Is the Market Pronatalist? Inequality, Differential Fertility, and Growth Revisited*

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Abstract

A negative relationship between income and fertility has persisted for so long that its existence is often taken for granted in the literature. One economic theory builds on this relationship and argues that rising inequality leads to greater differential fertility – the fertility gap between rich and poor. We show that the relationship between income and fertility has flattened between 1980 and 2010 in the US, a time of increasing inequality, as high income families increased their fertility. These facts challenge the standard theory. We propose that marketization of parental time costs can explain the changing relationship between income and fertility. We show this result both theoretically and quantitatively, after disciplining the model on US data. We explore implications of changing differential fertility for aggregate human capital. Additionally, policies, such as the minimum wage, that affect the cost of marketization, have a negative effect on the fertility and labor supply of *high* income women. We apply the insights of this theory to the literatures of the economics of childlessness and marital sorting.

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1 Introduction

A negative relationship between income and fertility has persisted for so long that its existence is often taken for granted in the literature (Jones & Tertilt 2008). This relationship has been typically explained by either the tradeoff between the quantity and quality of children, the opportunity cost of parental time, or both. Some of the many examples include Becker & Lewis (1973), Galor & Weil (1996), Galor & Weil (2000), and Doepke (2004). These mechanisms have led researchers to conclude that rising inequality would lead to more differential fertility, i.e. a greater gap in fertility between the poor and rich households (de la Croix & Doepke 2003, Moav 2005).[1] Hereafter, we refer to this as the standard theory.

However, even as recent decades have seen a dramatic rise in income inequality in the US (Autor, Katz & Kearney 2008, Heathcote, Perri & Violante 2010), the relationship between income (or education) and fertility has flattened as high income families increased their fertility, and even become U-shaped (Hazan & Zoabi 2015), challenging the standard theory. We argue that the ability to outsource (marketize) parents' time costs by purchasing babysitters, housekeepers, and prepared food, lessens children's opportunity cost of parental time. As inequality grows, the cost of marketization for the rich shrinks relative to their income, allowing them to have more kids without sacrificing time and careers.

In this paper, we show that changes in inequality, along with a decline in the price of market substitutes for parents' time with children, can quantitatively account for much of the changing relationship between income and fertility over time. We explore quantitatively and empirically the implications of our findings for aggregate human capital accumulation and policy (minimum wage).

Our point of departure is the standard model of fertility and educational investment in children, as in Galor & Weil (2000), applied to the case of inequality as in de la Croix & Doepke (2003) and Moav (2005). This model features both a quantity-quality tradeoff with respect to children as well as an opportunity cost

[1] Galor & Moav (2002) argue that the opposite is true before the demographic transition. Relatedly, Vogl (2016) indeed finds that the income-fertility gradient was positive in less developed countries before they experienced the demographic transition.

of parental time in childcare. We analyze this model under the assumption that the cost of children can be marketized. We show that this one assumption is crucial for understanding the effects of inequality on differential fertility.

Turning towards our quantitative analysis, we calibrate the model to the US in 1980, when fertility and income had a negative relationship. We discipline the model by matching the salient features of cross-sectional US data. Namely, we match the income profiles of fertility rates, mother's labor supply, marketization expenditures, and college attainment rates.[2] The model successfully fits the empirical targets with 8 parameters chosen to match 40 moments.

We then feed into the calibrated model the observed cross-sectional wages for 2010 and a price decline of home production substitutes. The model predicts the 2010 relationships between income and fertility and between income and mother's time at home. In the model (data), the fertility of the top two deciles increases by 43.5% (40%) between 1980 and 2010. Our measure of differential fertility, which compares the average fertility of the top two deciles to that of the second decile, increases by 41% (38.5%). An alternative measure of differential fertility, comparing the fertility of the top half and bottom half of the income distribution, increases by 24.4% (18.6%). All of these results are untargeted.

Decomposing the mechanisms at work, we find that it is the change in the price of market substitutes relative to parental income, rather than the general income effect, that is the main driver behind our findings. Furthermore, this result depends critically on increasing inequality in parental wages. Our results imply that a naïve modeler, working in 1980 under the view of the standard literature, which ignores marketization, would have predicted a significant decline in high income fertility over time if she had been given perfect foresight over actual income distributions.

One implication of our theory is that rising inequality increases aggregate human capital, and thus growth. This is due to the fact that the rich tend to provide more human capital to their children, as represented by college graduation rates. Thus,

[2]We measure marketization expenditures as the relative expenditures on childcare, as calculated from the Survey of Income and Program Participation (SIPP). See Appendix A for details.

as inequality increases, the average human capital of the next generation grows as *relatively* more kids are born to richer families.

Turning towards policy implications, according to our theory, anything affecting the price of marketization should have an indirect effect on the labor supply and fertility, especially of high income women. One prevalent policy that may affect the price of marketization is the minimum wage. We show empirically that the minimum wage level indeed has a large pass-through effect on wages in the home production substitute (HPS) sector.[3] Evaluating this effect in the context of the calibrated model allows us to quantify the impact of minimum wage laws on fertility and labor supply of high income women. This analysis is presented in Section 5.

Accordingly, we show that a disproportionately large number of workers in the HPS sector receive the minimum wage. Using cross state time series variation in the minimum wage from 1980 to 2010, we show that the minimum wage has a statistically significant and economically meaningful effect of about 58 cents increase in HPS sector wages for every dollar increase of the state minimum wage. We take an instrumental variables approach, as in Baskaya & Rubinstein (2012), as OLS may be biased as states tend to raise the minimum wage during good economic times.

We employ this estimated effect to perform a policy experiment, using the model to measure the effects of raising the minimum wage to $15/hour, as per Bernie Sanders, on the labor supply and fertility of high income women.[4] These women reduce their labor supply and fertility as marketizing becomes more difficult.[5] We confirm the model prediction with respect to labor supply by estimating the empirical elasticity of the labor supply of high income women with respect to the minimum wage. We do so using cross state time series variation in the minimum wage from 1980 to 2010 and the instrumental variable approach discussed

[3] We define these sectors as in Mazzolari & Ragusa (2013).

[4] See http://berniesanders.com/issues/a-living-wage

[5] Doepke & Kindermann (2016) argue that policies that lower the childcare burden on mothers are significantly more effective at increasing fertility as compared to general child subsidies. We argue that the minimum wage is a policy that *increases* the childcare burden on mothers, and hence decreases fertility.

above. The empirical elasticity is also negative, but quantitatively larger in absolute value than that of the model.

We conclude by discussing that explicit modeling of outsourcing of home production can also help us understand additional aspects of household behavior, focusing on two important phenomena: trends in childlessness rates among highly educated women (Baudin, de la Croix & Gobbi 2015) and marital sorting (Greenwood, Guner & Vandenbroucke 2017).

Hazan & Zoabi (2015) was the first paper to document the flattening of the fertility profile by mother's education, due to rising fertility rates among highly educated women. They qualitatively study a similar model to the one presented here to show theoretically the role of marketization. Furthermore, they exploit cross-state variation in wages and find that the wages of childcare workers, relative to mothers' wages, are negatively correlated with the propensity to have an extra child. This reduced-form evidence supports the quantitative analysis done in this paper. However, we differ in several critical ways. First, we document the flattening of the fertility-*income* profile. Second, we quantitatively evaluate the role of rising wage inequality and decreasing prices of home production substitutes, through the mechanism of marketization, in explaining this pattern. Finally, we examine theoretically and quantitatively the implications of inequality and marketization for human capital accumulation and minimum wage policy.

This paper is related to a large literature on motherhood and labor supply. Attanasio, Low & Sanchez-Marcos (2008) builds a life cycle model of fertility and labor force participation. They argue that reductions in child care costs can quantitatively account for the increase in labor supply of young mothers. Furtado (2016) finds that an increase in unskilled migration lowers wages in the child care services sector, and increases both fertility and labor supply.[6] Interestingly, she finds that native women with a graduate degree increase their labor supply and fertility much more than native women with just a college degree. Similarly, Cortés & Tessada (2011) exploit cross-city variation in immigration concentration, and show that an increase in low-skilled immigration increases labor supply, of

[6]Notice that this tackles inequality from another direction. Rather than focus on a rise in inequality due to rising wages among high income households, she is studying an increase in the supply of low wage workers. Our mechanism is agnostic as to the source of rising inequality.

women in the top quartile of the wage distribution.[7] These women reduce time spent on housework and purchase more services as substitutes. Interestingly, Cortés & Pan (Forthcoming) show that increased marketization of household work allows women both to enter occupations that demand high levels of effort, and lowers the earnings gap in those occupations. While the importance of marketization of home production has been widely recognized (e.g. Greenwood, Seshadri & Vandenbroucke 2005, Greenwood, Seshadri & Yorukoglu 2005), the consequences of rising inequality on differential fertility in the presence of the possibility to outsource home production have not been widely studied.[8]

We continue as follows. Section 2 presents our motivating evidence. Section 3 describes the theoretical framework of our analysis. Section 4 provides details on the parameterization of the model, along with quantitative results. Section 5 analyzes the effects of the minimum wage on labor supply and fertility through the lens of the calibrated model. Section 6 discusses implications of marketization on the literatures on childlessness and marital sorting. We conclude in Section 7.

[7] Using data from Hong Kong, Cortés & Pan (2013) show that the ability to hire foreign workers as live-in help increases labor force participation of mothers. They argue that child care cost reduction through immigration is a market alternative to child care subsidies.

[8] The literature on women's labor force participation is too vast to summarize here. However, a few papers showing how women's labor supply is related to structural transformation and taxes are worth noting, as they illuminate further potential effects of marketization on the economy. Akbulut (2011) argues that work at home, in which women have a comparative advantage, and work in services are quite similar. Thus, when demand for services rises, women's labor force participation rises as well. Buera, Kaboski & Zhao (2017) develop this argument further: they use a quantitative model of sectoral reallocation and specialization between men and women to evaluate various causes of structural transformation. Cerina, Moro & Rendall (2018) argue that the rise of high skill women entering the labor force, due to the increased skill premium, contributed to job polarization. When these women enter the labor force the high skill employment shares increase. As a side effect of their employment, these women also must now marketize their home production, leading to an increase in low skill employment in the HPS sector. Rendall (2018) argues that women's labor force participation and the service sector are strongly affected by taxes. Kaygusuz (2010) and Bar & Leukhina (2009) study the effects of changes in married couples' taxation on the rise of married female labor force participation in the US, while Guner, Kaygusuz & Ventura (2012) argue that participation would be even higher if America moved to a system of individual based taxation of married households. Duernecker & Herrendorf (2017) argue that labor productivity in home production in the US has stagnated in recent decades, while it has risen in other places such as Germany. Their result is based on the fact that wages of household workers, what we call HPS workers, have stagnated in the US but risen in Germany. They explain the US stagnation with the prevalence of cheap immigrant labor, which has become more widely used by richer Americans in home production.

2 Motivating Evidence

In this section, we describe our motivating evidence. We first show data on cross-sectional fertility changes and inequality. We then use cross-state variation in the relative wage of high income women to HPS sector workers, and show that states that had a larger increase in this ratio saw a larger increase in high income fertility.

Figure 1 shows fertility rates in the US in 1980 and 2010 for all native-born women, separated into five education groups: less than a high school degree (<12 years), a high school degree (12 years), some college (13-15 years), a college degree (16 years), and an advanced degree (>16 years).[9] We measure fertility of a given education group using "hybrid fertility rates" (HFR) (Shang & Weinberg 2013), which augments the total fertility rate (TFR) for women over 25 with children ever born (CEB) at age 24.[10] Fertility rates in 1980 are strongly negatively correlated with education, as has often been noted by the literature. However, in 2010, fertility rates are much flatter, and even rising between the "college degree" and "advanced degree" groups.

In this paper, we are concerned with the impact of inequality and marketization on the relationship between *income* and fertility. As such, we measure inequality by 10 income deciles, rather than 5 education groups. Furthermore, we restrict attention to white, non-Hispanic Americans in order to abstract from changes in demographics over time. Additionally, we focus on married couples for two reasons. First, this allows us to abstract from differences between the fertility considerations of different types of households, and second it allows us to more easily calculate income deciles, without having to compare between single households and (potentially) dual income households. Couples are allocated into income deciles according to their income rank among couples of the same age. We measure decile-specific moments (e.g. male wages by decile) by deriving the

[9] Hazan & Zoabi (2015) show a very similar pattern when restricting the data to white non-Hispanic women.

[10] Formally, $HFR_t = n_{24,t} + \sum_{a=25}^{55} AFR_{at}$, where $n_{24,t}$ is the average number of children ever born at age 24 in year t and $AFR_{at}$is the age-specific fertility rate for women of age a in year t. We estimate HFR separately for each education group.

age-specific averages for the given decile, and then averaging across ages. These moments capture the experience of a hypothetical couple that goes through life maintaining its decile ranking among other couples of their cohort. Figure 2 reports fertility rates by income decile for our sample of white non-Hispanic married couples. In 1980, there was a clear negative relationship between income and fertility. Fertility rates in 2010 were little changed for the bottom half of the income distribution. However, fertility patterns have changed starting at the 5th decile, representing a flat, or even a somewhat U-shaped relationship between income and fertility. The difference between 1980 and 2010 is most pronounced for the top deciles. The increase in fertility among the most educated women (Figure 1) closely corresponds to the increase in fertility among couples from the higher deciles (Figure 2). In particular, 9th (10th) decile women saw an increase in fertility of 0.64 (0.83) children, while the highest education group saw an increase of 0.51 children.

This change in fertility occurred at a time of rising inequality. This is seen in Figures 3 and 4, which show wages for wives and husbands, respectively, for each decile in each year in real 2010 dollars.

The theory proposed in this paper suggests that women should increase their fertility when their wages relative to the price of home production substitutes increase. Empirically, this pattern can be seen in the US cross state time series. Figure 5 shows that states that have seen a greater percent change in the relative wage of high income women (9th and 10th decile of family income as defined above) to workers in the home production substitute sector, between 1980 and 2010, have also seen a greater percent increase in fertility of high income women. This supports the notion that, where market substitutes have become relatively cheaper (as measured by the change in relative wages of workers in the sector), high income women have increased their fertility by more. In Appendix B we show the robustness of this relationship to controlling for changes in male wages, differential regional trends, and dropping outlying observations.[11]

Additional data sources corroborate this story and paint a more thorough picture

[11] Additionally, when using the sample of all native-born American women, as in Figure 1, and replacing high income women with women with advanced degrees, the regression coefficient is practically the same as in Figure 5.

of changing time allocation patterns. Female labor supply and childcare expenditures rise with family income decile. All deciles saw an increase in female labor supply, especially the top ones. These patterns will be discussed in Section 4 in the context of comparing model and data. Finally, data from Time Use Surveys show that females in the top three income deciles reduced their home production activities by over 16 hours per week. In contrast, couples in the bottom three deciles reduced home production time by only 7 hours per week. Our model will not distinguish between various types of home production activities. We take the stance that all those activities are needed to run a household and raise children.[12]

3 Model

There is a unit measure of households composed of married females (f) and males (m) that are heterogenous on the wage offers that the members receive, denoted w_f and w_m, respectively. The household derives utility from consumption c, number of children n, and their quality w_k (income per child). This approach is as in Galor & Weil (2000) and Moav (2005). The income per child is uncertain, and given by

$$w_k = \begin{cases} \omega \cdot w_{nc} & \text{w.p. } \pi(e) \\ w_{nc} & \text{w.p. } 1 - \pi(e), \end{cases} \tag{1}$$

where w_{nc} is the income for non-college graduates, $\omega > 1$ is the college premium, and $\pi(e)$ is the probability of receiving a college degree as a function of their education good. The utility function, given the realization of the children's income, is assumed to be:

$$u = \ln(c) + \alpha \ln(n) + \tilde{\beta} \ln(w_k). \tag{2}$$

We assume that all siblings in a family have the same realization of education un-

[12]Time directly devoted to children actually remained roughly unchanged for the top income families, partly because their fertility increased and partly due to the more inclusive definition of time spent in direct child care activities applied in later survey years.

certainty. Thus, parents in this model maximize the following expected utility:[13]

$$\mathbb{E}\left[u\right] = \ln\left(c\right) + \alpha \ln\left(n\right) + \tilde{\beta} \ln\left(w_{nc}\right) + \tilde{\beta} \ln\left(\omega\right) \pi\left(e\right). \tag{3}$$

Notice that the non-college income appears in the utility as a constant, and does not affect the household's decisions. Hence, we drop this constant in the analysis below.

We assume that π takes the form of:

$$\pi(e) = \ln\left(b(e+\eta)^{\theta}\right). \tag{4}$$

We choose this functional form for the probability of a child graduating college as it generates a negative relationship between fertility and income through a quantity-quality tradeoff.[14] Notice that plugging (4) into (3) and dropping the constant term, $\tilde{\beta} \ln\left(w_{nc}\right)$, yields:

$$\mathbb{E}[u] = \ln\left(c\right) + \alpha \ln\left(n\right) + \beta \ln\left(b(e+\eta)^{\theta}\right), \tag{5}$$

where $\beta = \tilde{\beta} \ln(\omega)$, which is similar to the objective function used in de la Croix & Doepke (2003) and Moav (2005). We continue our analysis on the basis of (5).

Parents are required to spend the same amount of resources on the quality of each child. Thus, the budget constraint is given by:

$$c + p_n n + p_e e n = w_f + w_m, \tag{6}$$

where p_n, defined below, captures both the time and market goods costs associated with raising a child, regardless of quality. p_e is the exogenously given price of a unit of education (quality).

[13] An alternative formulation would allow for the uncertainty over college to be resolved child-by-child. The advantage to our approach is that it allows for a closed-form solution to the model.

[14] Notice that this function is not bounded between 0 and 1. However, this is not an issue in our calibration, as for any range of e chosen, it is possible to pick parameters such that $\pi(e) \in [0,1]$. Jones, Schoonbroodt & Tertilt (2010) discuss conditions necessary on the π function such that it would yield a negative relationship between income and fertility, specifically that the elasticity of the human capital production function with respect to e is increasing. Our functional form both meets this criteria and yields a closed form solution.

We assume a technology for child rearing that includes marketization. Accordingly, we assume that kids require family resources combining mother's time, t_f, with market substitutes for home production, m, according to:

$$n = A\left(\phi t_f^{\rho} + (1-\phi)\, m^{\rho}\right)^{\frac{1}{\rho}}, \tag{7}$$

where $0 < \phi < 1$ controls the relative importance of mothers' time in the production of children, $\rho \leqslant 1$ controls the elasticity of substitution between the mother's time and home production substitutes, and A determines the total factor productivity (TFP) of child production. This production function explicitly takes into account the ability to marketize parental time in child rearing.

Given a level of fertility, n, let $TC(n)$ be the total cost of n children, independently of their education. $TC(n)$ is then the solution to the cost minimization problem given by:

$$TC(n) \quad = \quad \min_{t_f, m}\{t_f \cdot w_f + m \cdot p_m\} \tag{8}$$

such that (7) holds, where p_m is the price of the market substitutes.

The results, in terms of conditional factor demand and total cost function, are given by:

$$t_f = \frac{\left(\phi/w_f\right)^{\frac{1}{1-\rho}}}{A\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} n, \tag{9}$$

$$m = \frac{\left(\frac{1-\phi}{p_m}\right)^{\frac{1}{1-\rho}}}{A\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} n, \tag{10}$$

$$TC\left(n, w_f, p_m\right) = \frac{1}{A}\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} n \equiv p_n n. \tag{11}$$

Using (5) and (6) to solve for the utility maximization problem gives the following optimal solutions for e and n:[15]

$$e^* = \max \left\{ \frac{\frac{p_n}{p_e}\frac{\beta\theta}{\alpha} - \eta}{1 - \frac{\beta\theta}{\alpha}}, 0 \right\}, \tag{12}$$

$$n^* = \begin{cases} \left(1 - \frac{\beta\theta}{\alpha}\right)\left(\frac{\alpha}{1+\alpha}\right)\left(\frac{w_f + w_m}{p_n - \eta p_e}\right) & if\ e^* > 0 \\ \frac{\alpha}{1+\alpha}\left(\frac{w_f + w_m}{p_n}\right) & if\ e^* = 0 \end{cases} \tag{13}$$

The solution for n, t_f, and m imply that an increase in $\frac{w_f}{p_m}$ yields a decrease in $\frac{t_f}{n}$ and an increase in $\frac{m}{n}$, as families marketize the time costs of children more.[16] The ability of parents to substitute their own time with market goods and services leads to the following claim:

Claim 1 *When part of the time cost of children can be marketized, and $\rho \in (0, 1)$, rising inequality may lead to the fraction of children born to high income families to rise.*

This follows from the fact that n^* is either decreasing or U-shaped in w_f, in the interior solution region. This is shown in Appendix C.2, where we formally discuss the shape of fertility across deciles, which differ on w_f and w_m, and over time. When the dispersion of w_f rises, differential fertility could change in either direction; there could be relatively more children born to poor households, if the downward sloping section of the U shape is dominant. However, there could also be relatively more children born to rich households. Changes in fertility patterns have implications for aggregate human capital levels.

Claim 2 *When part of the time cost of children can be marketized, and $\rho \in (0, 1)$, rising inequality may lead to higher levels of average human capital in the next generation through differential fertility.*

[15] We show the existence of a unique solution to the household problem in Appendix C.1.

[16] Additionally, if $\rho > 0$, there is an increase in relative spending on market substitutes, i.e. $\frac{p_m m}{w_f t_f}$ rises.

This claim holds in the case where rising dispersion of w_f increases the fraction of children born to high income households, and thus increases the average human capital of the subsequent generation.

Much of the literature has abstracted from the assumption that some of the time cost of children can be marketized, and assumed that p_n is proportional to w_f. If one makes such an assumption, Equations (12) and (13) reveal that fertility is strictly decreasing in w_f in the interior solution. We refer to this special case as the "Standard Theory."[17]

Finally, a word must be said about two ways of modeling men and fertility. First, if men do not spend time raising children, as in our benchmark, then we say that there are "traditional gender roles". Men's wages under traditional gender roles act as any other form of wealth. A higher male wage yields more fertility, as can be seen directly in (13), through an income effect. Under this framework, it is possible that the changing fertility patterns in US data, where now high income households are likely to have relatively more children, can be explained by rising inequality among men, regardless of the ability to marketize. This is the assumption we make in our quantitative analysis below, as it allows for an alternative explanation for the emergence of the U-shape seen in the data.

Alternatively, we could assume "modern gender roles," in which men do engage in child care. Thus, p_n does depend on w_m. Clearly, this could be modeled in a large number of ways.[18] To understand the intuition of how modern gender roles interact with inequality and marketization, consider the extreme example of a Leontief function that aggregates time that husbands and wives spend in childcare into one "parental services" variable. For example, if men are required to spend one hour of time in child care for every hour that their wife spends in child care, then couples can be seen as one person with $w = w_m + w_f$ with all the same implications for the interaction between inequality and marketization. This assertion applies more generally when men and women are not perfect complements in the production of children (Siegel 2017).

[17] Notice that if p_n is proportional to w_f, and $w_m = 0$, then (13) collapses to the optimal fertility solution as in de la Croix & Doepke (2003) and Moav (2005).

[18] For an analysis on how parents bargain over the allocation of time to childcare, see Gobbi (2018).

4 Quantitative Exercise

In this section, we discuss the calibration of the model, the model fit, and breakdown of the mechanisms driving changing fertility patterns over time. We calibrate the model to 1980, and then study its implications under the 2010 wages and prices of home production substitutes. We begin by discussing the parameterization of the model and the model fit. We then test the model predictions for 2010 and break down quantitatively the various forces at work.

Throughout the quantitative exercise, we assume 10 representative couples that we map to income deciles, as described in Appendix A.

4.1 Parameterization

The model has 10 parameters, $\Omega \equiv \{\alpha, \beta, \eta, \theta, b, \phi, \rho, p_e, A, p_{m,1980}\}$. We now describe how we pick these parameter values, which are reported in Table 1.

p_e and $p_{m,1980}$ are normalized to one without loss of generality.[19] The remaining 8 parameters are picked to match model moments to data moments from 1980. In particular, we match the profile of fertility, the profile of mother's time at home, the profile of college attainment rates of children born to different income deciles in 1980 , and the index of relative expenditures on home good substitutes.[20] Each profile contains 10 moments, representing the 10 deciles, yielding 40 moments. See Appendix A for a description of the empirical moments. The model has a closed form solution which can be inverted to infer parameter values from the data. Due to the high number of moments relative to parameters, we minimize the distance between the model moments and the data moments in order to obtain the best fit.

[19]We show this formally in Appendix E.

[20]Regarding the index of marketization, we use the childcare module of the Survey of Program Participation and Income (SIPP) to estimate relative uses of market substitutes. Our index measures are based off expenditures on childcare hours purchased in the marketplace. Since this is only one aspect of marketization, we use this to target the relative use of marketization across deciles, rather than taking the absolute expenditure levels literally. The implicit assumption is that there is a strong correlation between the use of childcare and other market substitutes for parents' time. See Appendix A for more details.

Formally, we pick parameters to minimize the mean squared error of the loss function:

$$\{\alpha, \beta, \eta, \theta, b, \phi, \rho, A\} = \arg\min \sum_i \left(\frac{M_i(\Omega) - D_i}{D_i} \right)^2, \tag{14}$$

where $M_i(\Omega)$ is the value of the model moment i when evaluated at parameter values Ω. D_i is the data value of moment i.

While all of these 8 parameters are picked together, certain moments inform on them more than others. With a slight abuse of language, we describe a parameter as being picked to match a target, while it is understood that all parameters are jointly determined against the empirical moments. Table 1 shows the results of our identification strategy described below.

We begin by discussing α, β, and η which are picked to match fertility rates by decile. As can be seen in Equation (13), α plays a large role in determining the level of fertility, and can thus be thought of as being identified off the level of the fertility profile. The slope of fertility with respect to income depends on both an income effect, as kids are a normal good, and a substitution effect, as higher wages imply a higher opportunity cost of time with kids. In this model, fathers' wages, w_m, are purely an income effect, while mothers' wages contain both effects. β is important in determining the strength of the income effect. η controls the strength of the substitution effect. Thus, these three parameters are identified off the level and slope of the fertility profile with respect to both parents' wage offers.

Turning to θ and b, these parameters are closely related to education. First, however, notice that β and θ are inseparable in the utility function. However, θ affects the mapping between education expenditures, e, and college attainment, $\pi(e)$, while β does not. Thus, θ can be thought of as being identified off the slope of the profile of college attainment by decile, while β is identified off of the slope of the fertility profile, as described above. As seen in Equations (12) and (13), b does not affect the amount invested in children or quantity of children. It does, however, impact the education obtained. Therefore, it can be identified by the level of the profile of college attainment.

ϕ, ρ, and A are the parameters of the production function for kids. ϕ and ρ control the tradeoff between mother's time and home production substitutes, m, in the production of children. ϕ controls the relative importance of the mother's time in child care, while ρ controls the substitutability between mother's time and market goods. A controls how much resources are needed for childcare, in particular the amount of market goods needed. These three parameters thus determine how many resources of each type are needed and available, per child, across the income distribution. As such, they can be thought of being identified off both the level and slope of the profile of mother's time at home and the index of marketization.

4.2 Parameters and Model Fit

Table 1 shows the calibrated parameter values. Notice that the parameter values found here are consistent with much of the literature. For instance, the calibrated value of α suggests that $\frac{\alpha}{1+\alpha} = 31\%$ of household resources are dedicated towards children. Lino, Kuczynski, Rodriguez & Schap (2017) find that families with 2-3 children spend 37–57% of their expenditures on their children. Assuming that households have children at home for half of their adult life (de la Croix & Doepke 2004), our number of 31% is roughly consistent with the upper range of these estimates. While ϕ is somewhat high, this actually is conservative, as it reduces the importance of marketization in the calibration. Our value for ρ implies an elasticity of substitution between mother's time and market goods of 2.5, which is consistent with the higher estimates reported in Aguiar & Hurst (2007).

Figure 6 shows the model fit, matching 40 moments with 8 parameters. The model successfully fits empirical targets for 1980, by decile, despite its parsimonious nature. The top left panel shows the model and data for mother's time at home. The top right panel shows the model fit for fertility. The bottom left figure shows the model fit for college attainment rates of children born to families in different deciles in 1980. Finally, the bottom right shows the model fit for the index of marketization.

Overall, the model fit is excellent. Beginning with women's time at home, the match between the model and data is close to perfect. Turning towards fertility, both the model and data exhibit a strongly negative relationship between income decile and fertility rates, with the exception of the first decile.[21] The model is also able to capture the level of college attainment, by decile, almost perfectly. Finally, the index of relative marketization is well matched, showing that relative marketization rates in the model are similar to those in the data.

The average fraction of household income spent on market substitutes is 4.7%. This seems quite reasonable; expenditures on market substitutes are a relatively small fraction of total household income.

4.3 Change in p_m

We next turn towards the calculation of the change in p_m between 1980 and 2010. There is no consensus in the literature what this price change is, or even what exactly comprises m. We consider m to be composed of two types of market substitutes for home production: home production durables, d, such as dishwashers and washing machines, and time of HPS workers, t.[22] We first discuss the price change that we take of each type of input, and then discuss our choice of a 2% annual price reduction.

Greenwood, Guner, Kocharkov & Santos (2016), report a range of estimates from the literature of 2-13% annual price declines of home production durables, and in turn use 5%. We use 4% in order to be more conservative. The price of HPS workers is harder to measure. On one hand, measured wages of HPS workers in the Current Population Survey (CPS) have remained roughly constant. On the other hand, if the productivity of these workers has increased, then the price of their output should decline even if wages remain constant. Attanasio et al. (2008) argue that there was a 25% drop in the cost of childcare services for kids

[21]The imperfect fit results from a corner solution in education for the first two deciles.

[22] Notice that the cost of marketization, p_m, in our model is not directly linked to inequality in our model. One might expect it to be linked to the wages of lower decile workers in the income distribution. However, it would be hard to endogenize p_m given the current framework, as a large fraction of HPS workers are not married, Hispanic, not white, or not in our age range. As discussed below, we discipline p_m taking into account the wages of HPS workers.

under age 4, and a 5% drop for kids age 4-7 between 1980 and 2000. We take the price decline to be 10% over our time period, or 0.35% per year between 1980 and 2010. These price declines are not enough to calculate the price change of m, as we need some indication of the substitutability (or complementarity) between d and t, as well as their relative importance. However, they do indicate that a price decline of m of roughly 2% a year is reasonable; a 4% decline in durables and a 0.35% decline in HPS worker costs suggests 2% as a midpoint. We next do a more formal analysis of the interaction of durables and HPS workers in home production in order to explore changes in p_m.

We begin by assuming a functional form for the aggregation of d and t. In particular, we assume that m is aggregated using a constant elasticity of substitution (CES) production function of t and d:

$$m = (\phi_m d^{\rho_m} + (1-\phi_m) t^{\rho_m})^{\frac{1}{\rho_m}}, \tag{15}$$

where ρ_m controls the elasticity of substitution and ϕ_m the relative weight. We denote the price of durables to be p_d, and the price of HPS workers' time to be w. The price of this composite good p_m is thus given by:

$$p_m = \left(\phi^{\frac{1}{1-\rho_m}} p_d^{\frac{\rho_m}{\rho_m-1}} + (1-\phi_m)^{\frac{1}{1-\rho_m}} w^{\frac{\rho_m}{\rho_m-1}}\right)^{\frac{\rho_m-1}{\rho_m}}. \tag{16}$$

Pulling out w allows us to write p_m as a function of $\frac{p_d}{w}$:

$$p_m = w\left(\phi^{\frac{1}{1-\rho_m}} \left(\frac{p_d}{w}\right)^{\frac{\rho_m}{\rho_m-1}} + (1-\phi_m)^{\frac{1}{1-\rho_m}}\right)^{\frac{\rho_m-1}{\rho_m}}. \tag{17}$$

Minimizing costs yields expenditures on durables relative to expenditures on HPS workers, which are given by:

$$\frac{p_d d}{wt} = \left(\frac{p_d}{w}\right)^{-\frac{\rho_m}{1-\rho_m}} \left(\frac{1-\phi_m}{\phi_m}\right)^{\frac{1}{1-\rho_m}}. \tag{18}$$

In order to calculate the empirical counterpart to (18), we take the Survey of Consumer Expenditures (CEX) in 1980 and 2010. Our sample is married white households ages 25-55.[23] We find that expenditures on durables relative to HPS workers is 3.61 in 1980 and 1.45 in 2010.[24]

Dividing (18) in 2010 by (18) in 1980, yields:

$$\frac{\left(\frac{p_d d}{wt}\right)_{2010}}{\left(\frac{p_d d}{wt}\right)_{1980}} = \left(\frac{\left(\frac{p_d}{w}\right)_{2010}}{\left(\frac{p_d}{w}\right)_{1980}}\right)^{-\frac{\rho_m}{1-\rho_m}}. \tag{19}$$

Using (19), the fact that the ratio of relative expenditures in the data is $\frac{1.45}{3.60}$, and the change in relative prices of durables to HPS workers (declined by 67%), we can infer that $\rho_m = -4.33$. This implies strong complementarity between the two inputs, with an elasticity of substitution of approximately 0.2.

We are still missing two unknowns necessary to calculate the change in p_m over time: $\left(\frac{p_d}{w}\right)_{1980}$ and ϕ_m. We use the fact that (18) is equal to 3.6 in 1980 as one more equation. However this is not sufficient for identification. If we assume a low weight on durables $\phi_m = .05$, then we can solve for $\left(\frac{p_d}{w}\right)_{1980} = 2.45$ and, using (17), the implied change in p_m to be 1.94% per year between 1980 and 2010. If instead we assume a high weight on durables, $\phi_m = 0.95$, we can solve for $\frac{p_d}{w}_{1980} = 9.55$, and the implied annual change in p_m is 3.54%. We take as a benchmark $\phi_m = .059$, which implies a 2% annual price reduction between 1980 and 2010 and $\left(\frac{p_d}{w}\right)_{1980} = 2.55$. We refer to this exercise as our benchmark case. We perform robustness tests increasing and decreasing ϕ_m by 33%. We refer to these robustness exercises as "high ϕ_m" and "low ϕ_m," respectively.

[23] There is well known bias in CEX data, such that comparing the CEX and the National Income and Product Accounts (NIPA) over time shows substantial divergences. Attanasio, Hurst & Pistaferri (2012) surveys some of the literature on this subject. As a result, we only use CEX to examine *relative* expenditures on different types of goods, rather than absolute expenditures.

[24] For durables, we calculate expenditures using house furnishing and equipment expenditures ("houseeqcq"). For demand for HPS workers, we use babysitters and housekeepers expenditures ("domsrvcq" in 2010, and "housopcq" in 1980).

4.4 Results

4.4.1 Main Experiment

We assess the implications of changing wages and p_m by introducing their 2010 values into the benchmark model. This is our main experiment. We measure the contribution of these changes to explaining fertility and time allocation trends by comparing the main experiment predictions to the actual 2010 data.

Figure 7 repeats Figure 6, using the prediction of the main experiment and the 2010 data.[25] We report the results of the main experiment for the benchmark case as well as the low and high ϕ_m cases. The top left panel shows the model's prediction for women's time at home, and includes the 1980 data for comparison. The model's prediction is quite close to the actual data, though the model somewhat understates time spent at home for the first decile, and somewhat overstates it for the top two deciles. Overall, the model accounts for the change in female labor supply quite well, and is not very sensitive to changes in ϕ_m.

The top right panel shows the model's prediction for fertility, and again shows the 1980 data for comparison. With the exception of the rise in fertility between the first and second deciles, which is due to corner solutions in the model, the model accurately captures the declining fertility rates through the fifth decile, and the subsequent flattening/rising fertility rates. Consistent with the data, the main experiment generates little changes in fertility over time for low income couples and large increases in fertility for the high income couples. Here, the level of fertility, but not the general shape, is more sensitive to changes in ϕ_m, as can be seen in the high and low ϕ_m cases. The rise in fertility of the top decile is overstated, with fertility in the model being higher than that of the data by

[25] There is one point worth discussing about time allocations. Our model focuses on understanding time allocation between home production and work, implicitly assuming that the total time on non-leisure activities has not revealed a systematic trend. American Time Use Survey (ATUS) data, however, suggests that leisure may have slightly declined between 1975 and 2003 for the group of married females that we consider: by 6 hours per week for the top deciles and 3.5 hours for the bottom deciles. These are based on our own calculation, and we note that the 1975 ATUS gets reduced to a very small sample once we apply our sample restrictions. If this extra time is devoted towards *quantity* of children, rather than *quality* (time spent reading to children, other education), then our results may be slightly biased for 2010. However, the basic point that the model broadly captures trends in the data is unaffected by this potential mismeasurement.

approximately 0.4 children. Overall, the main experiment goes a long way in generating the observed changes in fertility rates and labor supply of married women between 1980 and 2010.

The bottom left panel shows the model prediction for college graduation rates of children born to couples from different deciles in 2010. There is no data on the graduation rates of these children, as they are still too young, so we show the comparison to the 1980 data, which is almost identical to the 1980 model as seen in Figure 6. This panel shows that college graduation rates barely change over time in the model, implying that high income couples raised fertility without sacrificing quality investments in children.

The bottom right panel shows the index of marketization in 2010. The model prediction for the lower half of the income distribution is quite good. For deciles 5–8, the model somewhat understates the rate at which households increase their marketization. Notice that this is also the interval in which fertility in the model is somewhat lower than in the data. There is a sharp kink in the index of marketization at decile 9, exactly where the model begins to overstate fertility rates. Given that this index is a measure relative to the first decile, it is unsurprising that it is insensitive to ϕ_m. The level of market expenditure on children grows by a factor of 3.2 for the top income couples and a factor of 2.5 for the second lowest income decile. Overall, the model does an impressive job accounting for the 2010 data patterns with changes in wages and the price of home production substitutes alone.[26]

We can also quantitatively compare the model with empirical results in the literature. Mazzolari & Ragusa (2013) study the effects of inequality on demand for home production substitutes. They look at cross-city variation in US employment growth in the home production substitutes sector between 1980 and 2005. Thus, they are estimating changes in demand for home production substitutes during our time period. They find that a one standard deviation (four percentage points) increase in a city's top decile wage bill is associated with a 8-16% growth

[26]In Appendix D we include changes in college tuition and the college premium as additional exogenous forces. The results remain largely unchanged, as these shocks exhibit offsetting effects and do not interact with wages or p_m.

in the number of hours in the home services sector.[27] Our model's counterpart is 13%, when taking an average of the corresponding result in the benchmark model (1980) and the main experiment (2010).

We next break down the results of the main experiment and explore the implications of differential fertility for human capital.

We use the following measures in our discussion. We measure "High Income Fertility" as the average fertility of the top two deciles. We use two measures of fertility gap between high and low income couples. MDF1 is computed as the ratio of top two decile fertility to 2nd decile fertility. We choose to focus on the 2nd decile rather than the bottom decile because the latter is affected by various welfare programs that we do not model. MDF2 is computed as the ratio of fertility in the top half of the income distribution to fertility in the bottom half of the income distribution.[28]

Finally, we introduce a fertility-driven measure of aggregate college attainment, which we compute by weighing the 1980 empirical college attainment profile by the appropriate fertility profile (in both the model and data by year).[29] We keep the relationship between income decile and college graduation fixed at the 1980 level for two reasons. First the data on college attainment rates for children born in 2010 will not be available until around 2035. Second, this measure allows us to isolate the effects of changing differential fertility on aggregate college attainment.

Table 2 summarizes the data, the main experiment results, and the breakdown of model mechanisms. The first column shows the data percentage changes in high income fertility, MDF1, MDF2, and the percentage point (p.p.) change in fertility-driven aggregate college attainment (see Footnote 29). High income fertility rose by 40%. Low income (second decile) fertility remained almost constant. These

[27]This is the range of their IV estimates. See their Table 2.

[28] Formally, high income fertility is expressed as $\frac{n(10)+n(9)}{2}$, where $n(i)$ is the fertility rate of decile i. MDF1 is expressed as $\frac{n(10)+n(9)}{2}/n(2)$, and MDF2 is expressed as $\sum_{i=6}^{10} n(i)/\sum_{i=1}^{5} n(i)$.

[29]Formally, the fertility-driven measure of college attainment is computed as $CG = \sum_d \frac{n(d)}{\sum_d n(d)} \pi_{1980}^{data}(d)$, where $\pi_D^{1980}(i)$ is the empirical college graduation rate of children born in decile i in 1980.

two facts combine to imply that MDF1 increased by almost 40% (38.5% to be precise). MDF2 increased by 18.6%. Overall, changes in differential fertility imply a 1.70 p.p. increase in college attainment rates of the next generation. The second column reports implications of the main experiment. In the model, high income fertility rises by 43.5%, MDF1 increases by 41%, MDF2 increases by 24.4%, and college attainment rates of the next generation rise by 2.4 p.p.[30] Recall that none of the data moments reported here are targeted. These results show that changes in wages together with falling prices of HPS goods (introduced by the main experiment) go a long way in explaining the observed trends in fertility, namely the rise in high income fertility and the decline in differential fertility.

4.4.2 Decomposing the Main Experiment

There are two mechanisms through which rising inequality leads to changing differential fertility in the main experiment. The first is increased marketization, as measured by changes in $\frac{w_f}{p_m}$. The second is the income effect on demand for children, as measured by changes in w_m. We now evaluate each in turn.

Contribution of Marketization

To assess the contribution of marketization, we recompute the main experiment except we adjust p_m, by decile, so that decile-specific $\frac{w_f}{p_m}$ are at their 1980 levels. We do so by varying p_m by decile.[31] We refer to this experiment as the "Main Experiment: No change in Marketization". Relative to the main experiment, this counterfactual shuts down the movement in the relevant measure of marketization cost, thereby allowing us to explore its importance for our results. Figure 8 (left panel) depicts the results.[32] As can be clearly seen, without a decreasing

[30]When calculating the college attainment rates in the model using the π in the main experiment, the college attainment rate rises from 38.3% in 1980 to 42.8% in 2010, an even larger increase.

[31]As can be seen from Equations (9) and (10), this implies that $\frac{t_f}{m}$ remains constant, by decile, over time.

[32]We leave out the first decile in Figure 8, because the model generates a corner solution and because couples in this decile qualify for various welfare programs that our model does not capture. This is not crucial as the effects of marketization on the first decile are minimal.

relative price of marketization, high income fertility falls drastically. This is the exact opposite of what happened in the data.[33] This is directly along the lines of the standard theory; without marketization, increases in inequality decrease fertility among high income families.

Column 3 of Table 2 reports the results for this counterfactual experiment. We see that, without the fall in the price of marketization, high income fertility counterfactually falls by 27%. More importantly, and consistent with the standard model, MDF counterfactually falls, using both measures. This in turn decreases college attainment by 0.50 p.p. as opposed to the 1.70 p.p. *increase* the data measures. This is despite the fact that rising male inequality is at work here. Thus, as we observe above, a naïve modeler, working in 1980 and ignoring marketization, would have predicted a widening of differential fertility and thus a decline in college attainment rates over time if she had been given perfect foresight over actual income distributions.[34] Adding this counterfactual decrease implied by the standard theory to the increase seen in the data, we find that the bias from ignoring changes in marketization is 2.2 percentage points of college attainment. This estimate implies that differential fertility's impact on education is substantial. To put things in perspective, 2.2 percentage points is equivalent to roughly one-quarter of the rise in college attainment between the 1950 and 1980 cohorts of white, non-Hispanic non-immigrant Americans (27% and 37.9%, respectively). Thus, the bias induced by ignoring marketization is both quantitatively large and changes the sign of the estimated implications of inequality on differential fertility, and thus education.

Contribution of Male Income Growth

To assess the contribution of changing male wages, we recompute the main experiment holding the decile-specific male incomes (w_m) at their 1980 levels. We

[33]Notice that the *level* of fertility is lower for all deciles. This is due to the fact that female wages grew more than male wages. Specifically, as can be seen from Equation (13), the positive income effect generated by male wage growth is counterbalanced by a larger increase in the price children driven by rising female wages.

[34]Notice that the standard theory does not allow for any marketization, while in our counterfactual exercise we do not allow the relevant cost of marketization to fall over time. Thus, while the two exercises are not perfectly comparable, the underlying economics is similar.

refer to this experiment as the "Main Experiment: No change in w_m." Figure 8 (right panel) illustrates our findings. As can be seen, the prediction for 2010 is quite similar to that of the main experiment, with somewhat lower fertility rates for high income households. The intuition is clear; those households saw a great rise in male income which, through the income effect, should increase fertility. Shutting down this mechanism leads to less fertility.[35]

Column 4 of Table 2 summarizes the results for this counterfactual experiment. When abstracting from changes in male income, high income fertility still rises 30%, which is 69% of the 43.5% increase in the main experiment. This means that the income effect can explain at most 31% of the increase in high income fertility. MDF1 (MDF2) increases 24% (15.1%), which is 59% (62%) of the 41% (24.4%) increase in the main experiment, implying that the male income effect can explain at most 41% (38%) of increased MDF. Finally, the college attainment rates of the next generation rise by 1.60 p.p., which is 67% of the increase in the main experiment, implying that the male income effect can explain at most 33% of the increase in college attainment attributed to changing differential fertility.[36]

We note two more interesting facts about this exercise. The first is that the findings are under the extreme assumption of traditional gender roles. If men bore a time cost of children as well, then marketization would presumably be an even stronger force for differential fertility in the model. Thus, our findings are conservative. Second, we note that this measure of the impact of the income effect on differential fertility captures all of the empirical mechanisms causing an increase in male wages by decile, including sorting. To see this point, imagine that sorting increases, with no other change in inequality. Then the higher deciles would begin to measure higher male wages. Thus, this exercise can be thought of as capturing the upper bound of the effect of rising marital sorting on differential fertility.

[35]The opposite happens for the low end of the distribution where male real incomes actually fell over time.

[36]Notice that these two exercises show that marketization and the income effect do not add up to the total effect. This is because there is an interaction between the two mechanisms; when p_m decreases, the positive effect of w_f on the price of children (p_n) weakens, as seen in Equation (C.6), thereby allowing the income effect of wages on fertility to grow in strength.

A Further Look into the Marketization Mechanism

Delving deeper into our results, we perform two more exercises in order to disentangle the roles of falling p_m and rising female wage inequality on fertility. First, we expand on the exercise described above as "Main Experiment: No change in Marketization," which illustrated the importance of changes in $\frac{w_f}{p_m}$, by separately analyzing the effects of changing w_f and p_m. Figure 9 (left panel) shows the main experiment's fertility rates, by decile, in 1980 and 2010. It then adds two curves. The curve "1980 with 2010 w_f" shows the 1980 model with women's wages from 2010. The curve "1980 with 2010 p_m" shows the 1980 model with marketization prices from 2010. As can be seen, simply changing w_f lowers fertility rates. However, the relationship between income and fertility flattens greatly after the 5th decile, as in the data, and even turns positive for between the 9th and 10th deciles. As opposed to this, if only p_m changes, fertility increases. Here, the first deciles have a positive relationship between income and fertility due to the corner solution in e. However, by the 4th decile, the relationship becomes negative, flattening out only after the 8th decile.

We conclude two things from this exercise. First, inequality in women's wages was a significant force for the flattening relationship between income and fertility. Second, the interaction of changes in w_f and p_m is what allows the model to match both the level and shape of the fertility profile in 2010. The importance of interaction effects is seen mathematically in Equation (C.6) which derives the (positive) effect of w_f on the cost of children (p_n). When inputs in home production are substitutes, the fall in p_m decreases the magnitude of this effect, thereby weakening the negative effect of w_f on fertility and allowing the positive income effects of w_f and w_m to get relatively stronger.

The second exercise is to show, mechanically, what is causing the change in fertility patterns between the two inputs into child production, viz. mother's time (t_f) and market substitutes (m), as in (7). Figure 9 (right panel) shows the 1980 and 2010 fertility profiles in the main experiment. The curve "1980 m with 2010 t_f" shows what fertility would have looked like had the 1980 levels of m been combined with the 2010 levels of t_f. Since mother's time at home is decreasing for all deciles between 1980 and 2010, the level of fertility is lower. However, for our purposes, it is important to note that fertility would still have been negatively correlated with income. The curve "1980 t_f with 2010 m" shows what fertility would have looked like had the 1980 levels of t_f been combined with the 2010 levels of m. Since all deciles purchase more market substitutes in 2010, fertility is higher. However, it is clear that it is the differential rise in the use of home production substitutes that led to a flat, or even increasing, relationship between income and fertility.

5 The Minimum Wage, Revisited

In this section, we first discuss the theory as to why the price of marketization (p_m) has a greater effect on higher income couples. We then show empirically, using cross state variation, that the minimum wage has a large effect on wages in the home production substitutes sector. Our theory then implies that changes in the minimum wage will have an impact on fertility and labor supply of high income couples. We use the benchmark model to quantify these effects. We end by turning to a reduced form empirical analysis to estimate the effect of the minimum wage on the labor supply of high income women and find even larger effects than those implied by the model.

5.1 Minimum Wage: Theory

The effects of minimum wage laws have been widely studied, but these studies focus on the labor supply of low wage workers (Manning 2016). The theory presented thus far makes a stark prediction; anything that changes the price of home production substitutes, such as caretakers for children, should affect the labor supply and fertility of *all* households. Thus, the minimum wage should also affect the labor supply of women whose own wages are not directly impacted by the minimum wage. We focus our attention on these women in order to completely abstract from the direct effect of minimum wage laws on wage offers. We show that the labor supply of these women is affected through the indirect impact of minimum wage laws on the price of market substitutes for home production, as represented by p_m in the model.

Claim 3 *If $\rho \in (0,1)$, an increase in the minimum wage decreases labor supply, when fertility cannot adjust, that is, $\frac{\partial t_f}{\partial p_m}|_{n=n_0} > 0$. Moreover the effect is differential across the income distribution. A sufficient condition for the effect to be increasing with wages is $\rho > \frac{1}{2}$. That is, $\frac{\partial^2 t_f}{\partial p_m \partial w_f}|_{n=n_0} > 0$ if $\rho > \frac{1}{2}$.*

Proof. Follows directly from differentiating (9) with respect to p_m, and then again with respect to w_f, holding n constant. ■

One can think of the effect of the minimum wage on labor supply holding fertility constant as a short run effect. That is, if fertility decisions have already been completed, then labor supply changes as described by Claim 3. However, the minimum wage will also affect fertility for families that can still adjust their fertility choices.

Claim 4 *Increases in the minimum wage decrease fertility. That is, $\frac{\partial n}{\partial p_m} < 0$.*

Proof. Follows directly from differentiating (13) with respect to p_m. ■

The magnitude of the effects of the minimum wage on fertility are differential across the income distribution, but it is theoretically ambiguous whether the magnitude increases or decreases with income. We show below that, in our calibration, the richer households see the greatest decline in fertility. Notice that an increase in the minimum wage increases the mother's time allocated per child, but decreases overall fertility. Therefore, the net effect on labor supply is theoretically ambiguous. Again, we show that in our calibration, an increase in the minimum wage lowers female labor supply, and more so for high wage women.

5.2 Minimum Wage: Quantitative Analysis

What are the effects of minimum wage changes on marketization? To answer this question, we first estimate the passthrough rate of the minimum wage to

HPS sector wages by exploiting cross-state variation in the minimum wage over time. We show that the minimum wage has a strong impact on average wages of workers producing home production substitutes. We then use our estimates to conduct a policy experiment in the model by calculating a change in the price of these goods following an increase of the federal minimum wage to $15/hour, as suggested by Bernie Sanders during the 2016 presidential election. We ask the model how a change in p_m in line with this minimum wage increase would affect labor supply and fertility across the income distribution. We end with a further comparison of the model-implied labor elasticity with our own IV estimates based on US cross-state data.

Using CPS data from 1980-2010, we compute the real wage of workers in the industries of the economy associated with home production substitutes.[37] Figure 10 shows the distribution of the real wage, relative to the minimum wage, both for the industries of the economy associated with home production substitutes and other sectors of the economy. The figure clearly shows that workers in industries of the economy associated with home production substitutes are much more likely to earn wages that are close to the minimum wage.

In order to infer the effect of the minimum wage on the wages of home production substitute sector workers, we estimate regressions of the following structure:

$$w_{ist}^{\text{HPS}} = \alpha + \beta w_{st}^{\min} + \gamma \bar{w}_{st} + \delta_{below} + \delta_t + \delta_s + \delta_{age} + \delta_{educ} + \delta_{Hispan} + \delta_{race} + \delta_{occ} + \epsilon_{ist}, \tag{20}$$

where w_{ist}^{HPS} is the real wage of individual i working in the HPS sector, in state s in year t, $w_{st}^{\min}$ is the real minimum wage in state s in year t. This is computed as the maximum between the state and the federal minimum wage.[38] $\bar{w}_{st}$ is the average wage of workers outside of the HPS sector in year t and state s. This allows us to control for state level economic fluctuations that may affect wages in the HPS sector.[39] $\delta_t, \delta_s, \delta_{age}, \delta_{educ}, \delta_{Hispan}, \delta_{race}$, and δ_{occ} are year dummies, state dummies, and demographic controls including age dummies, educational dum-

[37] The selection of these industries follows Mazzolari & Ragusa (2013).

[38] The data source for the minimum wage by state and year is Vaghul & Zipperer (2016).

[39] Our results below show that this variable is not important quantitatively or statistically for our findings.

mies, a dummy for being Hispanic, race dummies, and occupational dummies, respectively. δ_{below} is an indicator that is equal to one if that person is making at least the minimum wage and zero otherwise. We include this variable to control for the fact that there are many workers, roughly 30%, for whom the minimum wage does not seem to be binding. While we are not proposing a theory as to why these workers are paid less, we want to include them separately in our regression.[40] ϵ_{ist} is an error term.

Estimating (20) using OLS may yield an upward biased estimate of β if states tend to raise the minimum wage during good economic conditions, when wages in general are rising. We take two approaches to address this issue. First, we estimate (20) including on the right hand side the average wage in state s and year t.[41] The idea is that if HPS sector workers' wages have similar cyclicality as the rest of the workers in the economy, then the estimate of the relative wage implicitly controls for economic conditions. Second, we take an instrumental variables approach along the lines of Baskaya & Rubinstein (2012). The approach relies on two assumptions. The first is that the federal minimum wage is exogenous to local economic conditions, and therefore exempt from the critique above. However, whether or not the federal minimum wage binds is endogenous to the state. Accordingly, the second assumption is that the level of liberalism in the state determines how likely the federal minimum wage is to bind. Thus, our instrument for the minimum wage in state s and year t is the interaction between the federal minimum wage in year t and an index of state s liberalism from before the sample time period (Berry, Ringquist, Fording & Hanson 1998, Berry, Fording, Ringquist, Hanson & Klarner 2010).[42]

The coefficient of interest is β, which shows the dollar change in HPS sector wages when the minimum wage increases by a dollar. Table 3 reports the results of the estimation. Column 1 controls for year and state fixed effects and for

[40]For example, about 9 percent of workers in this sector are in managerial occupations, of whom 90 percent earn wages above the minimum wage with an average of 2.5 times the minimum wage.

[41]We calculate this average wage without workers in the home production substitute sector in order to avoid the reflection problem (Manski 1993).

[42]We use the average of their nominate measure of state government ideology from 1960–1980. The index of state liberalism has a range of 1 to 100, with more liberal states receiving a higher score, with an average (standard deviation) of 62.3 (11.3).

having a wage that is below or above the minimum wage. Column 2 adds the state average of real wages. Column 3 repeats Column 1 but replaces year fixed effects with region-year fixed effects. Column 4 adds to Column 3 demographic controls. Column 5 adds to Column 4 the state real wage. As can be seen by comparing these columns, the estimate of the impact of the minimum wage on the wages in the HPS sector is relatively stable, declining slightly only when adding the demographic controls. The OLS estimates thus imply that a $1 increase in the minimum wage yields approximately a 65-77 cent increase in wages in the HPS sector. Columns 6–10 repeat Columns 1–5, but instruments for the effective minimum wage in the state using the interaction of state liberalism and the federal minimum wage as described above. The IV estimates indicate that a $1 increase in the minimum wage yields approximately a 55-75 cent increase in HPS wages.[43]

To calculate how a change in the minimum wage to $15/hour affects the average wage in the HPS sector in 2010, we proceed as follows. First, we calculate the average wage in the HPS sector. Then, we create a counterfactual wage for everyone. This wage is equal to the actual wage if the person earned less than the minimum wage. That is, we assume that people who earn less than the minimum wage are unaffected by changes in the minimum wage.[44] For everyone else, their counterfactual wage is equal to their old wage + (15-minimum wage)*0.58. That is, we increase their wages by the estimated β from Column 10, our most demanding specification in Table 3, multiplied by $15 less the minimum wage in that individual's state in 2010. We then compare the average of this counterfactual wage to the average observed wage, and find it to be 21.1 percent higher. Using the price of m, as given by (16), along with the inferred parameter values described in Section 4, we find that a 21.1% increase in HPS wages would imply

[43]We also estimated (20) in log-log specifications which follow Table 3. In all specifications we obtain estimates that are highly significant and approximately equal to 0.5, with no clear difference between the OLS and the 2SLS estimates. An elasticity of 0.5 would imply a somewhat larger effect of changing the minimum wage on p_m than the one implied by the level regressions reported in Table 3.

[44] We are unsure why a person in our sample is earning less than minimum wage. It could be that this is a result of misreported data, lack of enforcement of the minimum wage, or an uncovered sector (waiters). To be conservative, we assume these people are unaffected by the minimum wage. Had we assumed them to be affected, then the counterfactual wage estimated here would be even higher, yielding a greater estimated impact of the minimum wage on home production substitute sector wages.

a 13.8% increase in p_m. Thus, for our exercise, we increase p_m by 13.8%. We focus on couples in the top half of the income distribution whose own wages are presumably unaffected by changes in the minimum wage.

The results are shown in Figure 11. The top panel shows fertility under the higher minimum wage relative to the benchmark model fertility in 2010. The bottom panel shows the relative mother's time at home. A higher minimum wage decreases fertility, and more so for higher income households. It also increases mother's time at home, and more so for higher income households. The magnitudes are large. A 10^{th} (5^{th}) decile household decreases fertility by 12.8% (9.4%), while the mother spends 9.7% (2.5%) more time at home. Notice that these numbers are for women under the assumption that they can adjust fertility. What about those who are "locked in" their fertility choice? We recalculate changes in mother's time at home keeping the model's fertility fixed. A 10^{th} decile mother increases time at home by 25.9%, while a 5^{th} decile mother increases it by 13.1%. These numbers are larger as the family has not had a chance to scale back fertility. The short run effect on labor supply is also significant. The average reduction in labor supply by women in the 9^{th} and 10^{th} deciles is 3.5%. We find the elasticity of high income female labor supply with respect to the minimum wage to be roughly -0.1 in the model.

In order to verify this model prediction, we estimate directly from the data the effect of the minimum wage on the labor supply of high income women. Specifically, we estimate regressions of the following structure:

$$\log Hours_{ist} = \alpha + \beta \log w_{st}^{\min} + \delta_t + \delta_s + \delta_{age} + \delta_{educ} + \delta_{Ind} + \delta_{occ} + \epsilon_{ist}, \tag{21}$$

where $\log Hours_{ist}$ is the log of yearly hours supplied by woman i, living in state s, in year t. All other variables have already been described. Notice that β is the elasticity of labor supply with respect to the minimum wage. We use CPS data for the years 1980–2010. Our sample is comprised of white non-Hispanic married women aged 25 to 54, whose real hourly wage is in the 9^{th} or 10^{th} decile in each five year age group, state, and year.[45] Again, as in the estimation of β in Equation (20), estimating (21) with OLS might induce an upward bias if hours of high income women and the state minimum wage are procyclical. To overcome this issue we estimate (21) using 2SLS when, again, state s minimum wage in year t is instrumented with the interaction between the federal minimum wage in year t and an index of state s liberalism from before the sample period.

Table 4 reports estimates of β. Column 1 only controls for year and state fixed effects. Column 2 repeats column 1 but replaces year fixed effects with region-year fixed effects. Column 3 adds to Column 2 age and education fixed effects. Column 4 adds to Column 3 industry fixed effects, Column 5 replaces the industry fixed effects in Column 4 with occupation fixed effects. Finally, Column 6 includes both industry and occupation fixed effects. As can be seen from the table, all of the OLS estimates are very close to 0 and none are even remotely significant. Columns 7–12 repeat Columns 1–6, but instrument for the state minimum wage. All of the estimates are statistically significant and economically meaningful. They imply that the elasticity of labor supply of high income women with respect to the minimum wage is in the range of -0.66 to -0.41. This empirical elasticity is thus larger than that of the model.

Finally, Table 5 repeats Table 4 for men. As can be seen from the table, all the OLS and the 2SLS estimates are close to 0 and none are even remotely significant. This is exactly what is expected under the assumption of traditional gender roles.

6 Additional Implications of the Rise in Marketization

In this section, we discuss additional implications of the rise in marketization. We first discuss how marketization affects childlessness rates of women by education (a proxy for income), and then ask how marketization affects the endogenous incentives for marital sorting on education.

[45] We drop 1% of outlying observations, specifically those groups with top wages of less than 10.9 dollars per hour or more than 60 dollars per hour, measured in 1999 dollars.

6.1 Childlessness

How does the ability to marketize the cost of children affect fertility along the extensive margin (childlessness rates) among the more educated women?

Baudin et al. (2015) estimate childlessness by woman's education, for those over 45, in the 1990 US census. They find that highly educated women (> 16 years of schooling) have relatively high rates of childlessness. In particular, they show that childlessness rates among married women with a college degree or less range between 6 to 10 percent, while childlessness rates among married women with master degrees and doctoral degrees are 13.7 and 19.1 percent, respectively. Baudin et al. (2015) attribute these high rates of childlessness to the high opportunity cost of these women raising children. According to our theory, this opportunity cost should be decreasing over time, as women marketize the cost of children more and more. Indeed, in Figure 12 (left panel), we show that the rates of childlessness for women with advanced degrees relative to other women is decreasing over time.[46] Indeed, this ratio falls from over two to almost one, yielding no difference in childlessness rates by 2014. The change is driven by decreasing childlessness among women with advanced degrees, as the childlessness rates of other women remained stable (see the right panel of Figure 12).

A natural question to ask is, how much of the changes in fertility rates of these highly educated women can be accounted for by changes along the extensive margin (childlessness), versus the intensive margin (the number of children born to a mother conditional on having at least one)? Figure 13 (left panel) shows the average number of children ever born to all married women with advanced degrees between 1990 and 2014. It is increasing over time, at a rate of about 0.01 children per year. Figure 13 (right panel) shows the average number of children born to these women, conditional on them having at least one child (i.e. the intensive margin of fertility). The intensive margin has remained flat. Thus, the increase in fertility shown in the left panel is mostly explained by the decrease in childlessness, or increased fertility along the extensive margin, rather than an increase in fertility along the intensive margin.

[46] We do not restrict our sample to white non-Hispanics in order to be consistent with Baudin et al. (2015). The results are not qualitatively sensitive to this sample selection.

Our model is not equipped to differentiate between fertility changes along the intensive and extensive margins. We leave this as a promising path for future research.

6.2 Endogenous Sorting

We discuss how the rise in marketization can help explain the rise in marital sorting. Greenwood et al. (2016) show how a narrowing gender wage gap, rising skill premium, and technological improvement in home goods (cheaper marketization) lead to, among other things, a rise in sorting. The intuition is as follows. When the gender gap is narrow, women's wages are relatively more important for the household, increasing the desire for men to marry higher wage women. The same is true as the skill premium rises. More relevant for our story, they find that cheaper marketization leads to a rise in married women's labor force participation, which they argue is important for the desire to sort. "A skilled man is indifferent on economic grounds between a skilled and unskilled woman if neither of them works, assuming that skill doesn't affect a woman's production value at home. When both work, however, the skilled woman becomes the more attractive partner, at least from an economic point of view" (Greenwood et al. 2016, p. 35). Fertility is not discussed in Greenwood et al. (2016). However, if children comprise an additional benefit to marriage, the mechanism proposed in this paper would reinforce the mechanisms they study.

To see this point, consider a man who is choosing between two women, one with a high wage and the other with a low wage. In 1980, the man would face a tradeoff. The high wage woman would provide more income, and thus consumption, but at a cost of fewer children. In 2010, the high wage women could marketize her time with children, such that there is no more tradeoff. That is, the man would not have to choose between high wages and a large family, yielding more of an incentive to marry a high wage woman. This argument is consistent with the fact that marriage outcomes for college educated women have improved relative to non college educated women, measured by the fraction of those ever married or currently married (Figure 14, for data on white non-Hispanic women, ages 35–44).

While these data are not conclusive, they are suggestive of a path for promising future research.

7 Conclusions

In this paper we have shown that the relationship between income and fertility has flattened between 1980 and 2010 in the US, a time of increasing inequality, as the rich increased their fertility. These facts challenge the standard theory according to which rising inequality should steepen this relationship. We propose that marketization of parental time costs can explain the changing relationship between income and fertility. We show this result both theoretically and quantitatively, after disciplining the model on US data. When abstracting from changes in marketization, the model behaves according to standard theory, generating a drop in differential fertility contrary to what happened in the data. We discuss implications for college attainment.

We have used the calibrated model to shed new light on the effects of changes in the minimum wage. Specifically, we have shown that an increase in the minimum wage to $15/hour, as per Bernie Sanders, would imply an increase in the cost of market good substitutes for home production of about 14 percent. This increase would have a significant detrimental effect on the labor supply and fertility of women, whose own wages are not directly affected by the minimum wage increase. The response is higher for high wage women.

We ended with a discussion on the insights our theory offers for the literatures of the economics of childlessness and marital sorting. These are promising avenues for future research.

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Table 1: Calibrated Parameter Values

Parameter	Interpretation	Value	Indentification
α	Weight on # of children	0.45	Fertility
β	Weight on quality of children	0.67	Fertility
η	Basic education	2.06	Fertility
θ	Exponent π	0.43	College Attainment
b	Scaling	0.87	College Attainment
ρ	Elasticity wife/m	0.59	Labor Supply
ϕ	Share of mother's time	0.90	Labor Supply
A	TFP child production	3.77	Index of Marketization
$p_{m,1980}$	Price of market substitutes 1980	1	Normalization
p_e	Cost of education	1	Normalization

Table 2: Results: Model Mechanisms

	Data	Main Experiment		
		Baseline	No change in Marketization	No change in w_m
	(1)	(2)	(3)	(4)
%Δ High Inc Fert	40.0%	43.5%	-27.0%	30.0%
%Δ MDF1	38.5%	41.0%	-1.8%	24.0%
%Δ MDF2	18.6%	24.4%	-4.1%	15.1%
Δ CG (pp)	1.70	2.40	-0.50	1.60

Notes: "High Inc Fert" is the number of children born to the top 2 deciles. "MDF1" is the fertility of the top two deciles relative to the fertility of the 2nd decile. "MDF2" is the fertility of the top half of the income distribution relative to the fertility of the bottom half of the income distribution. "CG" is the fertility-driven measure of aggregate college attainment. See Footnotes 28 and 29 for the formal definitions of these variables. All changes refer to between 1980 and 2010. Column (1), "Data", reports data changes. Column (2) , "Baseline", reports changes implied by the Main Experiment. Column (3) "No change in Marketization" and Column (4), "No change in w_m" report changes for the two counterfactual experiments that supplement the Main Experiment, as described in Section 4.4.2.

Table 3: The Effect of the Minimum Wage on the Wage in Industries Associated with Home Production Substitutes

	Dependent Variable: The Real Wage									
	OLS					2SLS				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Minimum Real Wage	0.764***	0.771***	0.770***	0.665***	0.648***	0.747***	0.645***	0.550**	0.632**	0.582**
	(0.059)	(0.053)	(0.063)	(0.058)	(0.056)	(0.169)	(0.133)	(0.267)	(0.248)	(0.247)
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	No	No	No	Yes	Yes	No	No	No
Region × Year FE	No	No	Yes	Yes	Yes	No	No	Yes	Yes	Yes
Average State Wages	No	Yes	No	No	Yes	No	Yes	No	No	Yes
Demographic Controls	No	No	No	Yes	Yes	No	No	No	Yes	Yes
1^{st} Stage F-Statistic	–	–	–	–	–	16.47	15.90	26.72	26.93	26.08
Obs.	228,197	228,197	228,197	228,197	228,197	228,197	228,197	228,197	228,197	228,197
R^2	0.258	0.259	0.259	0.372	0.372	0.258	0.258	0.259	0.372	0.372

Notes: Standard errors in parentheses are clustered at the state level.* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Sample comprises workers in industries of the economy associated with home production substitutes for the years 1980 to 2010 using CPS data. Demographic controls include age fixed effects, education fixed effects, occupation fixed effects, Hispanic and race fixed effects. The instrument for Columns 6–10 is the interaction between average state liberalism between 1960 and 1980 and the real federal minimum wage.

Table 4: The Effect of the Minimum Wage on the Labor Supply of High Income Women

	Dependent Variable: Log Yearly Hours											
	OLS						2SLS					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log min. wage	-0.032	-0.008	-0.022	0.038	0.021	0.039	-0.544***	-0.664***	-0.632***	-0.503**	-0.405*	-0.429*
	(0.087)	(0.069)	(0.065)	(0.049)	(0.053)	(0.052)	(0.177)	(0.250)	(0.225)	(0.208)	(0.217)	(0.233)
Year FE	Yes	No	No	No	No	No	Yes	No	No	No	No	No
Region× Year FE	No	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Age FE	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Education FE	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Industry FE	No	No	No	Yes	No	Yes	No	No	No	Yes	No	Yes
Occupation FE	No	No	No	No	Yes	Yes	No	No	No	No	Yes	Yes
1^{st} stage F statistic	–	–	–	–	–	–	15.72	24.13	24.25	24.39	24.46	24.62
Obs.	85,506	85,506	85,506	85,506	85,506	85,506	85,506	85,506	85,506	85,506	85,506	85,506
R^2	0.013	0.015	0.047	0.256	0.291	0.310	0.012	0.014	0.046	0.255	0.291	0.309

Notes: Standard errors clustered at the state level are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log of yearly hours worked. Sample of White non-Hispanic married women aged 25-54, whose real hourly wage is in the 9th and 10th deciles. Women are assigned to hourly wage decile by state, year and 5-year age group.

Table 5: The Effect of the Minimum Wage on the Labor Supply of High Income **Men**

	Dependent Variable: Log Yearly Hours											
	OLS						2SLS					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log min. wage	0.043	0.011	0.004	0.002	-0.009	-0.011	-0.118	-0.117	-0.036	0.031	-0.061	-0.032
	(0.034)	(0.031)	(0.028)	(0.026)	(0.027)	(0.027)	(0.115)	(0.149)	(0.123)	(0.122)	(0.122)	(0.119)
Year FE	Yes	No	No	No	No	No	Yes	No	No	No	No	No
Region× Year FE	No	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Age FE	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Education FE	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Industry FE	No	No	No	Yes	No	Yes	No	No	No	Yes	No	Yes
Occupation FE	No	No	No	No	Yes	Yes	No	No	No	No	Yes	Yes
1^{st} stage F statistic	–	–	–	–	–	–	15.27	25.10	25.18	25.42	25.32	25.63
Obs.	100,243	100,243	100,243	100,243	100,243	100,243	100,243	100,243	100,243	100,243	100,243	100,243
R^2	0.013	0.015	0.067	0.160	0.202	0.211	0.013	0.015	0.067	0.160	0.202	0.211

Notes: Standard errors clustered at the state level are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. The dependent variable is the log of yearly hours worked. Sample of White non-Hispanic married **men** aged 25-54, whose real hourly wage is in the 9th and 10th deciles. **Men** are assigned to hourly wage decile by state, year and 5-year age group.

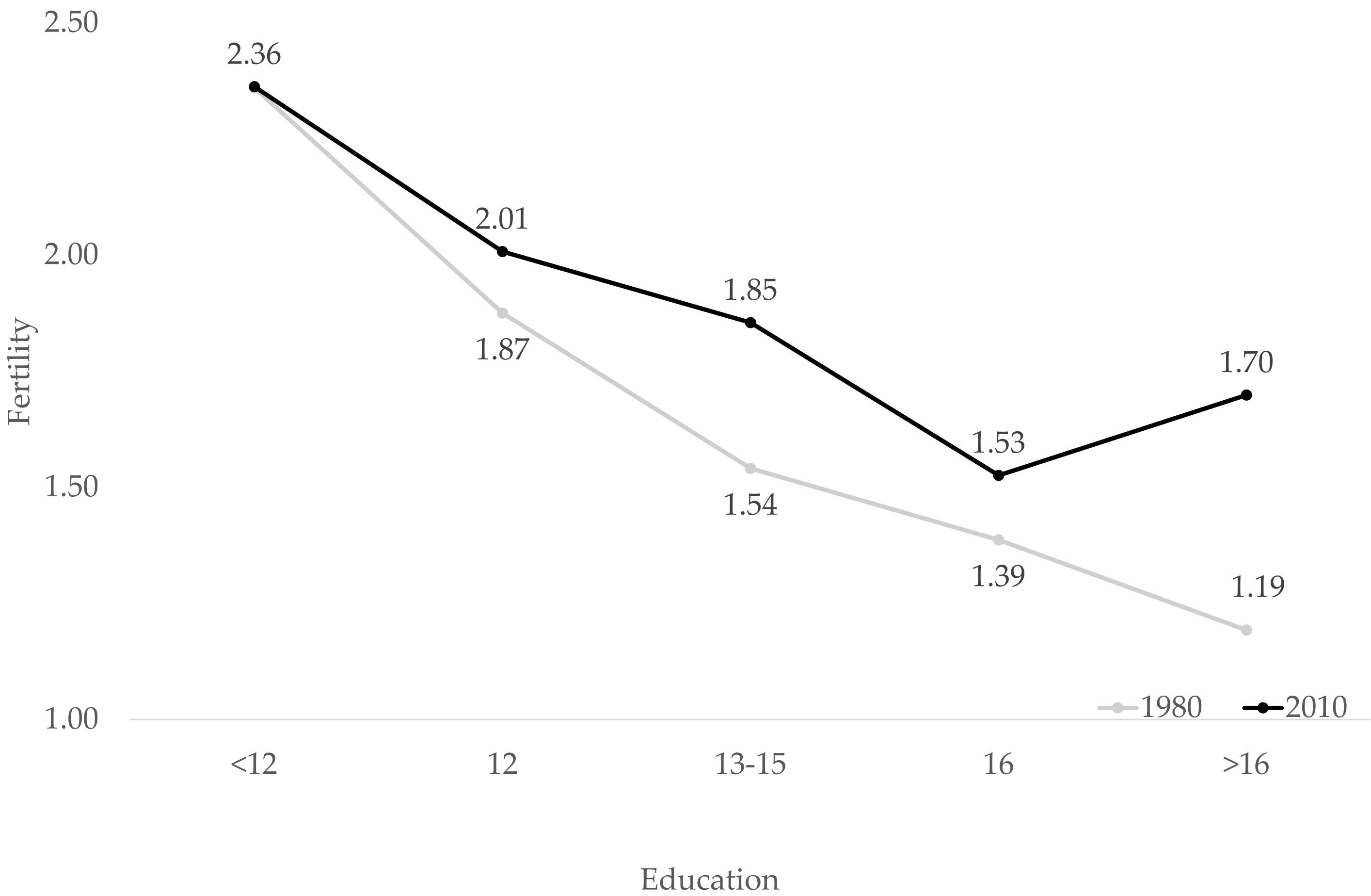

Figure 1: Fertility by Women's Education 1980 & 2010.
Notes: Authors calculations using Census and American Community Survey Data, using all native-born American women. Fertility rates are hybrid fertility rates. "< 12" refers to women with less than a high school degree. "12" refers to women who graduated high school. "13-15" refers to women with some college. "16" refers to college graduates. "> 16" refers to women with advanced degrees.

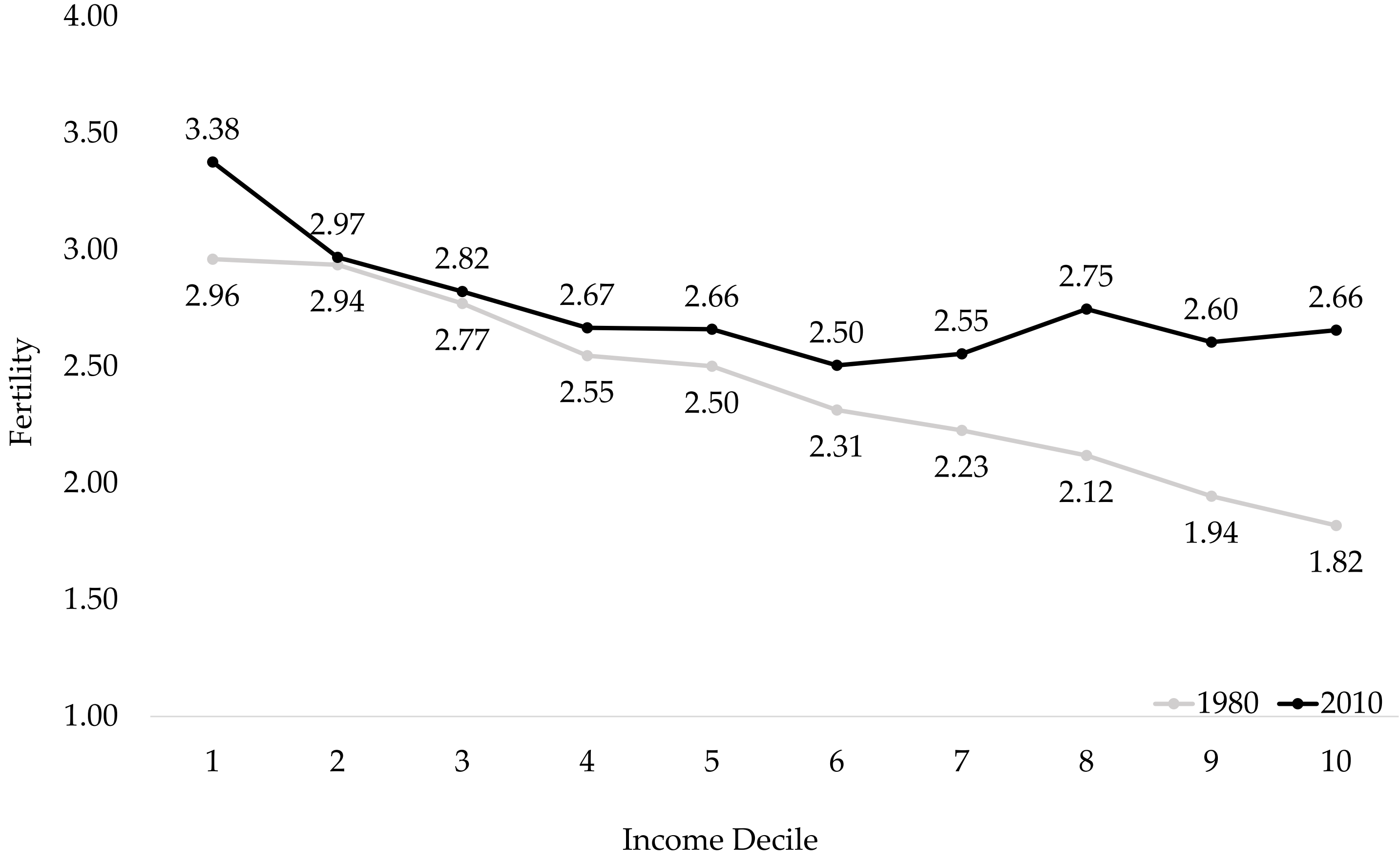

Figure 2: Fertility by Income Decile 1980 & 2010.
Notes: Authors calculations using Census and American Community Survey Data. The sample is restricted to white, non-Hispanic married women. Fertility rates are hybrid fertility rates, constructed by age-specific deciles. Deciles are constructed using total household income.

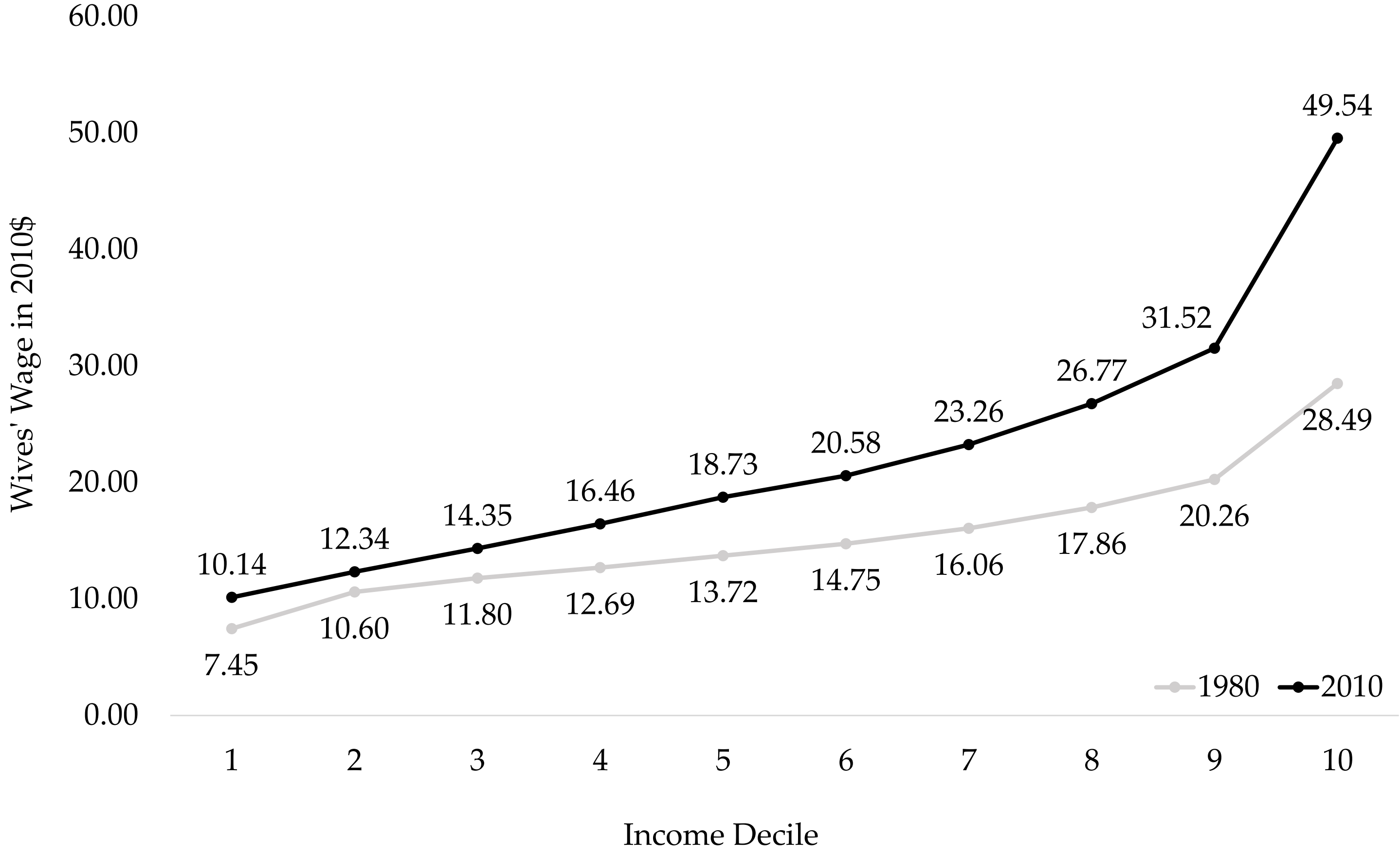

Figure 3: Wives' Wage by Income Decile 1980 & 2010.
Notes: Authors calculations using Census and American Community Survey Data. The sample is restricted to white, non-Hispanic married women. Deciles are constructed age-by-age, using total household income. Representative wages for each decile is the average of these decile-specific wages from ages 25 to 50. See Appendix A for more details.

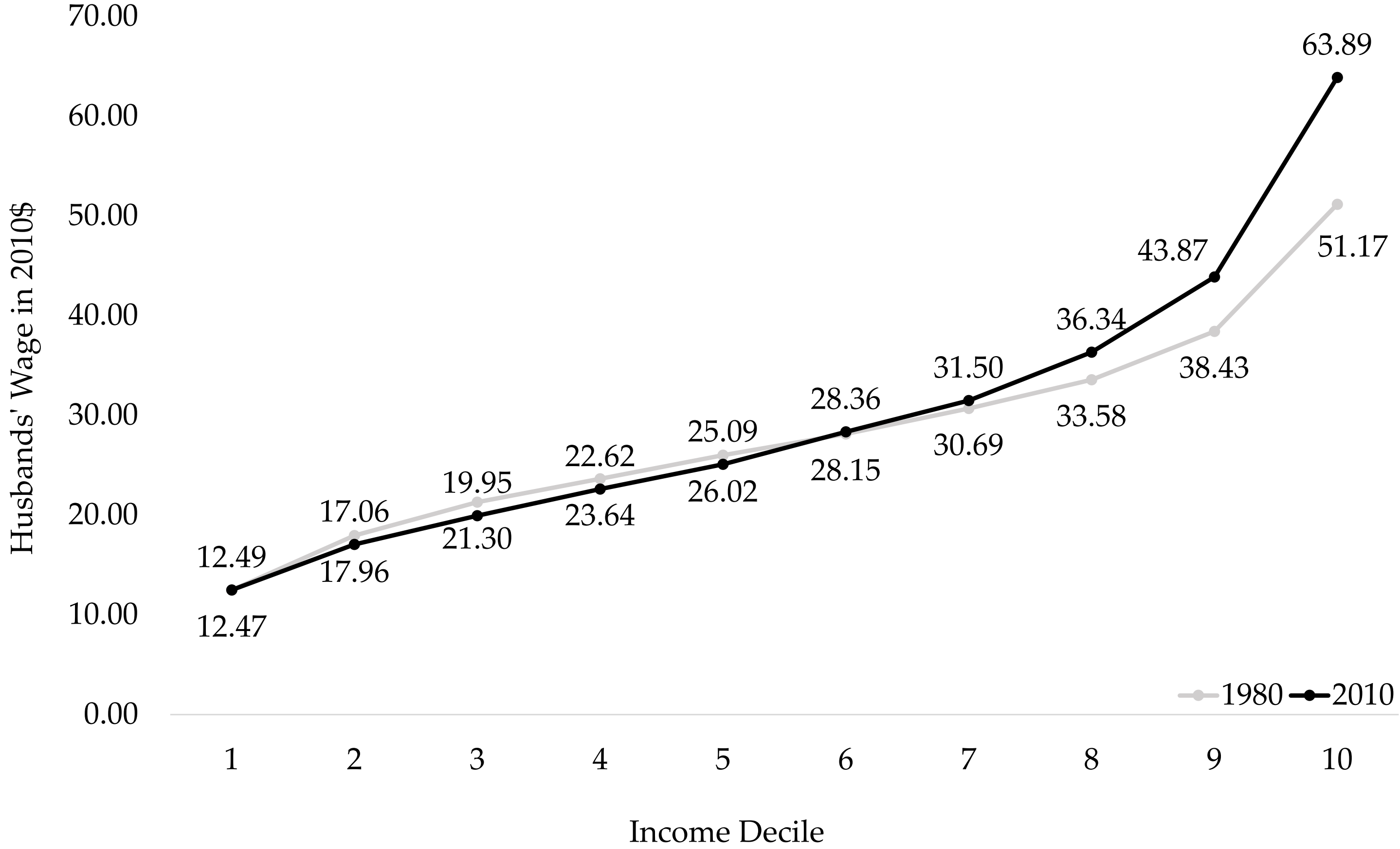

Figure 4: Husbands' Wage by Income Decile 1980 & 2010.
Notes: Authors calculations using Census and American Community Survey Data. The sample is restricted to white, non-Hispanic married men. Deciles are constructed age-by-age, using total household income. Representative wages for each decile is the average of these decile-specific wages from ages 25 to 50. See Appendix A for more details.

Figure 5: Cross State Inequality and Fertility.
Notes: The change in the relative wage of high income women to workers in HPS at the state level is defined as the percent change in the ratio of the average wage of women in the top two deciles to the average wage in the home production substitute sector. The change in fertility is defined as the percentage change in hybrid fertility rates for the top two decile women. Changes from 1980 to 2010. Deciles are constructed age-by-age by total household income, and wages of women are averaged over ages. Wages of HPS workers, as defined in Appendix A, are constructed by state-year. See Appendix A for more details on the exact definition of these variables. Data for high income women are restricted to white, non Hispanic, married women. Data on HPS sector workers are not restricted.

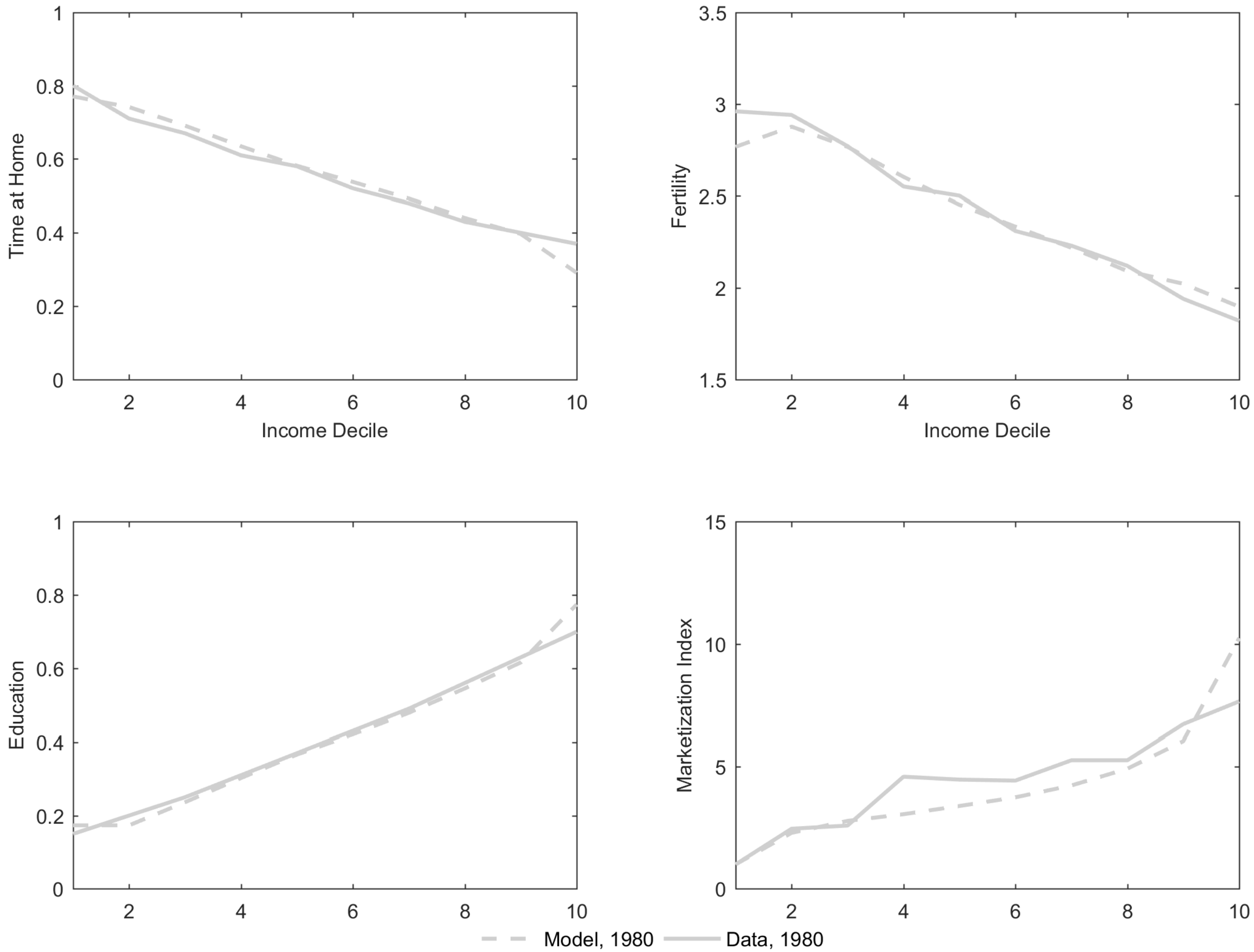

Figure 6: Model Fit

Notes: The top left panel, "Time at Home", is mother's time at home as measured by women's time not working in the data, and t_f in the model. The top right panel, "Fertility", is n in the model and hybrid fertility rates in data. "Education" is the fraction of children born to each decile who graduate college in the data and $\pi(e)$ in the model. "Marketization index" is the expenditures on babysitters, by decile, relative to the 1st decile in the data, and $\frac{p_m m(d)}{p_m m(1)}$ in the model, where $m(d)$ is the amount of market goods m purchased by decile d. "Model, 1980" refers to the calibrated model in 1980. "Data, 1980" refers to the relevant data described in this note and the text.

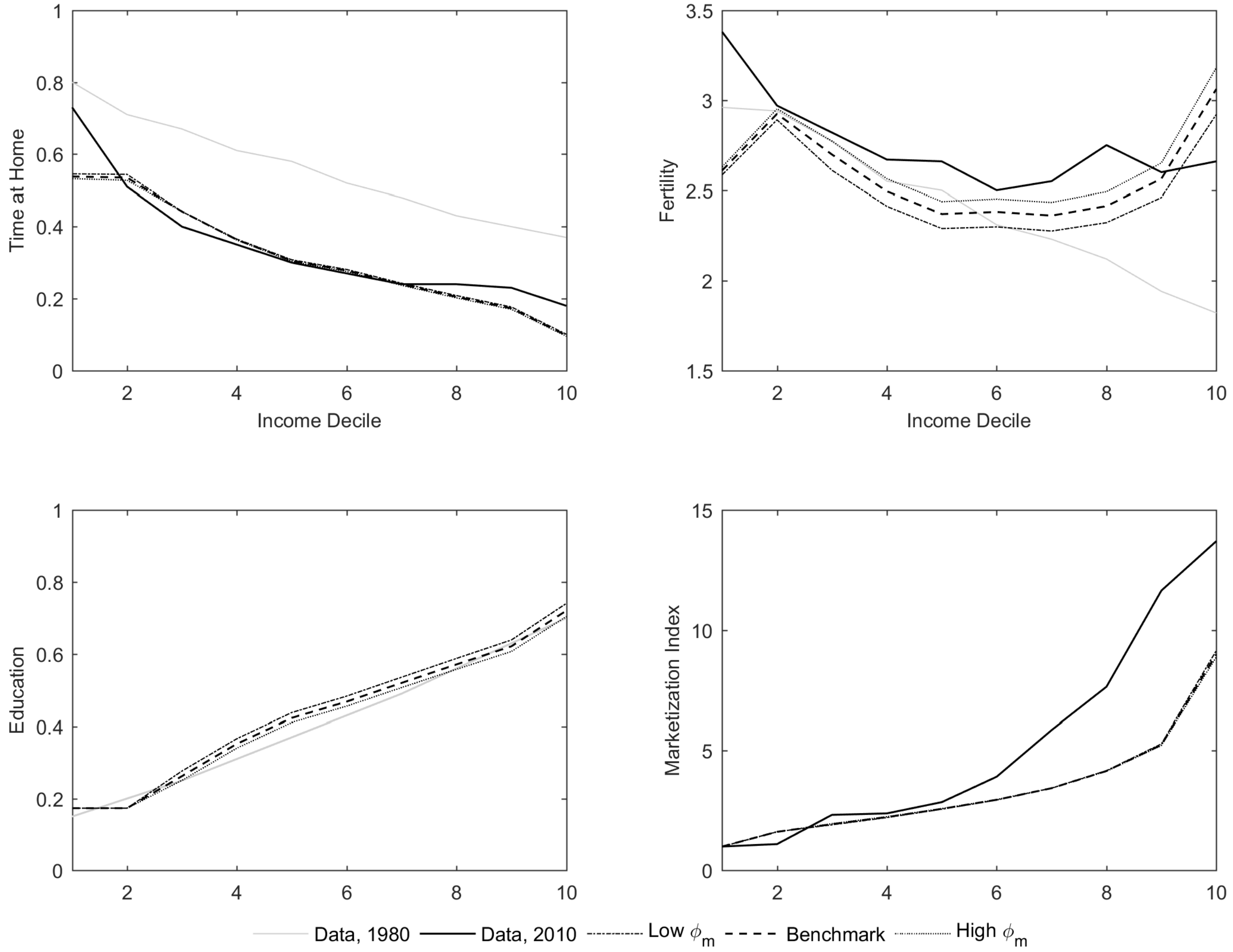

Figure 7: Main Experiment.
Notes: The top left panel, "Time at Home", is mother's time at home as measured by women's time not working in the data, and t_f in the model. The top right panel, "Fertility", is n in the model and hybrid fertility rates in data. "Data 2010" refers to the data, as described in the text. "Benchmark", "Low ϕ_m", and "High ϕ_m" refer to the main experiment's prediction for 2010, for the benchmark case, the "low ϕ_m" case, and the "high ϕ_m" case, respectively. The "main experiment" refers to introducing the 2010 values of w_m, w_f, and p_m into the calibrated model.

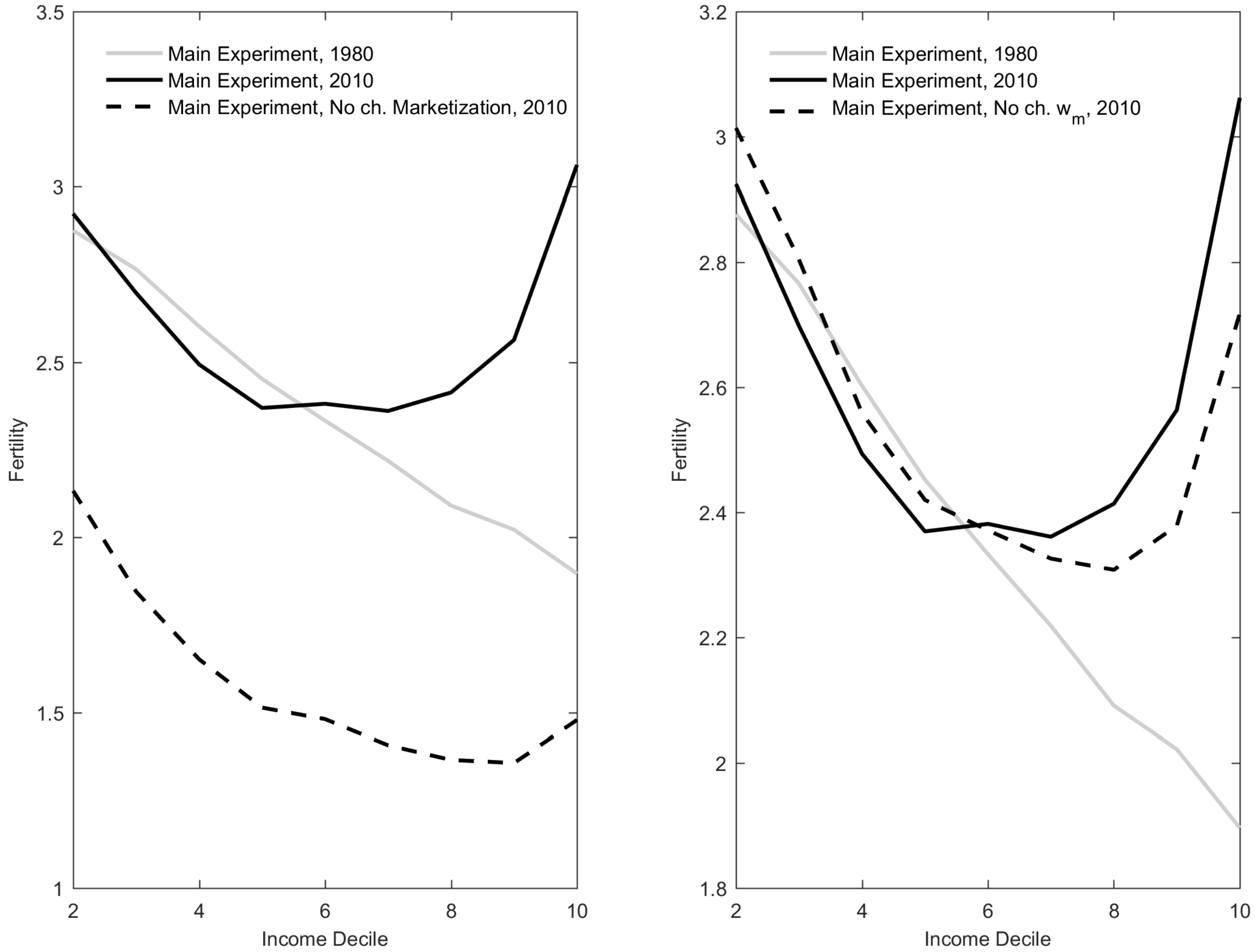

Figure 8: Counterfactuals.
Notes: Fertility is n in the model. “Main Experiment 1980” is the model calibrated to 1980, while “Main Experiment 2010” is the Main Experiment in both panels. Left panel: The curve labeled “Main Experiment: No ch. Marketization” is fertility in the 2010 model using the same relative price of market substitutes ($\frac{w_f}{p_m}$), by decile, as in 1980. Right Panel: The curve labeled “Main Experiment: No ch. w_m” is fertility in the 2010 model using the male wages from 1980.

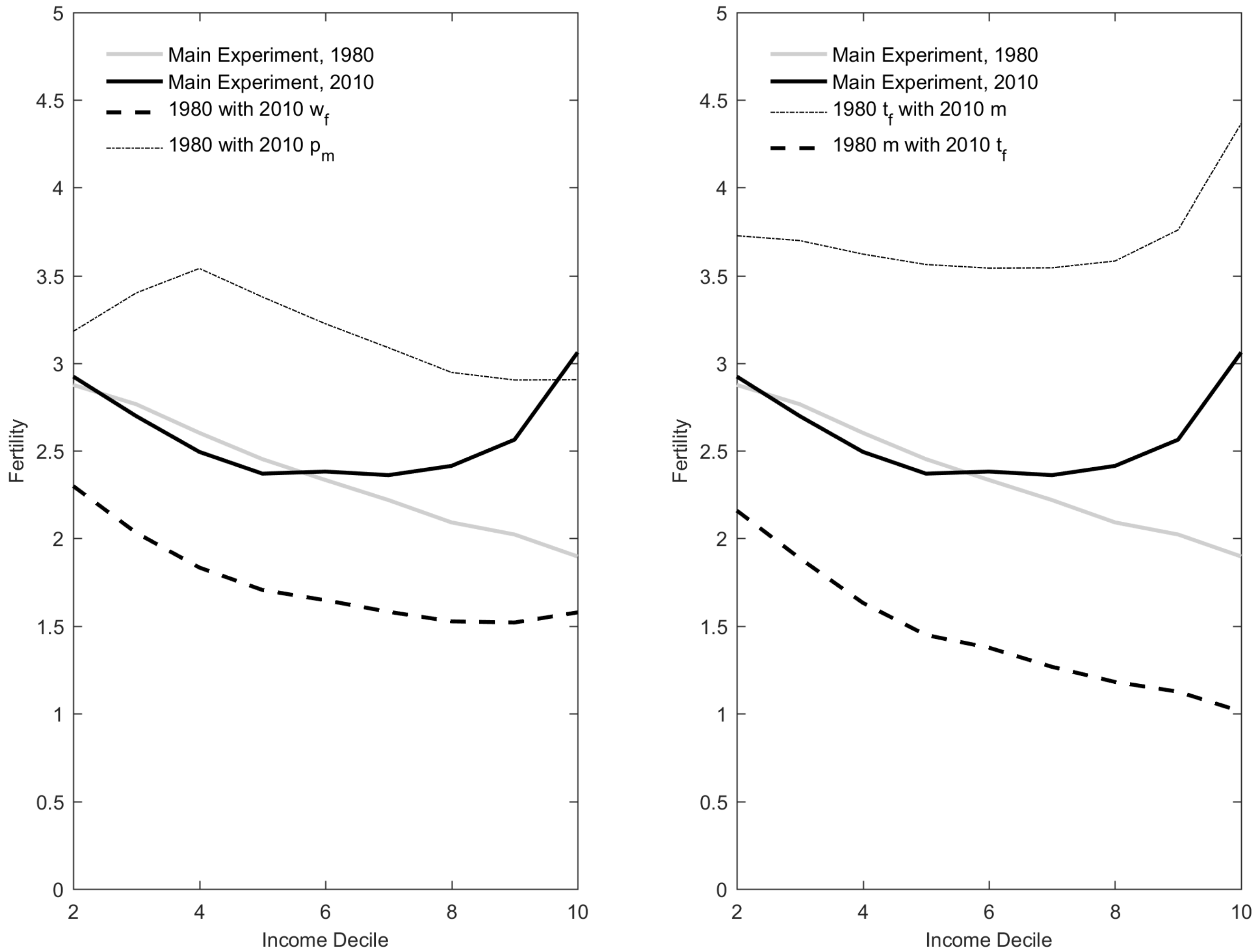

Figure 9: Disentangling Results.

Notes: Fertility is n in the model. "Main Experiment 1980" is the model calibrated to 1980, while "Main Experiment 2010" is the Main Experiment in 2010 in both panels. Left panel: The curve labeled "1980 with 2010 w_f" shows fertility in the 1980 model with women's wages from 2010. The curve labeled"1980 with 2010 p_m" shows fertility in the 1980 model with marketization prices from 2010. Right Panel: The curve "1980 m with 2010 t_f" shows fertility with the 1980 levels of m been combined with the 2010 levels of t_f. The curve "1980 t_f with 2010 m" fertility with the 1980 levels of t_f been combined with the 2010 levels of m.

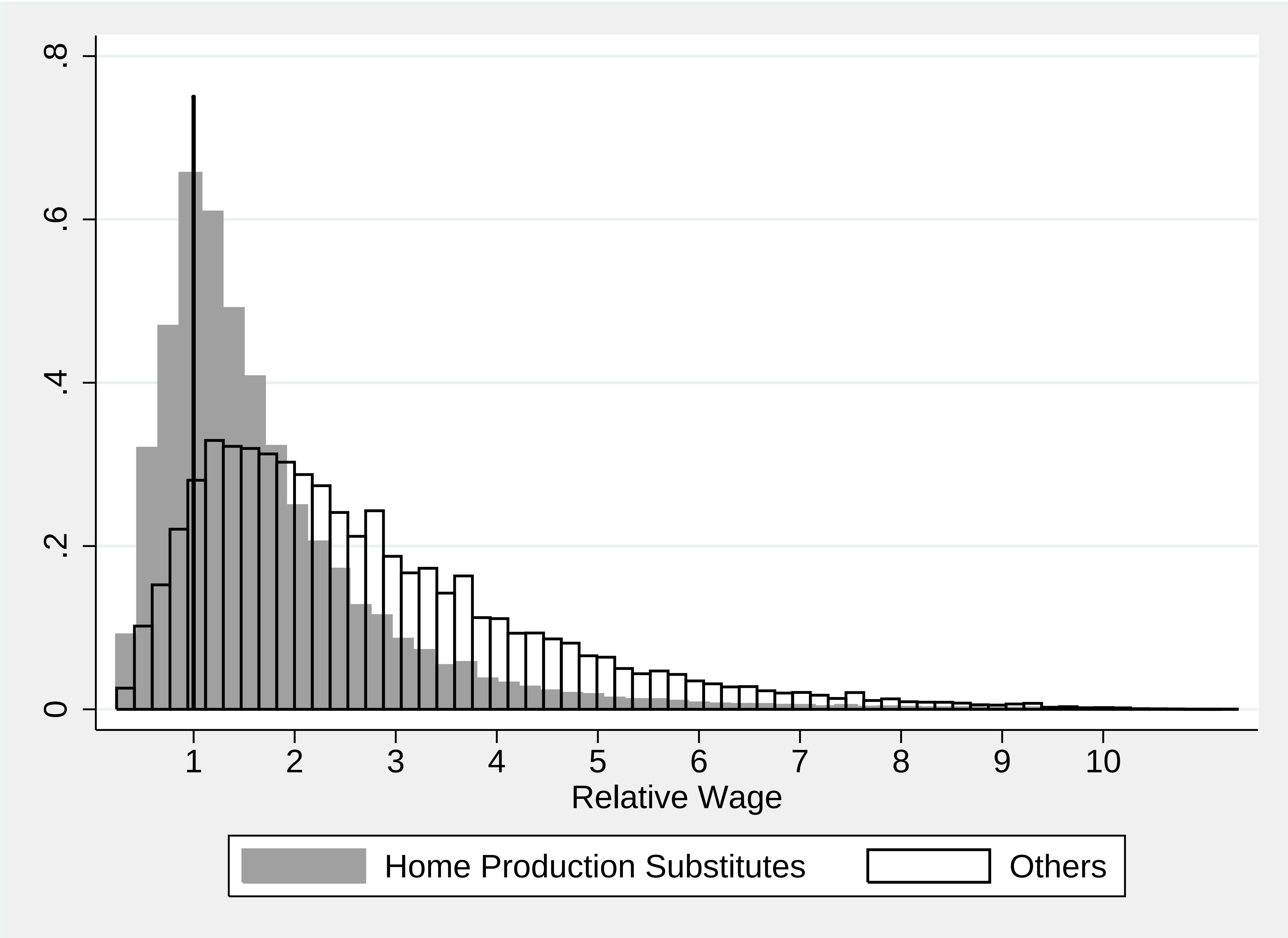

Figure 10: Wage Distribution

Notes: The distribution of real wages, relative to the effective real minimum wage in each state and year, by sector of the economy. Data from Current Population Survey, 1980–2010, using all workers. Home Production Substitute sector workers as defined in Appendix A.

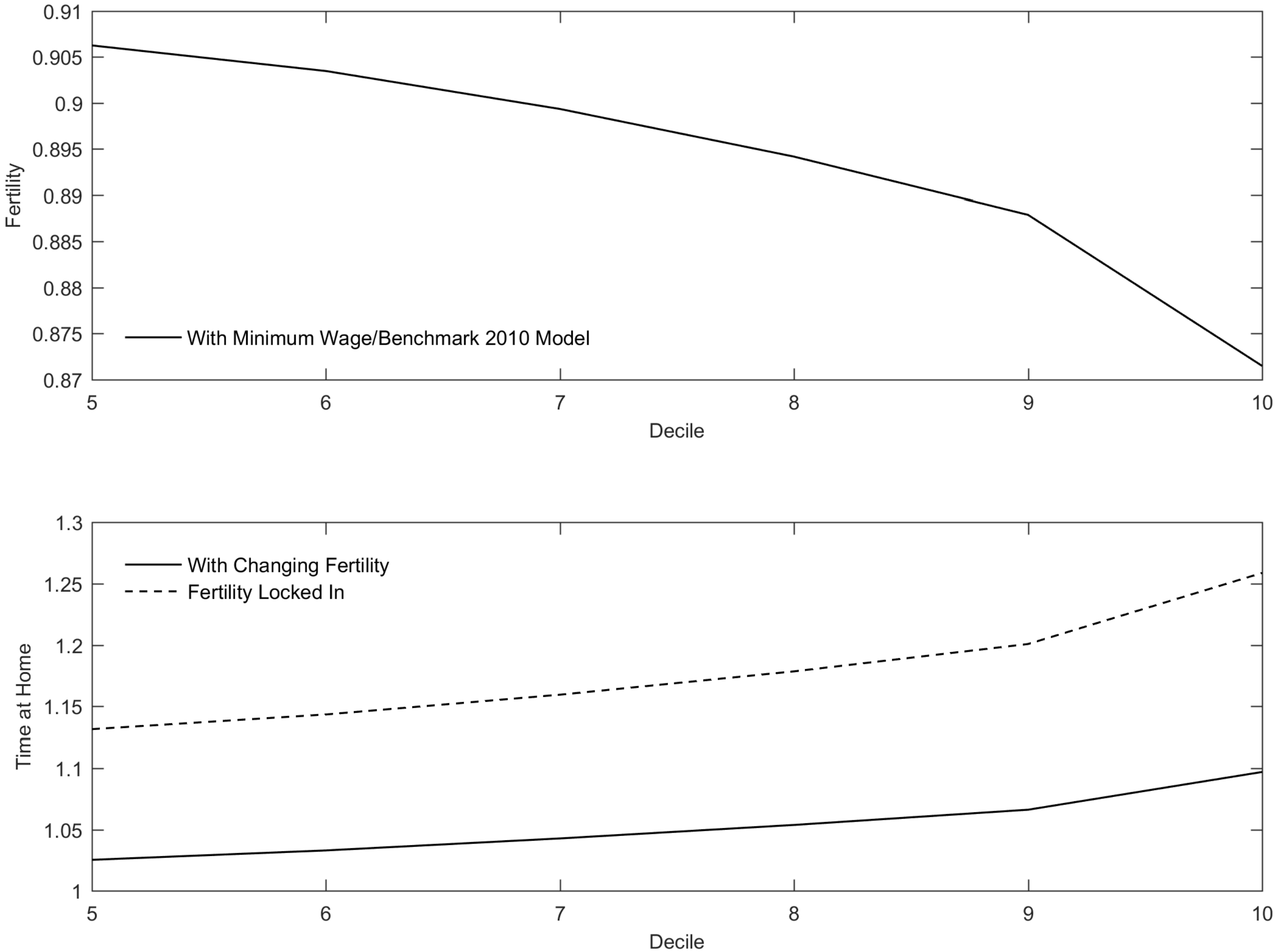

Figure 11: Minimum Wage: Quantitative Results.
Notes: The top panel shows fertility (n) in the 2010 version of the model, with a $15 minimum wage, divided by fertility in the benchmark 2010 model. The bottom panel shows mother's time at home (t_f) in the 2010 version of the model, with a $15 minimum wage, divided by mother's time at home in the benchmark 2010 model. The curve "With Changing Fertility" reports this ratio when fertility is allowed to change with the increased minimum wage, while the curve "Fertility Locked In" reports this ratio when households are forced to maintain the same fertility rate as in the benchmark 2010 model.

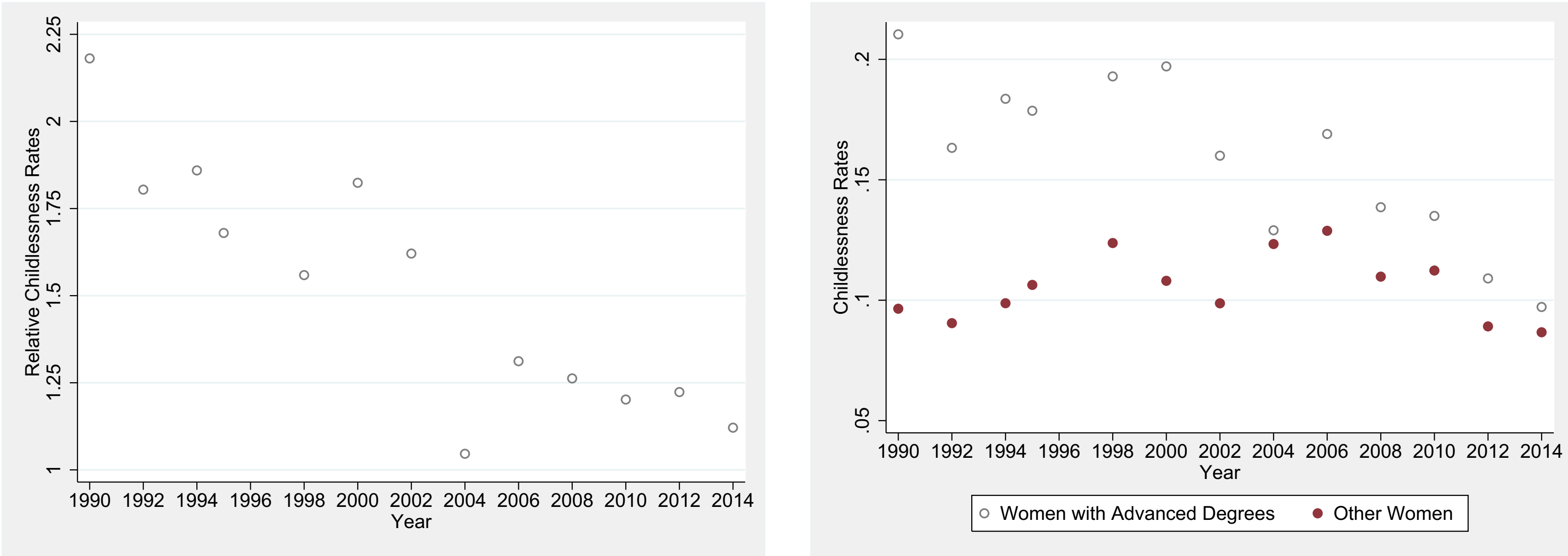

Figure 12: Childlessness by Education.
Notes: The left panel shows the childlessness rates of married women with advanced degrees (>16 years of school) relative to other women. The right panel shows the childlessness rates (the "extensive" margin of fertility) of married women with more than a college education labelled "Women with Advanced Degrees" and of women with up to and including a college education labelled "Other Women". Data is from the Fertility and Marriage supplement of the Current Population Survey (CPS) from 1990–2014, married women ages 40–44. Women over 45 are not asked about their fertility history in this survey.

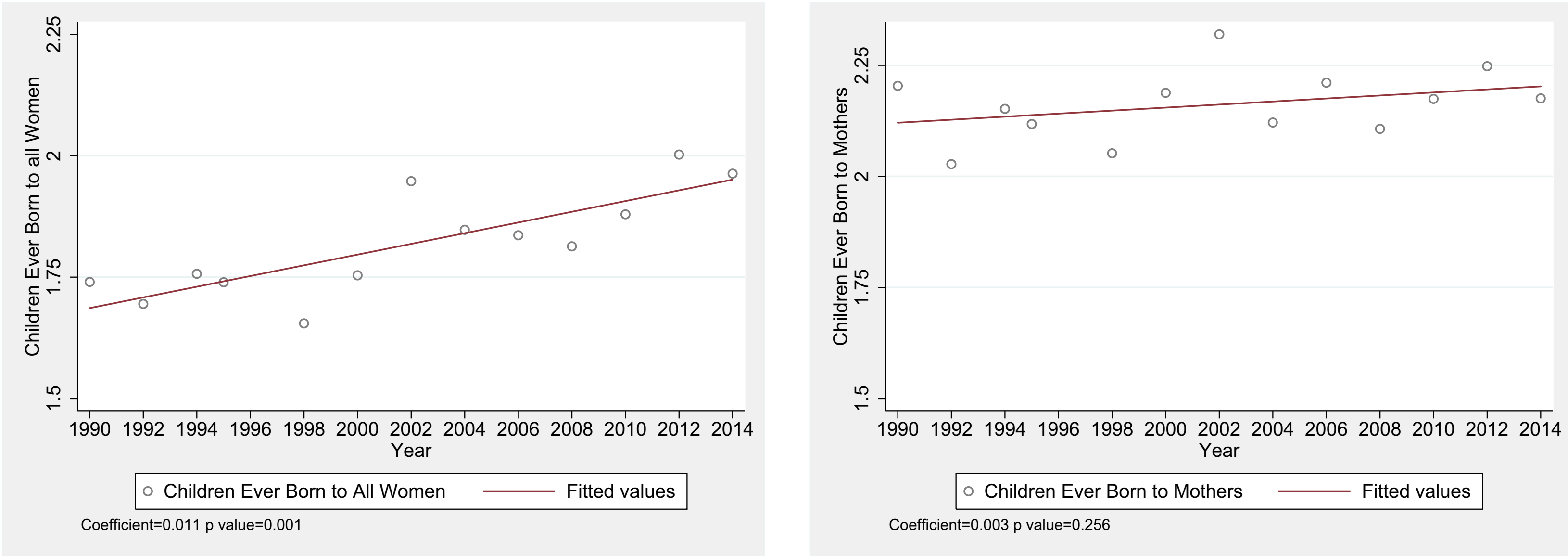

Figure 13: Children Ever Born to Women with Advanced Degrees.

Notes: The left panel shows the average number of children ever born for all married women with advanced degrees (>16 years of school). The right panel shows the number of children ever born to women with advanced degrees, conditional on having at least one child (the "intensive" margin of fertility). Data is from the Fertility and Marriage supplement of the Current Population Survey (CPS) from 1990–2014, married women ages 40–44. Women over 45 are not asked about their fertility history in this survey.

Figure 14: Marriage Markets.
Notes: The left panel shows the fraction of women with at least a college degree who are currently married divided by the fraction of other women who are currently married. The right panel shows the fraction of women with at least a college degree who have ever been married divided by fraction of other women who have ever been married. The data is from the US census and ACS. The sample is comprised of white, non-Hispanic women ages 35–44.

A Data

We employ the 1980 Census and the American Community Survey (ACS) 2010 (Ruggles, Alexander, Genadek, Goeken, Schroeder & Sobek 2010) for measuring incomes, fertility and work hours of each spouse. Additionally, we use the National Longitudinal Study of Youth 1997 (NLSY 97) for measuring educational attainment of children born around 1980, by family income. Finally, we employ the Survey of Program Participation and Income for measuring childcare expenditure by family income. In this study, we focus on the growth of inequality between 1980 and 2010. These years are chosen to allow us to follow the cohort from the NLSY 97 (born around 1980) for measuring their educational attainment by their parental income, while still studying the period of rising income inequality as defined by Autor et al. (2008).

A.1 Mapping of Model Objects to the Data

The mapping between the model and the data is not trivial. In the model, there is one period of adult life which aims to capture the entire working-age lifecycle. In the data, we observe choices of various couples of different age (fertility, work hours, etc) for a period of one year. To map the model to the data, we take the view that a model couple goes through its lifecycle by behaving according to the average age-specific behavior of those couples in the data that it represents.

There are ten types of couples in the model, each of measure 0.1. Each type of couple stands in for exactly 10% of the entire population of married couples of working age. Married couples in the data are allocated into these deciles according to their observed income. We do so based on the ranking of the couples' observed annual income in their group, defined by the wife's age.

From the 1980 Census and 2010 ACS data, we need to derive decile-specific empirical moments for household lifetime income, male lifetime income, male and female wages, male and female lifetime work hours, and couple's lifetime fertility, $I_{f,i}^{year}, I_{m,i}^{year}, w_{f,i}^{year}, w_{m,i}^{year}, hours_{f,i}^{year}, n_i^{year}, hours_m^{1980}$ for each decile $i \in [1, 2, ...10]$

and $year = 1980,\ 2010$. We state income and hours moments in annualized terms and report wages in hourly terms. This is done for clarity.

We restrict attention to white non-Hispanic married couples, aged 25-55, with the husband working at least 35 hours per week and at least 40 weeks per year, following Autor et al. (2008). We also drop the couples in the bottom and top 2% of the male income distribution.

All data couples assigned to a particular income decile are used to derive the average statistics for the model couple representing that decile. To compute the decile-specific lifetime income and hours moments for men, we first average the appropriate quantity within the decile-age cells. For each decile, we then sum across ages.

In the model, all men work full time throughout their life cycle, which is normalized to be 1. This corresponds to the average lifetime hours of full-time male workers in 1980, $hours_m^{1980}$ (~2,300 hours in annualized terms). We infer the data counterpart of $w_{m,i}^{year}$ as $I_{m,i}^{year}/hours_m^{1980}$. Note that the 1980 average hours are used to derive $w_{m,i}^{year}$ in each year. This method ensures that the observed variation of total male incomes across deciles and time will be fully reflected in the purchasing power of couples in the model.

Note that when we consider say a 37 year old woman in 1980 in a given decile, we observe her work hours, which partly reflect her number of children and their age distribution. Our goal here, however, is to derive average working hours for a *hypothetical* woman that experiences her lifecycle according to the cross-sectional profile. We need to proxy the hours each woman would work if she were to follow the 1980s cross-sectional fertility profile, not that of her own cohort. To this end, we regress female work hours in a given year on the actual age distribution of her children (i.e. number of children under 2, 2-3, 4-6, 7-10, 11 to 17), income decile and age dummies. We then predict the average adjusted female hours in each decile and for each age using the children's age distribution implied by the cross-sectional fertility profile. For each decile, we sum these average adjusted hours across age groups to obtain $hours_{f,i}^{year}$ and infer the data counterpart of time spent in home production $t_{f,i}^{year}$ as

$$1 - hours_{f,i}^{year}/hours_m^{1980}.$$

We infer the data counterpart of $w_{f,i}$ as $I_{f,i}^{year}/hours_{f,i}^{year}$ (about 2,050 hours in annualized terms).[47] We infer the empirical counterpart of n_i as a decile-specific hybrid Total Fertility Rate (TFR), as in Shang & Weinberg (2013). We first compute the average age-specific-birth-rate, based on all women in decile i. We then sum across all ages to compute decile-specific TFR. To obtain decile-specific hybrid TFR, we add on the average lifetime fertility among the 25 year-old women in the appropriate decile.[48]

We estimate college attainment for 1980 from NLSY97. Specifically, using the 2011 wave, we observe non-black non-Hispanic individuals, born between 1980 and 1982, and assign them into income deciles according to their parental household income in 1996. We assume that individuals with at least four years of college are college graduates. We measure college attainment π_i^{1980} as the fraction of children with a college degree among all children in the appropriate decile.

Finally, we use the childcare module of the Survey of Program Participation and Income (SIPP) to estimate relative uses of market substitutes.[49] Our index measures are based off of expenditures on childcare hours purchased in the marketplace. Since this is only one aspect of marketization, we use this to target the relative use of marketization across deciles, rather than taking the absolute expenditure levels literally. The implicit assumption is that there is a strong correlation between the use of childcare and other market substitutes for parents' time. To calculate childcare expenditures across deciles, we break households into 5-year age groups from 25–30 until 50–55. Within each group, we divide households

[47] Note that if instead we were to impute wages for non-working females via a Heckman procedure and then take average wages for each decile, our model would not be able to accurately match both female income and female hours. Both of these quantities are critical to our analysis.

[48] Why not include younger women in our analysis and use the standard measure of TFR? We model time allocation between work and home production, and therefore prefer to focus on couples that completed educational investments. Using the hybrid fertility measure then enables us to correctly account for the number of children.

[49] We use the 1990 childcare module as a proxy for the 1980 index of marketization, as this is the earliest available data. We use the 2010 module to derive the 2010 index.

into deciles according to their income. We then sum the childcare expenditures for each decile over the lifecycle. The index is this measure relative to the expenditures on childcare used by decile 1. As before, our sample is married, white, non-Hispanic households.

B Cross State Relationship Between the Relative Price of Marketization and High Income Fertility

In this appendix, we explore further the cross-state relationship between changes in high income fertility between 1980 and 2010 and the change in the relative price of marketization, as first introduced in Figure 5.

We estimate regressions of the following structure:

$$\Delta\% n_s = \Delta\% \left(\frac{w_{fs}}{w_s^{\text{HPS}}} \right) + \Delta\% w_{ms} + \delta_r + \epsilon_s, \quad \text{(B.1)}$$

where the dependent variable $\Delta\% n_s$ is the percentage change in hybrid fertility rates for the top two decile of white non-Hispanic married women in state s. The main explanatory variable of interest, $\Delta\% \left(\frac{w_{fs}}{w_s^{\text{HPS}}} \right)$, is the percentage change in the ratio of the average wage of white non-Hispanic married women in the top two deciles to the average wage in the home production substitute sector in state s. $\Delta\% w_{ms}$ is the percentage change of the average wage of white non-Hispanic married men in the top two deciles in state s. δ_r is a set of region fixed effects for each region $r \in$ {Northeast, South, Midwest, West}. ϵ_s is an error term. These variables are described in detail in Appendix A. All regressions are estimated with robust standard errors.

$\Delta\% \left(\frac{w_{fs}}{w_s^{\text{HPS}}} \right)$ captures the change, over our time period, in the relative price of marketizing a woman's time. Quantitatively, this variable is shown to be crucial for explaining changing fertility patterns in Section 4.4. $\Delta\% w_{ms}$ captures changes in the demand for children induced by increases in male wages and quantitatively evaluated in Section 4.4. The regional fixed effects are implicitly interacting differentially with time, as all our variables are changes between 1980 and 2010. This

Table B.1: The Effect of Marketization on High-Income Fertility

	Dependent Variable: Percent Change in High-Income Fertility					
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta\% \left(\frac{w_{fs}}{w_s^{\text{HPS}}}\right)$	1.064***	1.068***	0.952***	1.211***	1.247***	0.966*
	(0.279)	(0.294)	(0.332)	(0.433)	(0.424)	(0.486)
$\Delta\% w_{ms}$		-0.916	-1.186		0.450	0.187
		(1.453)	(1.511)		(0.779)	(0.876)
Region FE	No	No	Yes	No	No	Yes
Obs.	50	50	50	48	48	48
R^2	0.154	0.171	0.199	0.148	0.154	0.178

Notes: Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

allows us to control for differential regional trends.

Table B.1 describes the results. Column 1 regresses changes in fertility only on changes in the relative price of marketization. Notice that this regression describes the results shown graphically in Figure 5. Column 2 adds the change in male wages over time, while Column 3 adds region fixed effects. Columns 4–6 repeat Columns 1–3, but drop outlying observations, namely North Dakota and Wyoming.

All specifications show a statistically significant and economically meaningful elasticity between fertility and the relative price of marketization, of around 1. Controlling for changes in men's wages or regional fixed effects do not have meaningful effects on the point estimates or standard errors. Dropping outliers strengthens all specifications, but also increases the standard errors somewhat. In particular, the estimate in Column 6 has a point estimate that is basically the same as its counterpart in Column 3, but has a higher p-value of 0.053. All other specifications are significant at the 1% level. Changes in men's wages do not have a meaningful impact on changes in fertility rates, consistent with their relatively weak effect in the model, documented in Section 4.4

C Proofs

C.1 Existence and Uniqueness of the Solution to the Household Problem

Proposition 1 *The necessary and sufficient condition for existence of a unique solution to the household's problem is* $\frac{\beta\theta}{\alpha} < 1$.

Proof. The household's optimization problem can be written as follows:

$$\max_{e \geqslant 0} U(e) = -\ln\left(\frac{p_n}{p_e} + e\right) + \frac{\beta\theta}{\alpha}\ln(e+\eta)$$

There is a possibility that $U(e)$ is unbounded above, and therefore the household's problem has no solution. We can write the objective function as follows:

$$U(e) = \ln\left(\frac{(e+\eta)^{\frac{\beta\theta}{\alpha}}}{\frac{p_n}{p_e} + e}\right)$$

Taking the limit as $e \to \infty$,

$$\begin{aligned} \lim_{e\to\infty} U(e) &= \ln\left(\lim_{e\to\infty} \frac{(e+\eta)^{\frac{\beta\theta}{\alpha}}}{\frac{p_n}{p_e} + e}\right) \\ &= \ln\left(\lim_{e\to\infty} \frac{\frac{\beta\theta}{\alpha}(e+\eta)^{\frac{\beta\theta}{\alpha}-1}}{1}\right) = \begin{cases} \infty & \frac{\beta\theta}{\alpha} > 1 \\ 1 & \frac{\beta\theta}{\alpha} = 1 \\ -\infty & \frac{\beta\theta}{\alpha} < 1 \end{cases} \end{aligned}$$

The first step used chain rule of limits, and the second step used L'Hospital's rule since we have a limit of the form $\frac{\infty}{\infty}$. Intuitively, $\frac{\beta\theta}{\alpha}$ is the weight on quality in the utility function. When this weight is very high, it is possible that the household would like to choose $e \to \infty$ and $n \to 0$, which makes the problem unsolvable. Thus, in order to make the objective function bounded above, we have to impose the restriction $\frac{\beta\theta}{\alpha} \leqslant 1$.

Case 1: $\frac{\beta\theta}{\alpha} = 1$

$$\begin{aligned} U(e) &= -\ln\left(\frac{p_n}{p_e} + e\right) + \ln(e+\eta) \\ U'(e) &= -\frac{1}{\frac{p_n}{p_e} + e} + \frac{1}{e+\eta} \end{aligned}$$

In this case, the solution to the household's problem is as follows:

$$\begin{aligned} \frac{p_n}{p_e} &> \eta \Rightarrow U'(e) > 0 \ \ \forall e, \text{i.e. } U(e) \text{ is monotone increasing, } e^* \to \infty \\ \frac{p_n}{p_e} &< \eta \Rightarrow U'(e) < 0 \ \ \forall e, \text{i.e. } U(e) \text{ is monotone decreasing, } e^* = 0 \\ \frac{p_n}{p_e} &= \eta \Rightarrow U'(e) = 0 \ \ \forall e, \text{i.e. } U(e) \text{ is constant, } e^* \in (-\infty, \infty) \end{aligned}$$

Case 2: $\frac{\beta\theta}{\alpha} < 1$

In this case, the first order necessary condition for interior maximum is $U'(e^*) = 0$:

$$\begin{aligned} U(e) &= -\ln\left(\frac{p_n}{p_e} + e\right) + \frac{\beta\theta}{\alpha}\ln(e+\eta) \\ U'(e) &= -\frac{1}{\frac{p_n}{p_e} + e} + \frac{\frac{\beta\theta}{\alpha}}{\eta + e} = 0 \\ \frac{\eta + e}{\frac{p_n}{p_e} + e} &= \frac{\beta\theta}{\alpha} \\ e\left(1 - \frac{\beta\theta}{\alpha}\right) &= \frac{\beta\theta}{\alpha}\frac{p_n}{p_e} - \eta \\ e^* &= \frac{\frac{\beta\theta}{\alpha}\frac{p_n}{p_e} - \eta}{1 - \frac{\beta\theta}{\alpha}} \end{aligned}$$

The second order sufficient condition for e^* to be a local maximizer is:

$$\begin{aligned}
U''(e^*) &< 0 \\
\frac{1}{\left(\frac{p_n}{p_e}+e^*\right)^2} - \frac{\frac{\beta\theta}{\alpha}}{(\eta+e^*)^2} &< 0 \\
\left(\frac{\eta+e^*}{\frac{p_n}{p_e}+e^*}\right)^2 &< \frac{\beta\theta}{\alpha}
\end{aligned}$$

Using the first order condition:

$$\begin{aligned}
\left(\frac{\beta\theta}{\alpha}\right)^2 &< \frac{\beta\theta}{\alpha} \\
\frac{\beta\theta}{\alpha} &< 1
\end{aligned}$$

Thus, $\frac{\beta\theta}{\alpha} < 1$ guarantees that a solution to the household's problem exists, and the first order necessary condition is a local maximum. Moreover, since the critical point is unique, the local maximum must also be the unique global maximizer. ■

C.2 Shape of fertility across income deciles

We start with preliminary derivations needed for the proofs below. We focus on the region of parameter values where the solution is interior, i.e. $e^* > 0$. In this case, optimal fertility is given by

$$e^* = \max\left\{\frac{\frac{p_n}{p_e}\frac{\beta\theta}{\alpha}-\eta}{1-\frac{\beta\theta}{\alpha}}, 0\right\} \tag{C.2}$$

$$n^* = \left(1-\frac{\beta\theta}{\alpha}\right)\frac{\alpha}{1+\alpha}\left(\frac{w_f+w_m}{p_n-\eta p_e}\right) \tag{C.3}$$

$$\text{where } p_n = \frac{1}{A}\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}. \tag{C.4}$$

Notice that $e^* > 0$ and existence of solution to household's problem, $\frac{\beta\theta}{\alpha} < 1$, imply that $n^* > 0$ and $p_n - \eta p_e > 0$. To analyze the effects on fertility, it suffices to ignore the constant term and focus on the ratio term $\frac{w_f + w_m}{p_n - \eta p_e}$. Clearly, n^* is increasing in w_m as male wages work purely through the positive income effect appearing in the numerator. Female wages, however, affect both the numerator (the positive income effect) and the denominator (the negative price effect). Let $\mathcal{E}_{Y,X}$ denote the elasticity of Y with respect to X. It follows that, for small percentage changes in w_m and w_f, the approximate implied change in n^* is given by

$$\%\Delta n^* \approx \mathcal{E}_{num,w_m} \%\Delta w_m + \mathcal{E}_{num,wf} \%\Delta w_f - \mathcal{E}_{denom,w_f} \%\Delta w_f \tag{C.5}$$

where the elasticity terms are computed as follows:

$$\begin{aligned}
\mathcal{E}_{num,w_f} &= \frac{\partial \left(w_f + w_m\right)}{\partial w_f} \frac{w_f}{w_f + w_m} = \frac{w_f}{w_f + w_m} \\
\mathcal{E}_{num,w_m} &= \frac{\partial \left(w_f + w_m\right)}{\partial w_m} \frac{w_m}{w_f + w_m} = \frac{w_m}{w_f + w_m} \\
\mathcal{E}_{denom,w_f} &= \frac{\partial \left(p_n - \eta p_e\right)}{\partial w_f} \frac{w_f}{p_n - \eta p_e} = \mathcal{E}_{p_n,w_f} \frac{p_n}{p_n - \eta p_e}
\end{aligned}$$

and

$$\mathcal{E}_{p_n,w_f} \equiv \frac{\partial p_n}{\partial w_f} \frac{w_f}{p_n} = \frac{\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}}}{\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}} \in (0,1). \tag{C.6}$$

The question is how optimal fertility varies across couples that represent different income deciles for a given year, or the same decile across years. These couples differ on w_m and w_f. From (C.5) we see that for n^* to decline across income deciles in 1980, as was observed in the data, the price effect of $\%\Delta w_f$ must dominate the income effect of both $\%\Delta w_f$ and $\%\Delta w_m$, where the $\%\Delta$'s are taken across consecutive income deciles. Moreover, for n^* to increase between 1980 and 2010 for couples representing high income deciles, the price effect due to $\%\Delta w_f$ must yield to the income effect due to both $\%\Delta w_f$ and $\%\Delta w_m$. In this case, the $\%\Delta$'s refer to changes over time for a fixed decile. Because the effect of w_m on n is always positive ($\mathcal{E}_{num,w_m} > 0$), we focus on investigating the effect due to w_f alone

$(\mathcal{E}_{num,wf} - \mathcal{E}_{denom,w_f})$. This is done in the two propositions to follow. However, bear in mind that to understand the profile of optimal fertility across income deciles or over time, we need to consider the combined effects of both w_m and w_f.

Proposition 2 . *(Monotonicity and limit of $\partial n^*/\partial w_f$). If $\rho \in (0,1)$, i.e. inputs in the home production are substitutes, then (a) $\partial n^*/\partial w_f$ is monotonically increasing in w_f and (b) strictly positive for a large enough w_f, i.e. $\lim_{w_f \to \infty} \partial n^*/\partial w_f > 0$.*

Proof. Proof of (a). Differentiating (C.3) with respect to w_f and omitting the positive constant term gives

$$\frac{\partial n^*}{\partial w_f} \propto \frac{(p_n - \eta p_e) - \left(w_f + w_m\right)\frac{\partial p_n}{\partial w_f}}{(p_n - \eta p_e)^2} = \frac{1 - \left(\frac{w_f + w_m}{w_f}\right)\mathcal{E}_{p_n,w_f}\frac{p_n}{p_n - \eta p_e}}{p_n - \eta p_e}.$$

The denominator is positive. To show that the ratio is monotonically increasing, it suffices to show that the negative term in the numerator is made up of positive and monotone decreasing functions of w_f. This is seen from obtaining a negative derivative for each of the product terms:

$$\frac{\partial}{\partial w_f}\left(\frac{w_f + w_m}{w_f}\right) = -\frac{w_m}{w_f^2} < 0,$$

$$\frac{\partial}{\partial w_f}\mathcal{E}_{p_n,w_f} = \frac{\left(\frac{\rho}{\rho-1}\right)\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}-1}}{\left(\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}} p_m^{\frac{\rho}{\rho-1}}\right)^2} < 0, \text{ when } \rho \in (0,1),$$

and

$$\frac{\partial}{\partial w_f}\left(\frac{p_n}{p_n - \eta p_e}\right) = \frac{-\eta p_e \frac{\partial}{\partial w_f} p_n}{(p_n - \eta p_e)} < 0,$$

where the last inequality follows from showing that $\frac{\partial}{\partial w_f} p_n > 0$, as can be seen from Equation (C.4).

(b) Proof of (b). Because the limit of a product of functions is equal to the product of limits, we obtain

$$\lim_{w_f\to\infty}\left(\frac{w_f+w_m}{w_f}\right)\mathcal{E}_{p_n,w_f}\frac{p_n}{p_n-\eta p_e}=\lim_{w_f\to\infty}\left(\frac{w_f+w_m}{w_f}\right)\cdot\lim_{w_f\to\infty}\mathcal{E}_{p_n,w_f}\cdot\lim_{w_f\to\infty}\frac{p_n}{p_n-\eta p_e}.$$

Each limit can then be obtained. First,

$$\lim_{w_f\to\infty}\left(\frac{w_f+w_m}{w_f}\right)=1.$$

Second,

$$\lim_{w_f\to\infty}\mathcal{E}_{p_n,w_f}=\lim_{w_f\to\infty}\frac{\phi^{\frac{1}{1-\rho}}}{\phi^{\frac{1}{1-\rho}}+(1-\phi)^{\frac{1}{1-\rho}}\left(\frac{p_m}{w_f}\right)^{\frac{\rho}{\rho-1}}}=0,$$

which follows from $\lim_{w_f\to\infty}\left(\frac{p_m}{w_f}\right)^{\frac{\rho}{\rho-1}}=\infty$ implied by the assumption that $\rho\in(0,1)$. To derive the final limit, we first note that

$$\lim_{w_f\to\infty}p_n=\lim_{w_f\to\infty}\frac{1}{A}\left[\phi^{\frac{1}{1-\rho}}w_f^{\frac{\rho}{\rho-1}}+(1-\phi)^{\frac{1}{1-\rho}}p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}=\frac{1}{A}(1-\phi)^{-\frac{1}{\rho}}p_m>0$$

whenever $\rho\in(0,1)$. It follows that the third limit is

$$\lim_{w_f\to\infty}\frac{p_n}{p_n-\eta p_e}=\frac{\frac{1}{A}(1-\phi)^{-\frac{1}{\rho}}p_m}{\frac{1}{A}(1-\phi)^{-\frac{1}{\rho}}p_m-\eta p_e}\in(1,\infty).$$

The product of these three limits is zero. It follows that

$$\lim_{w_f\to\infty}\frac{\partial n^*}{\partial w_f}\propto\lim_{w_f\to\infty}\frac{1-\left(\frac{w_f+w_m}{w_f}\right)\mathcal{E}_{p_n,w_f}\frac{p_n}{p_n-\eta p_e}}{p_n-\eta p_e}=\frac{1}{(1-\phi)^{-\frac{1}{\rho}}p_m-\eta p_e}>0.$$

■

Corollary 1 *In the region of w_f where the solution is interior, n^* is either U-shaped or monotonically increasing in w_f.*

Proof. Observe from (C.2) that e^* is monotone increasing in w_f whenever e^* is interior. Thus, there exists a well-defined lowest wage marking the interior solution $\underline{w_f} \equiv \inf \{w_f | e^* (w_f) > 0\}$. If $\partial n^* / \partial w_f \left(\underline{w_f} \right) \geqslant 0$, i.e. n^* is non-decreasing near $\underline{w_f}$, then we know by proposition 2 that $\partial n^* / \partial w_f > 0$ for larger w_f. In this case, n^* is strictly increasing in the region of $w_f \geqslant \underline{w_f}$. If, however, $\partial n^* / \partial w_f \left(\underline{w_f} \right) < 0$, i.e. n^* is decreasing in w_f near w_f, then we know by proposition 2 that $\partial n^* / \partial w_f$ will monotonically increase with w_f becoming strictly positive for a large enough w_f. In this case, n^* is a U-shape function of w_f in the region of $w_f \geqslant \underline{w_f}$. ■

D Education Robustness

There has been increasing interest in rising returns to education and rising education costs in the literature. We have so far abstracted from these issues, using the empirical relationship between income and college attainment in 1980 in order to control for changing education rates over time, instead focusing on differential fertility. Is it possible, however, that changes in college returns and costs could be driving changes in differential fertility? In principle, rising education costs could lead to more fertility through a quantity-quality tradeoff, potentially yielding changing patterns of fertility by income. This effect might be mitigated by rising returns to education.

We now allow both the college premium ω, as described in Equation (1), and education costs (p_e) to change over time. Relative to our main experiment for 2010, described in Section 4.4, we only need to describe two things: how p_e changes over time and how ω changes.

Children born in 1980 attended college roughly between 1998 and 2002. Thus, we use 2000 as the year to determine costs and returns to college for the 1980 cohort of children. As the 2010 cohort of children has not yet gone to college, we use the 2015 college costs and premium for this cohort. Beginning with p_e, we normalize $p_{e,1980} = 1$ as before. Although education expenditures map into all possible education-related expenditures per child, we take the stance that college education cost changes accurately describe general changes over time. We therefore choose to proxy the increase in the price of education by the increase in the effective price of college. Using institutional survey data available through the National Center for Education Statistics, we obtain that an annual cost of a public 4-year college is approximately \$6,400 in 2000. This includes tuition and room & board, net of grants and scholarships. This quantity for the most recent year available is \$7,887, an increase of a 22%. We thus set $p_{e,2010} = 1.22$. ω in our model captures the lifetime return to college. This is different from the lifetime college premium which simply refers to the observed difference between the earnings of college graduates and other workers. Thus, we set $\omega_{1980} = 1.29$ and $\omega_{2010} = 1.31$, representing a 29% and 31% college premium, respectively (Valletta Forthcoming).[50] We use the same calibration of the model when performing this robustness exercise.[51]

The results are quite similar. High income fertility increases by 46% (as opposed to 43.5% in the benchmark model and 40% in the data). In the model, college attainment due to differential fertility rises modestly, by about 2.2 percentage points, but when recalculating holding the cost of marketization constant, this statistic falls by 1.6 percentage points, leading to a total bias from ignoring marketization of about 3.8 percentage points. This result is quantitatively similar to the main exercise.

E Normalization of Parameters

E.1 Normalizing p_e

Notice that in our model we can normalize $p_e = 1$ (or any other value), without affecting other meaningful quantities which are mapped to the data. Precisely, we will show that, for any change in p_e, parameters of the college attainment

[50]Notice that this small increase does not imply that there was a small increase in the returns to education in our time period, simply that the returns to education of children has not changed much. The inequality studied in this paper is focused on the parental generation, that is people who earned wages in 1980 and 2010, rather than people who were born in those years. The increase in inequality for the parental generation was driven at least partially by a large change increase in returns to college.

[51]Note that this involves setting $\tilde{\beta} = 2.63$. Recall from Section 3 that $\beta = \tilde{\beta}\ln(\omega)$. With this parameterization, β is the same as in the benchmark.

function π can be adjusted to ensure expenditures $p_n n$, $p_e en$ and our targeted moments π, n, $p_m m$, and t_f remain unchanged. At the interior solution we have

$$\begin{aligned} e &= \frac{p_n}{p_e}\frac{\beta\theta}{\alpha} - \eta, \text{ or} \\ p_e e &= p_n \frac{\beta\theta}{\alpha} - p_e \eta. \end{aligned}$$

The last equation shows that scaling up p_e by any factor, say ε, requires reducing e and η by the same factor to keep the expenditure on education $p_e e$ unchanged:

$$\begin{aligned} p_e e &= p_e \varepsilon \frac{e}{\varepsilon}, \\ p_e \eta &= p_e \varepsilon \frac{\eta}{\varepsilon}. \end{aligned}$$

It is then seen from Equation (13) that the solution to n will not change due to the scaling above as the product ηp_e is unchanged, so is p_n. Note that e itself has no data counterpart, but the quantity $\pi(e)$ is used to target college attainment rates in the data. However, the parameters inside $\pi(\cdot)$ can be scaled as follows, to keep it unchanged:

$$\pi\left(\frac{e}{\varepsilon}\right) = \ln\left(b\varepsilon^{\theta}\left(\frac{e}{\varepsilon} + \frac{\eta}{\varepsilon}\right)^{\theta}\right) = \ln\left(b\left(e+\eta\right)^{\theta}\right) = \pi(e).$$

Thus, the solution to the model, in terms of n and $\pi(e)$, is invariant to the following transformation of parameters:

$$\tilde{p}_e = p_e \varepsilon, \tilde{\eta} = \frac{\eta}{\varepsilon}, \tilde{b} = b\varepsilon^{\theta}, \tilde{e} = \frac{e}{\varepsilon} \;\; \forall \varepsilon > 0,$$

The remaining targets are $p_m m$ and t_f. It is seen from Equations (9) and (10) that these quantities remain unchanged as n remains the same.

E.2 Normalizing p_m

In this section we show that we can normalize p_m to any value, without affecting any of the meaningful quantities that have a data counterpart: expenditures $p_n n$,

$p_e en$ and our targeted moments: π, n, $p_m m$, and t_f. The solution to p_n given in Equation (11):

$$
\begin{aligned}
p_n &= \frac{1}{A}\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \\
&= \left[A^{\frac{\rho}{1-\rho}} \phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + A^{\frac{\rho}{1-\rho}} (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}.
\end{aligned}
$$

First we show that when scaling p_m by $\varepsilon > 0$, we can find adjustments to A and ϕ to keep p_n unchanged. Let the adjusted parameters be $\tilde{A}$ and $\tilde{\phi}$. We find them by requiring that p_n remains unchanged:

$$
\tilde{A}^{\frac{\rho}{1-\rho}} \tilde{\phi}^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + \tilde{A}^{\frac{\rho}{1-\rho}} \left(1-\tilde{\phi}\right)^{\frac{1}{1-\rho}} (p_m \varepsilon)^{\frac{\rho}{\rho-1}} = A^{\frac{\rho}{1-\rho}} \phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + A^{\frac{\rho}{1-\rho}} (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}.
$$

For this to hold, $\tilde{A}$ and $\tilde{\phi}$ must satisfy the following two equations:

$$
\begin{aligned}
\tilde{A}^{\frac{\rho}{1-\rho}} \tilde{\phi}^{\frac{1}{1-\rho}} &= A^{\frac{\rho}{1-\rho}} \phi^{\frac{1}{1-\rho}}, \\
\tilde{A}^{\frac{\rho}{1-\rho}} \left(1-\tilde{\phi}\right)^{\frac{1}{1-\rho}} \varepsilon^{\frac{\rho}{\rho-1}} &= A^{\frac{\rho}{1-\rho}} (1-\phi)^{\frac{1}{1-\rho}}
\end{aligned}
$$

Dividing through allows us to find $\tilde{\phi}$.

$$
\begin{aligned}
\frac{\tilde{A}^{\frac{\rho}{1-\rho}} \left(1-\tilde{\phi}\right)^{\frac{1}{1-\rho}} \varepsilon^{\frac{\rho}{\rho-1}}}{\tilde{A}^{\frac{\rho}{1-\rho}} \tilde{\phi}^{\frac{1}{1-\rho}}} &= \frac{A^{\frac{\rho}{1-\rho}} (1-\phi)^{\frac{1}{1-\rho}}}{A^{\frac{\rho}{1-\rho}} \phi^{\frac{1}{1-\rho}}}, \\
\left(\frac{1-\tilde{\phi}}{\tilde{\phi}}\right)^{\frac{1}{1-\rho}} &= \left(\frac{1-\phi}{\phi}\right)^{\frac{1}{1-\rho}} \varepsilon^{\frac{\rho}{1-\rho}}, \\
\frac{1-\tilde{\phi}}{\tilde{\phi}} &= \left(\frac{1-\phi}{\phi}\right) \varepsilon^{\rho}, \\
\tilde{\phi} &= \frac{1}{1+\left(\frac{1-\phi}{\phi}\right)\varepsilon^{\rho}} = \frac{\phi}{\phi + (1-\phi)\,\varepsilon^{\rho}} \in [0,1].
\end{aligned}
$$

Finally, solving for $\tilde{A}$ gives

$$\begin{aligned} \tilde{A}^{\frac{\rho}{1-\rho}} &= A^{\frac{\rho}{1-\rho}} \left(\frac{\phi}{\tilde{\phi}}\right)^{\frac{1}{1-\rho}} = A^{\frac{\rho}{1-\rho}} \left[\phi + (1-\phi)\, \varepsilon^{\rho}\right]^{\frac{1}{1-\rho}}, \\ \tilde{A} &= A \left[\phi + (1-\phi)\, \varepsilon^{\rho}\right]^{\frac{1}{\rho}}. \end{aligned}$$

Thus, scaling p_m by a factor $\varepsilon > 0$, and adjusting the share parameter and productivity as above, keeps p_n fixed. Since p_n is unchanged, the model solution remains the same, and so do the targeted moments for n and π.

It remains to show that the targeted moments t_f and $p_m m$ also remain unchanged. We express t_f and m in terms of p_n:

$$(A p_n)^{\frac{1}{\rho-1}} = \left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}.$$

Plug the bracketed term into t_f and m

$$\begin{aligned} t_f &= \frac{\left(\frac{\phi}{w_f}\right)^{\frac{1}{1-\rho}}}{A\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} n = \frac{\left(\frac{\phi}{w_f}\right)^{\frac{1}{1-\rho}}}{A\,(A p_n)^{\frac{1}{\rho-1}}} n = \frac{\left(\frac{1}{w_f}\right)^{\frac{1}{1-\rho}}}{A^{\frac{\rho}{\rho-1}} \phi^{\frac{1}{\rho-1}} p_n^{\frac{1}{\rho-1}}} \\ m &= \frac{\left(\frac{1-\phi}{p_m}\right)^{\frac{1}{1-\rho}}}{A\left[\phi^{\frac{1}{1-\rho}} w_f^{\frac{\rho}{\rho-1}} + (1-\phi)^{\frac{1}{1-\rho}}\, p_m^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}} n = \frac{\left(\frac{1-\phi}{p_m}\right)^{\frac{1}{1-\rho}}}{A\,(A p_n)^{\frac{1}{\rho-1}}} n = \frac{\left(\frac{1}{p_m}\right)^{\frac{1}{1-\rho}}}{A^{\frac{\rho}{\rho-1}} (1-\phi)^{\frac{1}{\rho-1}}\, p_n^{\frac{1}{\rho-1}}} \end{aligned}$$

We showed that the term $A^{\frac{\rho}{\rho-1}} \phi^{\frac{1}{\rho-1}}$ is unchanged due to scaling of p_m, which means that t_f is unchanged. However, the term $A^{\frac{\rho}{1-\rho}} (1-\phi)^{\frac{1}{1-\rho}}$ increases by a factor of $\varepsilon^{\frac{\rho}{1-\rho}}$. Thus, the effect of scaling p_m by a factor of $\varepsilon > 0$, and adjusting A and ϕ to keep p_n constant, gives:

$$m p_m = \frac{\left(\frac{1}{p_m \varepsilon}\right)^{\frac{1}{1-\rho}} (p_m \varepsilon)}{A^{\frac{\rho}{\rho-1}} (1-\phi)^{\frac{1}{\rho-1}}\, \varepsilon^{\frac{\rho}{1-\rho}} p_n^{\frac{1}{\rho-1}}} = \frac{p_m^{\frac{\rho}{1-\rho}} \varepsilon^{\frac{\rho}{1-\rho}}}{A^{\frac{\rho}{\rho-1}} (1-\phi)^{\frac{1}{\rho-1}}\, \varepsilon^{\frac{\rho}{1-\rho}} p_n^{\frac{1}{\rho-1}}} = \frac{p_m^{\frac{\rho}{1-\rho}}}{A^{\frac{\rho}{\rho-1}} (1-\phi)^{\frac{1}{\rho-1}}\, p_n^{\frac{1}{\rho-1}}}$$

Notice that ε cancels out, and therefore does not affect mp_m.