal Policy

We now analyze the case with moral hazard, where the entrepreneur can exert additional effort to increase the probability of success. The timing of moves is as follows: an entrepreneur learns

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[12]It is possible to exactly implement the first-best with the following direct revelation mechanism: ask entrepreneurs to report their type p. If an entrepreneur reports a type $p \in \left[\frac{1+\lambda}{R(1+\lambda)+\sigma}, \frac{1}{R} \right]$ then offer a loan with interest rate $r = R - 1$ and self-financing requirement $b = 0$. If the reported type p lies outside this interval, do not offer any loan. It is straightforward to verify that this mechanism is incentive compatible and ex-post efficient, and thus implements the first-best outcome. However, this mechanism may be difficult to implement in practice because, apart from requiring entrepreneurs to report their type which may be difficult to do in practice, entrepreneurs with types $p \in \left[0, \frac{1}{R} \right]$ are indifferent between reporting their types truthfully or not, but it is crucial for the efficiency of the mechanism that they report their types truthfully. We are grateful to Phil Reny for this observation.

its success probability p and decides whether to make an initial investment of c_I. Next the entrepreneur decides whether to apply for either a private or government loan that would help it complete its project, and whether to exert full effort. If the entrepreneur receives a loan, she implements her project. The payoff to the entrepreneur is whatever remains after she repays its loans. The loan is not repaid if the project fails.

As explained above, we assume the government may offer entrepreneurs to choose whatever combination (b_σ, r_σ) of self financing requirement and interest rate they want from a menu of such choices $\{(b_\sigma, r_\sigma)\}$. We denote the government contract that maximizes entrepreneur p's payoff by $(b_\sigma(p), r_\sigma(p))$. Again, for notational simplicity, we henceforth omit the subscript σ. Denote the payoff to entrepreneur p if it were to choose the government contract $(b(p), r(p))$ by $U_G(p)$. Observe that

$$U_G(p) \equiv \max\{p(R - (1 - b(p))(1 + r(p))) - b, \ kp(R - (1 - b(p))(1 + r(p))) + 1 - b(p) - c_I\}$$

where the first and second terms in the braces describe the expected payoff to entrepreneur p under government contract $(b(p), r(p))$ when she exerts full and partial effort, respectively. Note that $U_G(p)$ may be smaller than the expected payoff to entrepreneur p in the private market, $U_P(p)$, in which case type p prefers either to borrow in the private market, partially implement her project on her own, or drop the project.

Moral Hazard Constraint: A government contract (b, r) induces full effort from an entrepreneur of type p who receives government support if

$$p(R - (1 - b)(1 + r)) - b \geq \ kp(R - (1 - b)(1 + r)) + 1 - b - c_I \tag{7}$$

Participation Constraint: A government contract (b, r) induces an entrepreneur of type p to accept a government loan if the expected payoff to the entrepreneur under the government contract with either full or partial effort is larger than or equal to what the entrepreneur can obtain in the private market:

$$U_G(p) \geq U_P(p).$$

Incentive Compatibility Constraint: A menu of government contracts $\{(b(p), r(p))\}_{p \geq p^*}$ is incentive compatible for types $p \geq p^*$ if each type $p \geq p^*$ is induced to choose the government

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contract $(b(p), r(p))$ when its choice is restricted to government contracts and to exert full effort.

The following characterization of incentive compatible menus follows from standard arguments in mechanism design.

Proposition 3 *A menu $\{(b(p), r(p))\}_{p \geq p^*}$ is incentive compatible for types $p \geq p^*$ if and only if the induced expected payoff function of the entrepreneur $p\left(R - (1 - b(p))\left(1 + r(p)\right)\right) - b(p)$ is monotone increasing and convex for types $p \geq p^*$ and type p^* is induced to exert full effort. This is the case if and only if $b(p)$ is nondecreasing and $(1 - b(p))\left(1 + r(p)\right)$ is nonincreasing in $p \geq p^*$, respectively.*

The government's objective is to choose an incentive compatible menu $\{b(p), r(p)\}_{p \in [0,1]}$ that maximizes expected social welfare, taking into account that projects with $p \geq \frac{1}{R}$ will be financed by the private market or the entrepreneurs themselves if they do not obtain government support and that among the entrepreneurs who choose the same government contract some may exert full effort while others may exert only partial effort.

Proposition 4 *The optimal menu of government contracts consists of at most two contracts: one contract that induces full effort from some entrepreneurs who would otherwise develop their projects on her own and exert partial effort, and another contract that induces partial effort from some entrepreneurs who would not otherwise develop their projects even with partial effort. The optimal menu may also consist of just one of these two types of contracts.*

The intuition for this result is as follows: As explained above, the maximization of social welfare requires that entrepreneurs' payoffs be minimized. Proposition 3 shows that the incentive compatibility of government contracts implies that their induced payoffs to entrepreneurs is increasing and convex. The smallest possible increasing and convex function is linear, which is the payoff that is induced by a single government contract. The fact that if a certain type p is induced to exert full effort by some government contract then so do all higher types $p' > p$ who choose it, implies that there is no need for more than one government contract that induces full effort. A similar argument shows that there is no need for more than one government contract that induces partial effort.

Proposition 5 *The optimal government contract that induces partial effort is an epsilon contract $(b_\varepsilon, r_\varepsilon) \approx \left(0, \frac{R}{c_I} - 1\right)$, which attracts low probability projects that are not privately profitable but are socially profitable. The optimal government contract that induces full effort is given by $(\bar{b}, r)$ for some interest rate r*

The epsilon contract offers a loan of c_I (more precisely, $c_I - b_\varepsilon$) and induces projects that would not be implemented by the market to be partially implemented generating *additionality at the extensive margin*. The self financing requirement under this contract is approximately equal to zero. In contrast, the contract that induces full effort from some entrepreneurs who would not exert full effort without this contract is a (b, r) contract in which the self financing requirement b is set equal to the upper bound $\bar{b}$. This contract generates *additionality both at the extensive and intensive margins*.

The intuition is that an epsilon contract induces partial effort at nearly zero social cost so is (approximately) first-best. As for the contract that induces full effort from some types, denote the lowest type that exerts full effort under (b, r) by p^*. Because public funds are costly, maximization of expected social welfare requires that (b, r) maximizes the sum $b + p(1-b)(1+r)$ for types $p \geq p^*$ and $b + pk(1-b)(1+r)$ for types $p < p^*$ (i.e., minimize government expenditures for those projects), for those entrepreneurs that choose (b, r) contract subject to the moral hazard constraint $p^* \geq \frac{1}{R - (1-b)(1+r)}$. Recall that if the moral hazard constraint is satisfied for p^*, then it is also satisfied for all $p > p^*$. This implies that b should be increased to the maximum feasible level with the interest rate ensuring that the moral hazard constraint is binding at p^*: $b = \bar{b}$ and $(1-b)(1+r) = R - \frac{1}{p^*}$.

Inspection of the proof of Proposition 5 (Appendix A) shows that a contract that induces full effort from some entrepreneurs, who otherwise would exert only partial effort, will also be chosen by some entrepreneurs who would only exert partial effort under this contract and would have developed their projects on their own without this contract. This implies that such a contract may be very costly for the government, and would be offered only if k is relatively small (i.e., moral hazard is important), as the extra effort has a high payoff in raising the success probability and the set of such projects is small.

Whether the government prefers to offer only one of the contracts or both depends on

which policy generates higher welfare, which in turn depends on the parameters of the model: the values of k, private returns R, externality σ, and cost of public funds λ (as well as $\bar{b}$ and the distribution of p). We study this issue in the simulations in Section 5.

4.2 Extensions

4.2.1 Uncertainty about private returns

Our main result that the optimal contract includes at most one contract that induces full effort and another contract that induces partial effort (Proposition 4) is robust to adding uncertainty and asymmetric information about the return upon success R. To see this, suppose that R has two values, $R_H > R_L > 1$. Entrepreneurs and the private market know the realization of R but the government does not.

Observe that if type (p, R_L) prefers government contract (b, r) to government contract (b', r'), then so does type (p, R_H).[13] This implies that the government cannot screen entrepreneurs based on R. However, because an entrepreneur with a higher R obtains a larger expected payoff from any given government or private contract, entrepreneurs with different R's may have different incentives to choose any given government contract. Hence, a contract (b, r) is chosen by type (p, R_L) if and only if it is also chosen by type (p, R_H), provided that p is large enough. The next proposition is a corollary of this fact.

Proposition 6 *Suppose that R has two values, $R_H > R_L > 1$. The optimal menu of government contracts consists of at most two contracts: one that induces full effort from some entrepreneur who would not exert full effort under the contract offered to her in the private market, and another that induces partial effort from some entrepreneur who would not exert partial effort without this contract.*

For simplicity, this argument was made for the case in which R can have two possible realizations, but it also applies to any number of possible realizations larger than two.

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[13]Type (p, R_L) prefers government contract (b, r) to contract (b', r') if and only if $p(R_L - (1-b)(1+r)) - b \geq p(R_L - (1-b')(1+r')) - b'$ which holds if and only if $p(R_H - (1-b)(1+r)) - b \geq p(R_H - (1-b')(1+r')) - b'$ which holds if and only if type (p, R_H) prefers government contract (b, r) to contract (b', r').

4.2.2 Government budget constraint

We have assumed that the government does not face a budget constraint. Introducing a constraint does not fundamentally alter the analysis, if there is a continuum of projects (in terms of p and σ). In order to maximize expected welfare, the government should simply rank projects by the welfare per dollar of government money invested and then fund them in descending order until the budget is exhausted. Of course, to do this the government must first compute the optimal policy for each σ, as discussed before.

However, if there is a discrete number of indivisible projects, this criterion may create a 'knapsack problem' because it may be the case that the project that generates the highest expected social welfare per dollar invested requires a large investment that prevents the government from investing in other projects, whereas the project that generates the second highest expected social welfare per dollar invested is cheaper and allows for more welfare enhancing investments. However, this is a computational, rather than a conceptual, problem that can be addressed with existing algorithms.

5 Simulations

In this section we present calibrated simulations that compute the welfare generated by different R&D support policies and examine how they vary with parameters of the model. We assess how observed policies perform relative to the optimal policy derived in Section 4, and demonstrate that traditional metrics used to assess programs, such as additionality and redundancy, can be misleading.

5.1 Calibration

We calibrate the key parameters of the model using data from the U.S. and Israel. These parameters are: R, the distributions of $\{p_i\}$ and $\{\sigma_i\}$, c_I, λ and $\bar{b}$.

We use the distributions of private and social rates of returns to R&D for 609 publicly-traded U.S. manufacturing firms, φ^p and φ^s, estimated by Bloom, Schankerman and Van Reenen (2013).[14] The estimated φ^p vary from 0.11 to 0.30 (mean = 0.21); φ^s varies from 0.13

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[14]The estimated φ^p includes the direct effect of a marginal increase in a firm's R&D on output and the redistribution of output from other firms due to product market rivalry. The estimated φ^s includes the direct

to 1.0 (mean = 0.56). To map these to our model, we assume a lag of L periods between the investment cost (normalized to unity) and the private and social returns, R and $(R + \sigma)$ respectively. These internal rates of return are defined by $\frac{p_i R}{(1+\varphi_i^p)^L} - 1 = 0$ and $\frac{p_i(R+\sigma_i)}{(1+\varphi_i^s)^L} - 1 = 0$, which imply

$$p_i R = (1 + \varphi_i^p)^L \tag{8}$$

$$p_i(R + \sigma_i) = (1 + \varphi_i^s)^L \tag{9}$$

We use data on loans to start-up projects from the Israel Innovation Authority (2014) to calibrate the mean success probability, $\bar{p}$, and the mean commercialization lag $\bar{L}$. We measure $\bar{p}$ by the average fraction of supported projects that fully pay back the loans (plus interest), and $\bar{L}$ as the average time elapsed from initiation of the R&D project until the first payment back on the loan (among successful projects). The source reports averages for each of ten technology areas and we compute their weighted mean using the number of projects in each area as weights. The weighted means are $\bar{p} = 0.28$ and $\bar{L} = 2.5$ years. Substituting these values into (8) and (9) gives us $R = 5.7$ and $\bar{\sigma} = 5.1$.

To calibrate the distribution of success probabilities $\{p_i\}$, we use the beta distribution $B(\alpha, \beta)$. The Israeli data give us an estimate of the mean, $\bar{p} = 0.28$, and variance $V(p) = 0.0176$. From the formulas for the mean and variance of the beta distribution, we solve for its parameters $(\alpha, \beta) = (2.93, 7.53)$.[15] The kernel density of this distribution is presented in Appendix C.

To calibrate c_I (project cost with partial effort), we use data on the structure of venture capital funding from Gompers (1995). He breaks funding into the early rounds (seed + startup), usually made to very young companies, middle rounds usually made to young but further developed companies, and late stage financing. We use two measures of c_I: (1) ratio of the median of seed funding to total early round funding, 0.21; and (2) ratio of the median of early

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effect of R&D and the gains to other firms from knowledge spillovers. These parameters reflect the marginal returns from an extra dollar of R&D, which can be interpreted as the expected return.

[15] Because of data limitations, we can only estimate the variance *across* ten technology fields. Doing this implicitly assumes that the variance within a technology area is zero, and thus gives a lower bound to the total variance of p. If we assume instead that the within-field variance is a proportion μ of the between-variance, α and β will be lower if $\mu < 1$ (higher if $\mu > 1$), but the ratio α/β is preserved. To check robustness, we also run simulations using the alternative assumptions $\mu = 0.5, 1, 2$. The qualitative results are similar to those reported below, except that the optimal interest rate in the (b, r) contract is substantially higher. Details are reported in Appendix C.

round to total funding, 0.15.[16]

We calibrate λ based on estimates in the public finance literature (Dahlby, 2008). These vary depending on methodology, country coverage, and the choice of taxes used to generate the public funds. We focus on public funds raised by taxes on labor income; the estimated values of λ typically fall in the range of 0.25 to 1.5. The full cost of public funds is $1 + \lambda$.

Finally, we set the fraction of project cost with full effort that can be funded by the entrepreneur at $\bar{b} = 0.25$; the qualitative results reported below are robust to using other values of $\bar{b}$. (See Appendix C).

5.2 Simulation results

5.2.1 Optimal policy versus private market

We simulate the model with moral hazard to assess the performance of the optimal policy and to compare it to stylized policies that reflect observed practice. Computational details are provided in Appendix B. For each regime, we compute the welfare gain from the optimal policy (relative to relying solely on the private market), the fraction of projects implemented, and the share of implemented projects that are done with partial and full effort. In addition we compute measures of 'additionality at the extensive margin' (projects induced by the policy that would not be done otherwise), 'additionality at the intensive margin' (projects induced to shift from partial to full effort), and 'redundancy' (supported projects that would have been privately funded otherwise).

Table 1 presents results for different combinations of σ and λ. We fix $k = 0.21$ and $\bar{b} = 0.25$, but the results are robust to other values (see Appendix C).[17] For each case, we compute the optimal policy, the welfare gains it generates relative to no government intervention (Panel A), and other measures of performance (Panel B). For brevity, for each σ we present the results only for the values of λ up to where the optimal policy shifts from the (b, r) contract to

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[16]These values are similar to those computed from data on Israeli start up companies by Harel (2013). The mean (median) value of the share of seed money out of total invested funds is 0.23 (0.18) among the 1,149 firms that passed the seed stage. We thank Shai Harel for providing his data.

[17]We compute expected welfare using the same distribution of success probabilities of projects $\{p\}$ for different values of σ.This computational procedure implicitly assumes that p and σ are independent, but we emphasize this is not required for the theory. The correlation between the estimates of p and σ derived from the Bloom, Schankerman and Van Reenen (2013) data is small, -0.09.

Table 1. Performance Metrics of Optimal Policy

		Panel A			Panel B			
sigma	lambda	Optimal Policy	Optimal r (%)	Welfare Gain(%)	Projects Implemented (%)	Additionality Extensive (%)	Additionality Intensive (%)	Redundancy (%)
1	0.25	Epsilon	.	0.2	79.4	10.1	0.0	0.0
2.5	0.25	(b,r)	13	10.1	100.0	28.6	28.2	43.2
2.5	0.5	Epsilon	.	0.7	85.4	16.4	0.0	0.0
5	0.25	(b,r)	-82	19.4	100.0	28.6	34.6	36.8
5	0.5	(b,r)	13	8.3	100.0	28.6	28.2	43.2
5	0.75	Epsilon	.	1.4	90.2	20.8	0.0	0.0
7.5	0.25	(b,r)	-82	25.1	100.0	28.6	34.6	36.8
7.5	0.5	(b,r)	13	15.2	100.0	28.6	28.2	43.2
7.5	0.75	(b,r)	13	7.4	100.0	28.6	28.2	43.2
7.5	1	Epsilon	.	1.8	91.8	22.2	0.0	0.0
10	0.25	(b,r)	-82	28.6	100.0	28.6	34.6	36.8
10	0.5	(b,r)	-82	20.4	100.0	28.6	34.6	36.8
10	0.75	(b,r)	13	13.2	100.0	28.6	28.2	43.2
10	1	(b,r)	13	6.8	100.0	28.6	28.2	43.2
10	1.25	Epsilon	.	2.1	92.4	22.7	0.0	0.0

Note: Additional parameters used in the simulations are R=5.7, k=.21, bbar=.25.

the epsilon contract (for higher λ, the epsilon contract remains optimal).

The simulations in Panel A illustrate several important features. First, the welfare-maximizing policy depends on the cost of public funds, λ. When λ is high, it is optimal to use only the epsilon contract; at lower λ, the optimal policy is to offer the (b,r) contract. This is because the epsilon contract produces no redundancy; the (b,r) contract generates some degree of redundancy which is socially costly when λ is high. In all the simulations, when the joint – (b,r) plus epsilon – contract is optimal, we find that entrepreneurs choose only the (b,r) contract (thus in what follows we simply call it the (b,r) contract).

Second, for given λ, the (b,r) contract is optimal when the project externality, σ, is high. Offering the joint contract is costly because it involves redundancy. However, this contract widens the set of projects covered and also induces some partially implemented projects to switch to full implementation, both of which are more valuable when σ is high. For moderate levels of externality, the epsilon contract is optimal unless the cost of public funds is quite low.

Third, when the (b,r) contract is optimal, the associated interest rate increases with

λ and decreases with σ. This is because a higher interest rate reduces expenditure of public funds (more is repaid if the project succeeds), but it also reduces the set of (non-redundant) projects that seek support, which is costly for welfare when σ is large. The optimal interest rate is sensitive to these parameters. When λ is low, the interest rate is negative, implying a partial grant rather than a loan. Only when λ is high does the optimal (b, r) contract involve a loan, and the interest rate is modest (13 percent in our simulations).[18] If λ is high enough, the optimal policy shifts to the pure epsilon contract which involves an interest rate that captures nearly all of the return ($r = \frac{R}{c_I} - 1 - \varepsilon$).

Fourth, the welfare gains from using the optimal policy is substantial when the (b, r) contract is optimal – varying from about 7 to 29 percent. However, the gain is quite small when the epsilon contract is optimal. Thus the welfare gain depends heavily on λ and σ because these parameters determine whether the epsilon contract or the (b, r) contract is optimal. As noted earlier, when the (b, r) contract is optimal, the associated self-financing rate is always set to its maximum feasible level, $b = \bar{b}$ (see proof of Proposition 5 in Appendix A).

Turning to Panel B, we first note that, for all sets of parameters, the private market alone funds 71.4 percent of projects. About half of these projects are fully implemented, the others partially implemented (not shown). As column 1 shows, the optimal policy always increases the fraction of projects that are implemented, even when it is just the epsilon contract. Both the epsilon and the (b, r) contracts generate a lot of additionality (column 2) – inducing implementation of roughly 20-30 percent of projects that would otherwise not be undertaken – and this finding is not sensitive to most values of λ and σ. Moreover, the (b, r) contract generates substantial additionality at the intensive margin (column 3) – inducing about 30 percent of implemented projects to shift from partial to full implementation. The epsilon has no effect at the intensive margin since that policy appropriates all ex post returns so the entrepreneur only exerts partial effort.

These results suggest that neither redundancy nor additionality should be used as a criterion for optimality. The epsilon contract generates no redundancy but it is not always

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[18]In Appendix C we provide addititonal simulations which show that the interest rate for the optimal (b, r) contract is also sensitive to the value of $\bar{b}$. For example, when $\bar{b} = 0.5$, $\sigma = 5$, and $\lambda = 0.5$, the optimal interest rate is 69 percent; for $\bar{b} = 0.75$, it is 159 pecent. The actual policies we observe are inconsistent with this finding: loan schemes that have high self-financing rates always have very *low (if any)* interest rates.

welfare superior to the (b, r) contract. The same applies to the additionality measure: the (b, r) contract generates larger additionality than the epsilon contract, but it is not always better.

To examine this more closely, for each pair of parameters (σ, λ), we use the simulations to compute the policy that would be chosen if the objective were to minimize redundancy, or alternatively maximize additionality, and compare the welfare from such policies to the optimal policy that maximizes welfare. For brevity, we only summarize the findings here (the full set of results is available on request). When the criterion is to minimize redundancy, the choice is always the epsilon policy since it generates zero redundancy. Thus there is only a welfare loss in cases where the optimal policy is the (b, r) contract and, as shown in Table 1, that occurs when σ is high and/or λ is low. For these cases, we find that minimizing redundancy yields a level of welfare about 4-20 percent lower than the optimal policy, depending on parameters.

When the objective is to maximize additionality, the choice is less obvious since the epsilon policy also generates additionality, though only on the extensive margin. However, for all the parameter pairs in our simulations, under this criterion the (b, r) policy is always chosen. Thus it generates welfare loss only when the optimal policy is the epsilon contract, which occurs when σ is low and/or λ is high. In these cases, we find that maximizing additionality generates a level of welfare about 2-75 percent lower than the optimal policy, again depending on parameters.

In short, neither additionality nor redundancy is an appropriate criterion for evaluating R&D support schemes. Both criteria can be highly misleading and should be interpreted with care.

5.2.2 Optimal vs observed policies

In nearly all countries that have R&D loan schemes, the contract involves a single interest rate and matching requirement – in contrast to our optimal (b, r) contract where the interest rate depends on the externality from the project, σ.[19] Almost all of these are either full grant schemes or interest-free loans (repayment conditional on project success) – i.e., (b, r) policies

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[19]The only exception we know is a Flemish interest-free loan scheme that uses a different co-financing rate for 'research' projects (20 percent) and 'development' projects (80 percent).

with interest rates of $r = -1$ and $r = 0$, respectively. How do full grant and interest-free loan schemes (not conditioned on σ) perform relative to the optimal policy, and to each other?

To assess this, we simulate the welfare generated by a full grant scheme, an interest-free loan and, for comparison, a policy with an interest rate of 15 percent, for different values of σ and λ. In these we fix $b = \bar{b}$. It is not clear *a priori* which of these policies generates greater welfare. The advantage of a full grant is that it induces full effort for any projects that are implemented and have $p \geq \frac{1}{R}$ (those with $p < \frac{1}{R}$ will be partially implemented), whereas under the loan scheme a larger set of projects is implemented with only partial effort. However, the grant is costly as it gives inframarginal rent to projects that would be implemented anyway with the interest-free loan. This tradeoff suggests that grants are only likely to be better if σ is high and λ is low enough. A scheme with a small interest rate saves on costly public funds, but may reduce implementation at the extensive and/or intensive margins.

Table 2 summarises the results. Three key features stand out. First, both grants and interest-free loans are much worse than the optimal policy unless public funds are not costly ($\lambda = 0.25$), or the externality is very large. This is in line with the trade-off discussed above. In fact, these policies are often even worse than no intervention at all (not shown in the table). When $\lambda = 0.75$ and $\sigma = 2.5$ (implying a social rate of return about 45 percent higher than the private rate), the grant generates 32.6 percent less welfare than the optimal policy; for interest free loans, the shortfall is about 21.6 percent. The performance of both schemes deteriorates sharply with the value of λ. A similar pattern holds for higher values of σ.

Second, these results imply that shifting from full grants to interest-free loans would generate a substantial welfare gain unless λ is very low (results not shown).[20]. For example, when $\lambda = 0.75$ and $\sigma = 2.5$, the welfare gain is 16.4 percent. With $\sigma = 5$, the policy shift generates 6.6 percent higher welfare. The gains rise with λ and decline with σ. Given that the loan scheme only requires repayment if the project succeeds, a policy shift from grants to interest-free loans poses no risk to the entrepreneur and thus would seem to be an attractive reform.

Finally, and somewhat surprisingly, charging an interest rate of 15 percent makes little

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[20]The entries in Table 2 are percentage changes relative to the base welfare under the optimal policy. Straightforward manipulations allow us to deduce the gains relative to the full grant policy.

Table 2. Policies using fixed interest rates (and b=bbar=0.25)

	Welfare gain relative to optimal policy (%)		
	sigma		
lambda	2.50	5.00	7.50
	-1.9	-0.3	-0.2
0.25	-0.2	-0.7	-1.5
	-0.3	-1.0	-1.9
	-32.6	-10.0	-3.5
0.75	-21.6	-4.1	-0.4
	-20.3	-3.6	-0.3
	-92.5	-50.9	-29.3
1.5	-67.6	-35.5	-18.8
	-64.4	-33.6	-17.7

Notes: Top entry = Full grant vs optimal contract

Middle entry = Free interest loan vs optimal contract

Bottom entry = 15% interest loan vs optimal contract

Additional parameters used in the simulations are R=5.7, k=.21, bbar=.25.

difference relative to an interest-free loan; both perform similarly against the optimal policy (this holds for modest variations in the rate). This reason is that the welfare function is not sensitive to interest rates at low levels (however, this is not the case for negative or large positive interest rates, as the earlier simulation results confirm).

To summarize, the simulations show that policies commonly used by governments are likely to be particularly undesirable in settings where the social cost of public funds is high (and/or externalities are small). While many factors play a role, countries with weaker institutional capacity are likely to have less efficient tax systems, and so higher λ. Thus R&D loan policies we observe – and particularly full grants – are likely to be ill-suited for developing countries.

6 Concluding remarks

We study the optimal design of government financing for R&D projects that vary in risk and generate positive externalities. These programs are often used to support innovation by high-

tech, start-up companies. We show that, when there is adverse selection over project risk, the optimal contract requires a high interest rate but (virtually) zero self-financing. This contrasts sharply with observed policies, which are typically characterized by zero or negative interest rates and high self-financing provisions. When we add moral hazard, allowing the entrepreneur's effort to affect the probability of success, the optimal policy consists of a menu of at most two contracts, one with high interest/zero self-financing and a second with lower interest but some co-financing.

Calibrated simulations of the model show there are significant welfare gains from moving to the optimal policy. We also show that evaluating policies based on commonly-used metrics – such as additionality and/or redundancy – can be very misleading. The simulations also confirm, as implied by the model, that the gains from the optimal policy depend heavily on the level of the externality generated by projects and the cost of public funds.

There are two important policy implications of the paper. First, optimal policies should 'target the middle'. Projects with low risk are likely to be financed by the private market anyway, so government support is redundant. Projects with very high risk will not be privately funded but, unless they generate very large externalities, the expected social payoff from supporting them is likely to be small. This key message does not seem to be widely appreciated. The second policy message is that different designs are needed in different settings. If the (average) externality differs across technology fields, the parameters of the policy should ideally vary by field. The same principle applies across countries, where the extent of externalities and the cost of public funds are likely to differ.

Why do observed loan programs differ sharply from the theoretically optimal policy? First, governments may think that, in order to help entrepreneurs, they need to set low interest rates than those they could get in the private market, and then demand a high co-financing rate as a mark of 'seriousness' on part of the entrepreneurs. Our analysis undermines this presumption. The second explanation is that 'targeting the middle' is likely to be politically less attractive to governments than targeting the 'best' (low risk) projects, as is done in practice. Governments may be concerned that the general public does not appreciate the idea of redundancy and would interpret support for intermediate projects as the inability of the public agency to screen effectively. Finally, the public agency itself may worry about the government's

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commitment to fund it in the future, and hedge this risk by choosing profitable projects Whatever the reason, this paper shows that moving toward the optimal policy generates important welfare gains.

A Proofs of Propositions

Proof of Proposition 1. Follows from the discussion above, upon observation that an entrepreneur p develops the project if and only if p satisfies both the lender's and entrepreneur's participation constraints, taking into account the fact that for large p's entrepreneurs prefer bank loans at interest rates that induce them to exert full effort. ∎

Proof of Proposition 2. Follows from the arguments above the statement of Proposition 2. ∎

Proof of Proposition 3. An incentive compatible menu satisfies

$$U_G(p) \equiv p\left(R - (1 - b(p))(1 + r(p))\right) - b(p) \geq p\left(R - (1 - b(p'))(1 + r(p'))\right) - b(p')$$

and

$$U_G(p') \equiv p'\left(R - (1 - b(p'))(1 + r(p'))\right) - b(p') \geq p'\left(R - (1 - b(p))(1 + r(p))\right) - b(p)$$

for any two types $p > p' > p^*$. It follows that

$$(p - p')\left(R - (1 - b(p'))(1 + r(p'))\right) \leq U_G(p) - U_G(p') \leq (p - p')\left(R - (1 - b(p))(1 + r(p))\right)$$

from which it follows that $(1 - b(p))(1 + r(p))$ is non-increasing in $p \geq p^*$. Dividing the last inequality by $p - p'$ and taking the limit as $p' \searrow p$ implies that the derivative of $U_G(p)$ is equal to $(R - (1 - b(p))(1 + r(p)))$ whenever it is continuous in p, which because of monotonicity holds a.s. in $p \geq p^*$. And, the fact that the derivative of $U_G(p)$ is non-increasing implies that $U_G(p)$ is convex for $p \geq p^*$.

Conversely, if $U_G(p)$ is convex and type p^* is induced to exert full effort, then the payoff that any type $p' \geq p^*$ obtains from selecting the contract $(b(p), r(p))$ is obtained on a line at the point $(p, U_G(p))$ with slope $U_G'(p)$, at the point p' on that line. Convexity of $U_G(p)$ implies that this payoff lies below $U_G(p')$ which is the payoff that type p' obtains by being truthful.

Finally, rearrangement of the moral hazard constraint (7) shows that a government contract (b, r) induces full effort from type p if and only if

$$p \geq \frac{1}{R - (1 - b)(1 + r)}.$$

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It follows that if p^* is induced to exert full effort under (b, r), then so is every $p \geq p^*$ because convexity of $U_G(p)$ implies that $(1 - b(p))(1 + r(p))$ is nonincreasing in $p \geq p^*$. ■

Proof of Proposition 4. A government contract (b, r) induces an expected payoff to entrepreneurs of $p(R - (1 - b)(1 + r)) - b$ that is linear in p. Increasing b pivots this payoff function in the sense that it increases its slope $R - (1 - b)(1 + r)$ and lowers its intercept $-b$. Increasing b and r in such a way that keeps $(1 - b)(1 + r)$ fixed shifts the payoff function downwards in a parallel way.

Following the last part of the proof of Proposition 3, the moral hazard constraint (7) implies that if p is induced to exert full effort under (b, r), then so is every $p' > p$, and that both increasing b and increasing b and r in a way that keeps $(1 - b)(1 + r)$ fixed preserves p's incentive to exert full effort.[21]

Suppose that the optimal menu induces some entrepreneur $p \leq \frac{2 - \bar{b}}{R}$ who would not exert full effort under the private market to exert full effort. Denote the smallest type that is induced to exert full effort by the optimal menu by p^* and the government contract that is chosen by p^* by (b, r). The fact that p^* is induced to choose the contract (b, r) implies that $p^*(R - (1 - b)(1 + r)) - b \geq U_P(p^*)$.

Recall that

$$U_P(p) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{1}{R}\right] & \text{no implementation} \\ (pR - 1)k & \text{if } p \in (\frac{1}{R}, \frac{2 - \bar{b}}{R}] & \text{partial} \\ pR - 1 & \text{if } p \in (\frac{2 - \bar{b}}{R}, 1] & \text{full} \end{cases}$$

Distinguish the following three cases:

1. The function $p(R - (1 - b)(1 + r)) - b$ has a slope smaller than or equal to kR (i.e., flatter than $U_P(p)$ in the interval $(0, \frac{2 - \bar{b}}{R})$, and it intersects $U_P(p)$ above or to the right of the point $\frac{2 - \bar{b}}{R}$ (recall that $U_P(p)$ is discontinuous at $p = \frac{2 - \bar{b}}{R}$) or lies entirely above $U_P(p)$.

2. The function $p(R - (1 - b)(1 + r)) - b$ has a slope smaller than or equal to kR and it intersects $U_P(p)$ at a point in the interval $(\frac{1}{R}, \frac{2 - \bar{b}}{R})$.

3. The function $p(R - (1 - b)(1 + r)) - b$ has a slope larger than kR.

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[21] If type $p \leq \frac{2 - \bar{b}}{R}$ satisfies moral hazard then it has to be that $r \leq r_P(p) = \frac{1}{p} - 1$. It follows that $p(1 + r) \leq 1$ and so p's expected payoff under (b, r) is decreasing in b.

Case 1. b and r can be increased in such a way that keeps $(1 - b)(1 + r)$ fixed so that the function $p\left(R - (1 - b)(1 + r)\right) - b$ shifts down in a parallel way and the intersection point moves left. This change preserves moral hazard and increases social welfare because it lowers the entrepreneurs' payoff and so makes the loan less socially costly. It follows that if the function $p\left(R - (1 - b)(1 + r)\right) - b$ intersects the function $U_P(p)$ above or to the right of the point $\frac{2 - \bar{b}}{R}$ then $b = \bar{b}$.

Case 2. Such an intersection necessarily implies that $p\left(R - (1 - b)(1 + r)\right) - b$ is flatter than $U_P(p)$ between $\frac{1}{R}$ and $\frac{2 - \bar{b}}{R}$. Its intercept with the y-axis is higher than the induced intercept of $U_P(p)$ in this interval, which implies $b \leq c_I \, (= k)$. Its intercept with the x-axis is smaller than the intercept of $U_P(p)$, which implies $\frac{b}{R - (1 - b)(1 + r)} \leq \frac{1}{R}$. The last inequality implies that $1 + r \leq R$. Increasing b makes $p\left(R - (1 - b)(1 + r)\right) - b$ steeper and moves the intersection point between $p\left(R - (1 - b)(1 + r)\right) - b$ and $U_P(p)$ to the right.[22]

As explained above, increasing b preserves moral hazard, and the fact that the intersection point between $p\left(R - (1 - b)(1 + r)\right) - b$ and $U_P(p)$ moves to the right implies that some types that may have exerted partial effort now switch to full effort. This contributes to social welfare because, if it was optimal to induce full effort from type p with contract (b, r), it is efficient to induce full effort from larger types with a smaller loan and the same interest rate. The intersection point of $p\left(R - (1 - b)(1 + r)\right) - b$ with the x-axis also moves to the right when b is increased, which implies that some types that select (b, r) and exert partial effort now prefer to drop their projects. However, as shown below, it is anyway possible to induce such types to exert partial effort costlessly using an epsilon contract if this contributes to social welfare, so this does not decrease overall efficiency. Thus it follows that social welfare is increasing in b, so $b = \bar{b}$ in this case as well.

The fact that $b = \bar{b}$ implies that there is no need for another government contract that induces full effort except for $(\bar{b}, r)$. The contract $(\bar{b}, r)$ induces full effort from any entrepreneur who selects it, so the only possible reason to introduce another contract is to induce full effort from entrepreneurs of types $p > p^*$ who prefer to exert partial effort on their own in Case 2.

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[22]The intersection point between these two functions is $\frac{c_I - b}{(1 - b)(1 + r) - (1 - k)R}$. The derivative of this expression with respect to b is nonnegative provided that $1 + r \leq R$.

But convexity of $U_G(p)$ implies that such an additional contract has to induce a steeper payoff function than $(\bar{b}, r)$, which is impossible because $b = \bar{b}$ and so $(1 - b(p))(1 + r(p))$ attains its smallest possible value at $(\bar{b}, r)$.

Case 3. All the types $p > p^*$ exert full effort either under the contract (b, r) or under the private market. So, again there is no need for an additional contract because such a contract cannot increase overall effort, and would generate a larger payoff to the entrepreneurs who select it, at the expense of social welfare.

Finally, observe that a contract that induces full effort from some type $p \leq \frac{2-\bar{b}}{R}$ may also be accepted by types $p < p^*$ who would not be induced by it to exert full effort. Nevertheless, the government may still benefit from offering entrepreneurs another contract, which induces only partial effort because such a contract may increase participation from entrepreneurs who otherwise would drop their projects. As explained in the analysis of the problem without moral hazard, this additional contract would require an arbitrarily small self-financing, ε, and a payment of $R - \varepsilon$ upon success, which implies an interest rate of approximately $1 + r = \frac{R}{c_I}$. Such a contract would extract approximately the entire rent of entrepreneurs who would accept it and exert partial effort (approximate rather than exact because the contract has to provide some positive rent to induce participation from those types that generate positive expected social welfare, but not lower types). ∎

Proof of Proposition 5. Proposition 4 implies that it is optimal to offer at most two contracts: one that induces only partial effort, and another that induces full effort from some entrepreneur types. As explain in the proof of that Proposition, the optimal government contract that induces only partial effort is an epsilon contract. Suppose that the optimal contract that induces full effort from some entrepreneur types is given by (b, r). The proof of Proposition 4 shows that if the expected payoff to the entrepreneur under the contract (b, r), which is $p(R - (1 - b)(1 + r)) - b$, is flatter than the expected payoff to the entrepreneur under the private market on the interval $[\frac{1}{R}, \frac{2-\bar{b}}{R}]$, so $R - (1 - b)(1 + r) \leq R(1 - k)$, then it follows that $b = \bar{b}$.

Suppose that the expected payoff to the entrepreneur under the contract (b, r) is steeper than the expected payoff to the entrepreneur under the private market on the interval $[\frac{1}{R}, \frac{2-\bar{b}}{R}]$,

i.e., $R - (1 - b)(1 + r) > R(1 - k)$. Denote the smallest type that exerts full effort under (b, r) by p^*. Because public funds are costly, maximizing expected social welfare requires that (b, r) maximize the sum $b + p(1 - b)(1 + r)$ for types $p \geq p^*$ and $b + pk(1 - b)(1 + r)$ for types $p < p^*$ (i.e., minimize government funding to induce those projects), for those entrepreneurs that choose the (b, r) contract subject to the moral hazard constraint $p^* \geq \frac{1}{R - (1 - b)(1 + r)}$. (Recall that if the moral hazard constraint is satisfied for p^*, then it is also satisfied for all $p > p^*$). This implies that b should be increased so that the moral hazard constraint is binding at p^*, so $(1 - b)(1 + r) = R - \frac{1}{p^*}$ and $b = \bar{b}$.

Notice that, if it is the case that the moral hazard constraint binding at p^* implies that $(1 - b)(1 + r) > R(1 - k)$, this means that the expected payoff under the (b, r) contract is flatter than expected payoff under the market in the relevant interval. In this case, we have already proved that $b = \bar{b}$. ∎

Proof of Proposition 6. Denote the smallest (p, R_H) type that exerts full effort under the optimal menu of contracts by (p_H^*, R_H) and the contract chosen by this type be (b, r). The argument used in the proof of Proposition 4 can be used to show that there is no need for another contract in order to induce full effort from types (p, R_H) such that $p > p_H^*$.

The contract (b, r) may also be picked by some types (p, R_L). Denote the smallest (p, R_L) type that exerts full effort under (b, r) by (p_L^*, R_L). The argument used in the proof of Proposition 4 implies that there is no need for another contract in order to induce full effort from types (p, R_L) such that $p > p_L^*$. Hence, the only possible reason for introducing another contract is in order to induce full effort from some types (p, R_L) such that $p < p_L^*$.

However, moral hazard implies that in order to do so, another contract (b', r') has to be such that $(1 - b')(1 + r') > (1 - b)(1 + r)$. This is ruled out by incentive compatibility, which implies that $(1 - b(p))(1 + r(p))$ is nonincreasing in p.

There is also no need for more than one contract to induce partial effort. The government has to decide which is best, to extract the full rent from types (p, R_L) who would exert partial effort under the first-best contract and allow types (p, R_H) to capture a positive rent, or to extract the full rent from types (p, R_H) who would then exert partial effort, and exclude types (p, R_L) who would exert partial effort under the first-best contract. These are the only two

possibilities because, under the optimal contract, types (p, R_L) should have a rent that is approximately zero, either through an epsilon contract, or through exclusion, and if they are excluded, then the government may as well extract the entire rent from types (p, R_H) who would exert partial effort under the first-best contract. ∎

On-line Appendices: Not for Publication

B Simulation procedure

The first step to compute welfare is to derive individual utility (profits) for entrepreneurs from the various alternatives: private market funding, epsilon contract or (b, r) contract. The choices made by the entrepreneurs are based on comparing these utilities and choosing the action that gives the highest profit.

B.1 Utilities from contracts

1. Private market

The utility of entrepreneur p when funded by the private market is

$$
U_P(p) = \begin{cases} 0 & \text{if } p \in \left(0, \frac{1}{R}\right] & \text{no implementation} \\ (pR-1)k & \text{if } p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right] & \text{partial} \\ pR-1 & \text{if } p \in \left(\frac{2-\bar{b}}{R}, 1\right] & \text{full} \end{cases} \tag{10}
$$

Note that $U_P(p) \geq 0$.

2. Epsilon contract

The epsilon contract with moral hazard is $(b_\varepsilon, r_\varepsilon) = \left(\frac{\varepsilon k^2 (1+\lambda)}{\sigma + k\varepsilon(1+\lambda)}, \frac{R}{k} - 1 - \varepsilon \right)$. Recall that the loan given by the government is $k - b_\varepsilon$. The ε contract is designed in such a way that it screens out projects having negative expected welfare under partial effort. Specifically, projects with

$$
p < \frac{1}{R + \frac{\sigma}{1+\lambda}}
$$

will not take the contract and will not implement the project.

By design, the utility to the entrepreneur of taking this contract is close to zero (its tends to zero as $\varepsilon \downarrow 0$) when the entrepreneur exerts partial effort. The entrepreneur will not exert full effort under the ε contract because she may not have sufficient funds to do so and, even if she does, her expected profits tend to $-(1-k) < 0$ as $\varepsilon \downarrow 0$.

Thus, the utility of entrepreneur p from taking the ε contract is

$$
\begin{aligned}
U_\varepsilon(p) &= kp\left[R - (k - b_\varepsilon)(1 + r_\varepsilon)\right] - b_\varepsilon \tag{11} \\
&= \frac{\varepsilon}{\sigma + k\varepsilon(1+\lambda)} k^2 \left[pR(1+\lambda) + p\sigma - (1+\lambda)\right]
\end{aligned}
$$

with partial implementation. Note that

$$U_\varepsilon(p) \gtreqless 0 \text{ as } p \gtreqless \frac{1}{R + \frac{\sigma}{1+\lambda}}.$$

3. (b,r) + epsilon contract

The utility from taking the government loan (b, r) and partially or fully implementing the project is, respectively,

$$U_{br}^{Par}(p, b, r) = kp\left[R - (1-b)(1+r)\right] - (k - (1-b)) \tag{12}$$

$$U_{br}^{F}(p, b, r) = p\left[R - (1-b)(1+r)\right] - b \tag{13}$$

The utility from taking the ε contract is given by (11).

B.2 Computation of welfare and the optimal policy

Let θ denote the vector of parameters. Fix (θ, b, r) and draw $N = 500$ observations on p. For each p, compute $U_P(p)$, $U_{br}^{Par}(p, b, r)$ and $U_{br}^{F}(p, b, r)$. We compute welfare under three different scenarios. Scenario 1 is when the is no government intervention. Scenario 2 occurs when only the epsilon contract is offered, and scenario 3 occurs when only the $(b, r) + \varepsilon$ contract is offered.

1. *Welfare from private market funding without government intervention*

$$W_{P\_only}(p; \theta) = \begin{cases} k\left[p(R + \sigma) - 1\right] & p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right] \\ \left[p(R + \sigma) - 1\right] & p \in \left(\frac{2-\bar{b}}{R}, 1\right] \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

In these cases, the entrepreneur will implement the project partially if $p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right]$, and fully if $p \in \left(\frac{2-\bar{b}}{R}, 1\right]$.

2. *Welfare when only the ε contract is offered*

(a) Private market funding with ε contract: If $U_P(p) > U_\varepsilon(p)$ then the contribution of p to welfare is

$$\tilde{W}_{P,\varepsilon}(p;\theta) = \begin{cases} k\left[p(R+\sigma) - 1\right] & p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right] \\ \left[p(R+\sigma) - 1\right] & p \in \left(\frac{2-\bar{b}}{R}, 1\right] \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

In these cases, the entrepreneur will implement the project partially if $p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right]$, and fully if $p \in \left(\frac{2-\bar{b}}{R}, 1\right]$.

(b) Government funding with ε contract: If $U_P(p) \le U_\varepsilon(p)$ and $U_\varepsilon(p) \ge 0$ these projects take the ε contract and generate

$$\tilde{W}_{\varepsilon,\varepsilon}(p;\theta) = kp(R+\sigma) - k - \lambda(k - b_\varepsilon)\left(1 - pk(1+r_\varepsilon)\right)$$

(c) Total welfare from ε contract is

$$W_\varepsilon(p;\theta) = \tilde{W}_{P,\varepsilon}(p;\theta) + \tilde{W}_{\varepsilon,\varepsilon}(p;\theta)$$

3. *Welfare when the $(b,r)+\varepsilon$ contract is offered*

(a) Private market funding with $(b,r)+\varepsilon$ contract: If

$$U_P(p) > Max\left\{U_{br}^{Par}(p,b,r), U_{br}^{F}(p,b,r), U_\varepsilon(p)\right\}$$

then the contribution of p to welfare is

$$\tilde{W}_{P,br+\varepsilon}(p;\theta) = \begin{cases} k\left[p(R+\sigma) - 1\right] & p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right] \\ \left[p(R+\sigma) - 1\right] & p \in \left(\frac{2-\bar{b}}{R}, 1\right] \\ 0 & \text{otherwise} \end{cases}$$

In these cases, the entrepreneur will implement the project partially if $p \in \left(\frac{1}{R}, \frac{2-\bar{b}}{R}\right]$, and fully if $p \in \left(\frac{2-\bar{b}}{R}, 1\right]$.

(b) Government funding with $(b,r)+\varepsilon$ contract: If

$$U_P(p) < Max\left\{U_{br}^{Par}(p,b,r), U_{br}^{F}(p,b,r), U_\varepsilon(p)\right\}$$

then we have the following cases:

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If $U_\varepsilon(p) > Max\left\{U_{br}^{Par}(p,b,r), U_{br}^F(p,b,r)\right\}$ these projects take the ε contract part of the $(b,r) + \varepsilon$ contract when it is the only contract offered and generate

$$\tilde{W}_{\varepsilon,br+\varepsilon}(p,b,r;\theta) = kp(R+\sigma) - k - \lambda(k-b_\varepsilon)\left(1 - kp(1+r_\varepsilon)\right)$$

If $U_{br}^{Par}(p,b,r) > Max\left\{U_{br}^F(p,b,r), U_\varepsilon(p)\right\}$ these projects take the (b,r) part of the $(b,r) + \varepsilon$ contract when it is the only contract offered and generate

$$\tilde{W}_{Par,br+\varepsilon}(p,b,r;\theta) = kp(R+\sigma) - k - \lambda(1-b)\left(1 - kp(1+r)\right)$$

(c) If $U_{br}^F(p,b,r) > Max\left\{U_{br}^{Par}(p,b,r), U_\varepsilon(p)\right\}$ these projects take the (b,r) part of the $(b,r) + \varepsilon$ contract when it is the only contract offered and generate

$$\tilde{W}_{F,br+\varepsilon}(p,b,r;\theta) = p(R+\sigma) - 1 - \lambda(1-b)\left(1 - p(1+r)\right)$$

(d) Total welfare from $(b,r) + \varepsilon$ contract is

$$W_{br+\varepsilon}(p,b,r;\theta) = \tilde{W}_{P,br+\varepsilon}(p;\theta) + \tilde{W}_{\varepsilon,br+\varepsilon}(p,b,r;\theta) + \tilde{W}_{Par,br+\varepsilon}(p,b,r;\theta) + \tilde{W}_{F,br+\varepsilon}(p,b,r;\theta)$$

No implementation. This occurs for projects with $Max\left\{U_{Par,br}(p,b,r), U_{F,br}(p,b,r), U_\varepsilon(p)\right\} < 0$ and $U_P(p) = 0$. Their contribution to welfare is zero.

Averaging. Having computed welfare in each of the three scenarios for each p and parameters, we average over the N values of p to get expected welfare:

$$
\begin{aligned}
W_{P\_only}(\theta) &= \frac{1}{N}\sum_{i=1}^{N} W_{P\_only}(p_i;\theta) \\
W_\varepsilon(\theta) &= \frac{1}{N}\sum_{i=1}^{N} W_\varepsilon(p_i;\theta) \\
W_{br+\varepsilon}(b,r;\theta) &= \frac{1}{N}\sum_{i=1}^{N} W_{br+\varepsilon}(p,b,r;\theta)
\end{aligned}
$$

We now keep only the average values so we have just one observation per value of (θ, b, r).

The optimal values of b and r are obtained by selecting the b and r achieving the highest welfare

$$(b_{opt}(\theta), r_{opt}(\theta)) = \arg\underset{\{b,r\}}{Max} W_{br+\varepsilon}(b,r;\theta)$$

giving welfare

$$W_{br+\varepsilon}(\theta) = W_{br+\varepsilon}(b_{opt}(\theta), r_{opt}(\theta);\theta)$$

Finally, we compare the welfare from the single and joint contracts to select the optimal contract,

$$W(\theta) = Max\left\{W_\varepsilon(\theta), W_{br+\varepsilon}(\theta)\right\} \tag{16}$$

from which the optimal contract choice is also derived for each vector of parameters, θ.

C Additional Results

C.1 Calibration

Figure C.1 plots the estimated density from a random sample of 500 observation from the $beta(2.93, 7.53)$ distribution. We use equations (8) and (9) in the text to solve for σ_i and its

Figure C.1: Density of p from Beta(2.93, 7.53) Distribution

Figure C.2: Empirical Distribution of Sigma
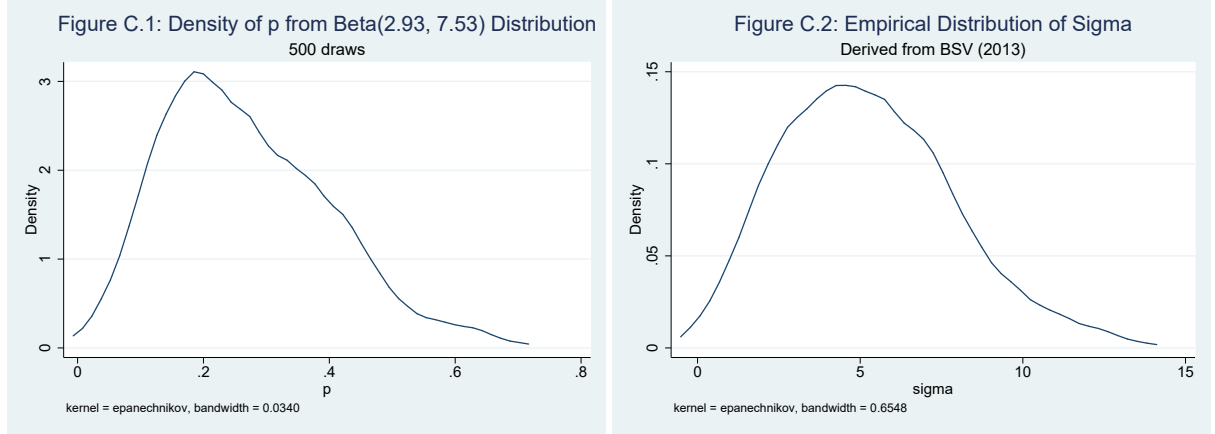
estimated density function is plotted in Figure C.2.

The table below summarizes the parameter values used to calibrate the simulations of the model.

Calibration Values	
Parameter	Values
R	5.7
σ	$1, 2.5, 5.0, 7.5, 10$
λ	$0.25, 0.5, 0.75, 1.0, 1.5$
$c_I = k$	0.21
L	2.5
$\bar{b}$	0.25
p	500 draws $Beta(2.93, 7.53)$

C.2 Simulations: robustness results

In the tables below, we present additional simulation results using alternative values of $\bar{b}$, k and the parameters of the Beta distribution of p.

Table C.1 replicates Table 1 for $\bar{b} = 0.5$ and $\bar{b} = 0.75$.

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Table C1. Robustness of welfare simulation to values of bbar

sigma	lambda	bbar=0.5			bbar=0.75		
		Optimal Policy	Optimal r (%)	Welfare Gain (%)	Optimal Policy	Optimal r (%)	Welfare Gain (%)
(1)	(2)	(3)	(5)	(6)	(7)	(9)	(10)
1	0.25	Epsilon	.	0.2	Epsilon	.	0.2
2.5	0.25	(b,r)	69	5.1	(b,r)	139	1.1
2.5	0.5	Epsilon	.	0.6	Epsilon	.	0.6
5	0.25	(b,r)	-73	10.9	(b,r)	-50	4.3
5	0.5	(b,r)	69	5.1	(b,r)	159	1.9
5	0.75	Epsilon	.	1.2	Epsilon	.	1.1
7.5	0.25	(b,r)	-73	14.6	(b,r)	-50	6.3
7.5	0.5	(b,r)	69	9.0	(b,r)	-50	3.8
7.5	0.75	(b,r)	69	5.2	(b,r)	159	2.4
7.5	1	Epsilon	.	1.6	Epsilon	.	1.5
10	0.25	(b,r)	-73	16.9	(b,r)	-50	7.4
10	0.5	(b,r)	-73	12.2	(b,r)	-50	5.4
10	0.75	(b,r)	69	8.3	(b,r)	139	3.7
10	1	(b,r)	69	5.2	(b,r)	159	2.6
10	1.25	(b,r)	69	2.1	Epsilon	.	1.7
10	1.5	Epsilon	.	1.8	Epsilon	.	1.6

Notes: Welfare gains are relative to no government intervention. Additional parameters used in the simulations are R=5.7, k=.21, bbar=.25.

Table C.2 replicates Table 1 for $k = 0.15$.

Table C.2 Robustness of welfare simulation to values of k

				Welfare Gain
			Optimal r (%)	(%)
s	lambda	Optimal Policy		
1	0.25	(b,r)	13	2.0
1	0.5	Epsilon	.	0.1
2.5	0.25	(b,r)	13	12.1
2.5	0.5	Epsilon	.	0.5
5	0.25	(b,r)	-82	21.9
5	0.5	(b,r)	13	10.1
5	0.75	Epsilon	.	1.0
7.5	0.25	(b,r)	-82	27.6
7.5	0.5	(b,r)	-82	17.2
7.5	0.75	(b,r)	13	9.1
7.5	1	Epsilon	.	1.3
10	0.25	(b,r)	-82	31.2
10	0.5	(b,r)	-82	22.8
10	0.75	(b,r)	13	15.0
10	1	(b,r)	13	8.4
10	1.25	(b,r)	13	1.9
10	1.5	Epsilon	.	1.5

Notes: Welfare gains are relative to no government intervention. Additional parameters used in the simulations are R=5.7, bbar=.25.

The baseline parameters of the Beta distribution assume that there is no within-technology field variation in p. For robustness we assume instead that the within-field variance is a proportion μ of the between-variance, and redo the analysis using three alternative values: $\mu = 0.5, 1, 2$. Table C.3 present these results.

Table C3. Robustness of welfare simulation to Beta distribution parameters

sigma	lambda	Beta (1.86,4.78) -- Within is half between variance			Beta (1.32,3.40) -- Within equals between variance			Beta (0.79,2.03) -- Within is twice between variance		
		Optimal Policy	Optimal r (%)	Welfare Gain (%)	Optimal Policy	Optimal r (%)	Welfare Gain (%)	Optimal Policy	Optimal r (%)	Welfare Gain (%)
(1)	(2)	(3)	(5)	(6)	(7)	(9)	(10)	(11)	(13)	(14)
1	0.25	Epsilon	.	0.2	Epsilon	.	0.1	Epsilon	.	0.0
2.5	0.25	(b,r)	64	5.9	Epsilon	.	0.4	Epsilon	.	0.2
2.5	0.5	Epsilon	.	0.4	Epsilon	.	0.4	Epsilon	.	0.1
5	0.25	(b,r)	-98	14.4	(b,r)	-70	7.2	(b,r)	25	3.9
5	0.5	(b,r)	64	4.7	Epsilon	.	0.9	Epsilon	.	0.4
5	0.75	Epsilon	.	0.8	Epsilon	.	0.8	Epsilon	.	0.4
7.5	0.25	(b,r)	-98	19.5	(b,r)	-77	11.9	(b,r)	-83	7.9
7.5	0.5	(b,r)	-98	10.3	(b,r)	49	4.0	(b,r)	75	1.8
7.5	0.75	(b,r)	64	4.0	Epsilon	.	1.2	Epsilon	.	0.6
7.5	1	Epsilon	.	1.1	Epsilon	.	1.1	Epsilon	.	0.5
10	0.25	(b,r)	-98	22.7	(b,r)	-77	14.8	(b,r)	-83	10.7
10	0.5	(b,r)	-98	15.2	(b,r)	-70	7.9	(b,r)	63	4.7
10	0.75	(b,r)	64	8.2	(b,r)	49	2.5	Epsilon	.	0.8
10	1	(b,r)	64	3.6	Epsilon	.	1.4	Epsilon	.	0.7
10	1.25	Epsilon	.	1.3	Epsilon	.	1.3	Epsilon	.	0.7

Notes: Welfare gains are relative to no government intervention. Additional parameters used in the simulations are R=5.7, k=.21, bbar=.25.

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