Essays on the Structure of Financial Markets

A thesis presented

by

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Abstract

Chapter I: Adverse selection in an insurance market may result in low-risk individuals remaining uncovered. In the framework of a monopolistic insurance market with private information, it is shown that government entry to the market as a competitor which sells insurance, results in all potential buyers actually purchasing insurance.
Chapter II: The welfare trade off between reduction in risk and enhanced market power, as depository institutions become larger but fewer, is studied. The main result is that when there are enough independent risks in the economy, it is possible to achieve high diversification through mergers between depository institutions at a very small cost in terms of greater market power.
Chapter III: Firms wishing to issue securities on the stock market are required to disclose private information which might be beneficial to competitors. Issuing securities publicly is more costly than doing so privately. In equilibrium, firms with sensitive information issue securities privately, while competitors cannot unambiguously infer that the information withheld is very sensitive. This suggests that one special role of banks and venture capital in financial markets, is to provide debt and equity financing, respectively, confidentially.
Chapter IV: A decentralized market with pairwise meetings of agents is studied. There is a one sided information asymmetry regarding the state of the world, which may be "low" or "high." The set of equilibria of the model is characterized, and its behavior is studied as the market becomes approximately frictionless. For any one sided information economy there is an equilibrium where trade occurs at the right price (an ex-post individually rational price). Moreover, there is an open set of economies where in all the equilibria trade occurs at the right price.

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Table of Contents

INTRODUCTION

Financial markets are more fragile than other markets. The objects which are traded in these markets are not ordinary commodities. They are contracts such as stocks, stock options, bonds, or certificates of deposit, which are all intrinsically valueless promises to deliver physical commodities (or other intrinsically valueless promises) at some future date, possibly contingent on some event which may or may not occur. The value of such promises is affected by the uncertainties associated with their fulfillment.

Often enough these uncertainties are sufficiently important for a promise to be traded at a premium - or at a discount, it all depends on one's perspective, reflecting the risk involved in accepting the promise in exchange for a physical commodity or money (which is nothing but another promise, one we usually tend to believe). These premia are a manifestation of the fact that, in comparison with a world where promises are always kept to the word, financial markets are often inefficient.

There are promises which it never pays to keep. This suggests that certain contracts, although potentially beneficial to all parties, are never made. The market for many financial assets is simply missing.

The inefficiency and incompleteness of financial markets is of particular concern to economists because the smooth operation of these markets is essential for the functioning of most other markets in the economy. Any insurance or risk sharing agreement, any intertemporal reallocation of resources (saving, borrowing and lending,) involves trade in financial assets. Moreover, most of the transactions in the economy are carried out with financial instruments (checks, money orders, credit cards, letters of credit).

Much of the research on financial markets has focused on identifying and understanding the problems which lead to their failure. Two major types of such problems are the presence of transaction costs and of asymmetric information (which can also be regarded as some sort of transaction cost - information can be made symmetric at a cost, sometimes at an infinite cost). Other strands of the literature have focused on the role of government in facilitating the operation of financial markets. The government can improve the efficiency of financial markets and encourage the opening of new ones by creating the right legal environment (laws, regulations, regulatory institutions and procedures, quality and speed of the judicial system). Macroeconomists have focused on the role of government and the central bank in promoting stability of prices and expectations, in maintaining the public's confidence in the financial system, in ensuring that there is enough money, and sufficient availability of credit for the smooth functioning of the economy. Optimal taxation theory has certainly not neglected financial markets. The debate on the desirability of capital gains taxes is a prominent example.

Less research has been done on the importance of the market structure of financial markets for their smooth operation. If the structure of financial markets indeed matters, the government may exert its influence on their operation by affecting their structure. The essays in this volume make a contribution to this line of research.

The first chapter focuses on a monopolistic insurance market suffering from an adverse selection problem. In such a market the buyers with the lower valuation for insurance, the low risk buyers, may be left out of the market with no coverage. The government may deal with this situation by entering the market as a competitor which sells insurance. It is shown that this results in all potential buyers actually purchasing insurance. Such regulation by entry, where a privately owned firm co-exists in the market with a public firm, reduces the inefficiencies caused by adverse selection.

The second chapter asks whether the size of financial intermediaries may compensate for missing insurance markets. The claim is that large depository institutions can more easily diversify their portfolio of loans. A small number of large depository institutions, though, may enjoy market power. The welfare trade off between reduction in risk and enhanced market power, as depository institutions become larger but fewer, is studied. The main result is that when there are enough independent risks in the economy, it is possible to achieve high diversification through mergers between depository institutions at a very small cost in terms of enhanced market power. When the number of independent risks in the economy is not large, the optimal market structure of depository institutions cannot be unambiguously determined, but the trade off between diversification and market power is still an important consideration. Some degree of market power for depository institutions may be socially desirable. This, of course, has implications for regulators who have a great deal of influence on the competitiveness of and the risk sharing arrangements between financial institutions.

The monitoring role of banks and venture capital is well understood. The third chapter points to another special role that banks, venture capital, and other institutions which provide cash to firms in exchange for private placements of securities, may have in financial markets. Firms wishing to issue securities on the stock market are required to disclose private information which might be beneficial to competitors. Firms with sensitive information may prefer to issue securities privately. As issuing securities publicly is more costly than doing so privately, competitors will not be able to unambiguously infer that the information withheld is very sensitive. This suggests that one special role of banks and venture capital in financial markets, is to provide debt and equity financing, respectively, confidentially.

The need for confidentiality arises due to the disclosure regulations, which in turn are a result of asymmetric information between borrowers and lenders, and between purchasers and issuers of stock. Without disclosure regulations stock markets may function less well. Securities which are traded when there are disclosure regulations might not be traded in the absence of such regulations. The effort by regulators to enhance the efficiency of stock markets and render them more complete creates a demand for a new, differentiated kind of service - financial intermediation away from the public eye.

Stock markets are centralized markets. The auctioneer in these markets is a real person (or a computer), not a metaphor. Banks, insurance companies, and other financial intermediaries operate in a decentralized manner. An important question in this context (originally formulated by A. Wolinsky) is whether, in a decentralized market with asymmetric information, where transactions are concluded in pairwise meetings of agents, and prices are not called, the process of trade fully reveals the private information of informed agents to the uninformed when the market becomes approximately frictionless, that is when agents become almost infinitely patient. In other words, the question is whether a frictionless decentralized market approximates an informationally efficient centralized market. This question is studied in the fourth chapter. Wolinsky showed that when there are uninformed agents amongst sellers and buyers information is not fully revealed to the uninformed. In contrast, it is shown in the fourth chapter that when one side of the market is informed, there is a wide class of economies where information is transmitted to the uninformed through the process of trade.

CHAPTER I

GOVERNMENT PROVISION OF INSURANCE[1]

1 Introduction

Approximately one sixth of the population under age sixty-five in the U.S. lack medical insurance coverage [Brown (1990), Davis (1989), Fein (1989), Gruber and Krueger (1990)]. One possible explanation for this phenomenon is that insurance companies offer contracts which separate unobservable types in the population, differing in their probabilities of accident [Rothschild and Stiglitz (1976), Stiglitz (1977)]. This results in high risk individuals purchasing full coverage and low risk individuals purchasing partial or no coverage.

This explanation is consistent with the available data on health insurance for the U.S. For example, amongst the uninsured who are older than 18, one half are less than 24 years old [Brown (1990)]. Thus, the common impression that the uninsured consist primarily of "high risk individuals who get rejected by insurance companies," is unfounded.[2] Also, over one half of the uninsured are in families where at least one family member has a full-time job [Davis (1989)]. This fact contradicts the view that the uninsured are "the very poor and unemployed" (In fact, the very poor are covered by Medicaid). The uninsured are definitely not "the old who did not save when they were young," as every individual above 65 years of age is covered by Medicare. What we have in mind is a young or middle aged individual, with a modest income, say $20,000 a year, who considers himself relatively healthy, and

[1] Joint work with Francesca Cornelli.

[2] Insurance companies may refrain from overtly discriminating customers according to their age for fear of the reaction of the regulatory authorities or the press. Instead they discriminate covertly by offering insurance policies which many young healthy people find too expensive.

does not wish to spend somewhere between \$2000-\$6000 to cover himself or his family.[3]

This situation calls for government intervention on the following grounds. First, although the medically uninsured receive some medical care, this care tends to be emergency care rather than routine and preventive care. Detection and treatment occur late, and the overall health state of the uninsured population (and the work-force) is worse than if they had access to routine health care. Second, there may be positive externalities associated with health insurance. For example, society may care "about preventing the spread of contagious diseases more than any individual does or would take account of," [Summers (1989), p.178].

A different and completely independent line of reasoning advocates government intervention on social fairness or social justice grounds. If information regarding the probabilities of incurring a loss were not private, or alternatively if there was only one type of individuals, everybody would be fully covered. When the probabilities of loss differ across individuals, and are private information, low risk individuals are offered a partial coverage contract, in order to prevent the high risk individuals from misrepresenting their type. This is an externality which high risk individuals impose on low risk individuals, altering the opportunities available to the latter in the market for insurance. According to this view, the government should intervene in order to facilitate the purchase of insurance by low risk individuals. Others simply regard certain services, such as health care, as indispensable services which every citizen must have access to.

[3] Note that from the perspective of such an individual this is a perfectly rational decision. The price of insurance has been set at a level which is too high for him as a result of the presence of unidentifiable people with a higher probability of becoming ill. However, it is also plausible that some people do not purchase insurance because they "irrationally underestimate the probability of catastrophic health expenses," [Summers (1989), p.178]. We focus on the adverse selection phenomenon described above.

It turns out that on this particular issue there is no contradiction between the two approaches. Setting aside moral hazard considerations (which we ignore in this paper), both would favor higher coverage for the uninsured.

Governments deal with the existence of uninsured people by heavily regulating the insurance industry, or by providing the services universally, financing the cost through taxation. Examples of such services are survivor benefits - a form of life insurance-, income compensation payments to workers who have been hurt on their jobs- a form of disability insurance-, and provision of health services at low or no cost- a direct substitute for medical insurance.

In this paper we study an alternative solution, arguing that government entry to the insurance market *as a competitor which sells insurance* results in increased coverage for individuals with no (or with partial) coverage, without loss in expected utility for any buyer of insurance.[4] We endow the government with preferences which are different than profit maximization, and we constrain it not to lose money on average. As we use a partial equilibrium setting, this is a reasonable assumption. If the government were permitted to cross-subsidize the insurance sector with funds from other sectors, the partial equilibrium setting would cease to be appropriate. One would have to evaluate the shadow cost of such intersectoral transfers.

Our analysis is related to the literature on public firms, and in particular public firms which coexist with private firms in the same market, possibly endowed with an objective function different than profit maximization (e.g. maximization of social surplus). See Beato and Mas-Colell (1984) and the references therein.

We turn to a description of the government entry process and how it succeeds in increasing the coverage of the uninsured.

[4]The entry of the government can be regarded as the creation of a public firm which enters the market and competes.

Consider a monopolistic insurance market, as in Stiglitz (1977).[5] We assume that the monopolist separates types by offering low risk individuals no (rather than partial) insurance. This is the extreme case which is in accordance with reality, and which most strongly calls for government intervention.[6]

The government is assumed to possess no informational advantage over the monopolist. Thus, following Holmström and Myerson (1983), the equilibrium is *interim incentive efficient*. This means that since the buyers of insurance privately know their own type, a planner cannot implement an allocation where all agents are better off. It follows that any intervention is bound to hurt someone. If the government wishes to intervene nevertheless, for the reasons presented above, the natural candidate for bearing the burden is the monopolist. This is in fact the case in our model. Yet, we shall show that the intervention scheme we propose attains an improvement in the Hicks - Kaldor sense.

We model government entry to the insurance market as a game, for which we characterize the subgame-perfect equilibria. We assign to the government the role of the Stackelberg leader. A possible justification for this timing is that the government's ability to commit to its policy is greater than the monopolist's, possibly because a change in government policy requires legislation. In all the subcases we consider, government entry results, for both types of individuals, in an expected utility level at least as high as in the pre-entry situation. The degree of coverage for the low risk individuals is increased in all subcases, while the high risk individuals

[5] The insurance industry in the U.S. is not monopolistic of course. We think of it as being monopolistically competitive. The monopolistic case should be regarded as a benchmark case.

[6] The analysis can easily be generalized to the case where low risk individuals are offered partial coverage and the monopolist serves both types of individuals making money on both. The analysis does not apply to the case where the number of high risk individuals is very small and the monopolist finds it optimal to cross subsidize high risk individuals, on which it loses money, with profits from contracts with low risk individuals. However, in that event it is hard to make the case for government intervention in the first place.

remain fully covered.

The central feature of the intervention policy is that in equilibrium the government provides full coverage to the high risk individuals, at zero profits, and the monopolist provides insurance to the low risk individuals. Note that prior to the intervention the low risk individuals were without insurance, and the high risk individuals were being served by the monopolist. When it enters the market, the government attracts the high risk individuals with an insurance policy that only they find lucrative. Having lost its high risk clients, the monopolist serves the remaining people in the market, which are of the low risk type.

At first glance it may seem unintuitive that the government should target the segment of the population which is already being served by the monopolist. The point is that targeting the uninsured segment, the low risk individuals, will not always work: Any contract accepted by low risk individuals will also be purchased by the high risk individuals. The monopolist will react by attracting those buyers of insurance which are profitable. The government may remain with the non-profitable buyers, losing money. Therefore the government must target the high risk individuals, as explained above. A similar approach may be appropriate whenever the government decides to intervene in markets which are "thin" as a result of an adverse selection problem: The government should attract the "lemons," leaving the rest of the clients for the private sector.[7]

In section 2 we formulate the model and in section 3 we define and solve the

[7]Bodie (1989, p.10) points out that the market for life annuity contracts suffers from an adverse selection problem, as people with above than average life expectancy tend to have a high valuation of annuities. The result is that the average individual finds the price of this kind of insurance unattractive. David Weil, in a private conversation, has suggested that a similar phenomenon occurs in the market for mortgages, where the presence of high risk borrowers renders the interest rate too high for the average borrower. The policy we propose in this paper can be adapted to these markets.

game. Section 4 is dedicated to a welfare analysis of the equilibrium. In section 5 and 6 we briefly address other, more traditional, forms of government intervention such as taxation. Section 7 concludes.

2 Model Formulation and the Pre-Intervention Equilibrium

Formulation

There are two types of individuals, differing only in the probability of having an accident (which can be identified with disability or any illness which affects income). They have an initial endowment of W; the loss, if an accident occurs, is ℓ, $W-\ell > 0$, and the probabilities of accident are p_H and p_L for high and low risk individuals respectively, $p_H > p_L$. All individuals are expected utility maximizers, with the same von Neumann-Morgenstern utility function $U(\cdot)$, satisfying $U'(\cdot) > 0$, $U''(\cdot) < 0$. The reservation utility level, $\bar{U}_i$, for an individual of type $i = L, H$ is defined by

$$\bar{U}_i = (1 - p_i)U(W) + p_i U(W - \ell). \tag{1}$$

At any point (C^{NA}, C^A) in consumption space the absolute value of the slope of an indifference curve of an individual of type $i = L, H$ is $\frac{(1-p_i)U'(C^{NA})}{p_i U'(C^A)}$, the indifference curve of a low risk individual being steeper. The superscripts "NA" and "A" denote "No Accident" and "Accident" respectively. Let μ_i denote the proportion of type $i = L, H$ individuals in the population.

An insurance contract is a pair of numbers: The first is $[W - C^{NA}]$, the payment to the monopolist in the event of no accident; The second is $[C^A - (W - \ell)]$, the payment by the monopolist if an accident occurs. As such a contract is fully characterized by (C^{NA}, C^{A}), we abbreviate notation by calling points in consumption space "contracts."

The monopolist knows the number of individuals of each type, and the probabilities of accident, but cannot distinguish between types. It maximizes expected profits subject to individual rationality and incentive compatibility constraints in the usual manner.

Expected profits from a contract (C_i^{NA}, C_i^{A}) with an individual of type $i = L, H$ are

$$E\tilde{\pi}_i = (1 - p_i)[W - C_i^{NA}] - p_i[C_i^A - (W - \ell)]. \tag{2}$$

This is the equation of a negatively sloped line in consumption space; the absolute value of its slope is $\frac{1-p_i}{p_i}$, with $\frac{1-p_L}{p_L} > \frac{1-p_H}{p_H}$. Setting expected per capita profits to zero in the iso-expected profits lines in (2), we see that the lines pass through the endowment point $(W, W - \ell)$. We denote them by $(\pi = 0)_L$ and $(\pi = 0)_H$ (see figure 1). From (2) it follows that expected profits are positive to the left and below these lines, and are negative to the right and above them.[8] From now on we will simply say "profits."

Profits from Full Insurance Contracts

In what follows we shall be particularly concerned with full insurance contracts, which are points on the 45° line in consumption space. Substituting (C_i, C_i) for consumptions in (2) yields, for $i = L, H$,

$$E\tilde{\pi}_i = W - p_i\ell - C_i. \tag{3}$$

[8] Average per capita expected profits from serving all buyers of insurance with the same contract (C_μ^{NA}, C_μ^{A}) are, using (2), $E\tilde{\pi} = \mu_L E\tilde{\pi}_L + \mu_H E\tilde{\pi}_H = (1-p)[W - C_\mu^{NA}] - p[C_\mu^A - (W - \ell)]$, where $p = \mu_L p_L + \mu_H p_H$. The absolute value of the slope of this line is $\frac{1-p}{p}$, with $\frac{1-p_L}{p_L} > \frac{1-p}{p} > \frac{1-p_H}{p_H}$.

From (3) we see that per capita profits decrease linearly with consumption along the 45° line. Also, for any contract on the 45° line, we have $E\tilde{\pi}_L > E\tilde{\pi}_H$.

The Pre-Intervention Equilibrium

Stiglitz (1977). The monopolist will offer two distinct contracts: a full insurance contract, purchased by the high risk individuals, and a partial or no insurance contract, depending on the parameter values of the model, purchased by the low risk individuals. More precisely, for given W, ℓ, p_L and p_H, there is a critical μ_L below which low risk individuals are offered a no-insurance contract. This can be seen as follows.

It is shown in Stiglitz (1977) that the solution to the monopolist's program satisfies (a) $C_H^{NA} = C_H^A = C_H$, i.e. full insurance for high risk individuals, (b) The individual rationality constraint for low risk individuals binds, (c) The incentive compatibility constraint for high risk individuals binds, and (d) The incentive compatibility constraint for low risk individuals is redundant. From (a) and (c) we have

$$U(C_H) = (1 - p_H)U(C_L^{NA}) + p_H U(C_L^A), \tag{4}$$

and from (b) we have

$$(1 - p_L)U(C_L^{NA}) + p_L U(C_L^A) = \bar{U}_L. \tag{5}$$

Per capita profits are then given by

$$\begin{aligned} E\tilde{\pi} &= \mu_L E\tilde{\pi}_L + \mu_H E\tilde{\pi}_H = \qquad (6)\\ &= \mu_H(W - p_H\ell - C_H) + \mu_L\{(1 - p_L)[W - C_L^{NA})] - p_L[C_L^A - (W - \ell)]\}. \end{aligned}$$

Differentiating (4) and (5) with respect to C_H yields $\frac{dC_L^{NA}}{dC_H} = -\frac{p_L U'(C_H)}{(p_H - p_L)U'(C_L^{NA})} < 0$ and $\frac{dC_L^A}{dC_H} = \frac{U'(C_H)(1-p_L)}{U'(C_L^A)(p_H - p_L)} > 0$. Differentiating (6) with respect to C_H and substituting for $\frac{dC_L^{NA}}{dC_H}$ and $\frac{dC_L^A}{dC_H}$ yields

$$\frac{dE\tilde{\pi}}{dC_H}(C_H, C_L^{NA}, C_L^A) = \mu_L \frac{(1-p_L)p_L U'(C_H)}{p_H - p_L}\left(\frac{1}{U'(C_L^{NA})} - \frac{1}{U'(C_L^A)}\right) - \mu_H. \quad (7)$$

As long as the derivative in (7) is negative, the monopolist will decrease C_H, decreasing the coverage of low risk individuals. From (7) it follows that for small enough μ_L we have $\frac{dE\tilde{\pi}}{dC_H} < 0$ for any triplet (C_H, C_L^{NA}, C_L^A). This means that the monopolist will decrease C_H until the two individual rationality constraints bind, serving the high risk individuals at point F and the low risk individuals at the endowment point E (see figure 1).[9]

A useful result. By differentiating (7) we obtain $\frac{d^2E\tilde{\pi}}{dC_H{}^2} < 0$, which implies

Lemma 1 *If* $\frac{dE\tilde{\pi}}{dC_H}(\bar{C}_H, C_L^{NA}, C_L^A) < 0$ *for some* $\bar{C}_H$*, then* $\frac{dE\tilde{\pi}}{dC_H}(C_H, C_L^{NA}, C_L^A) < 0$ *for any* $C_H > \bar{C}_H$*.*

Assumption. We assume that

$$\frac{dE\tilde{\pi}}{dC_H}(C_H^{(F)}, W, W-\ell) = \mu_L \frac{(1-p_L)p_L U'(C_H^{(F)})}{p_H - p_L}\left(\frac{1}{U'(W)} - \frac{1}{U'(W-\ell)}\right) - \mu_H < 0, \quad (8)$$

where $C_H^{(F)} = U^{-1}(\bar{U}_H)$ is the consumption of a high risk individual at point F, which is the certainty equivalent to his endowment point. The assumption implies that the monopolist will offer to the low risk individuals no (rather than partial) insurance. The assumption is convenient and realistic (as there are people who

[9]Note that the reservation indifference curve of the low risk individuals, $\bar{U}_L$, which is not shown in the figure, could pass above or below point H (it must pass below point L by risk aversion.) These two cases will yield different equilibria in the game below.

purchase no insurance), but is not essential for the analysis.

Example. The example illustrates that the adverse selection effect is strong: A relatively small percentage of high risk individuals in the population is sufficient to induce the monopolist to offer no insurance to the low risk individuals.

Let $U(C) = \frac{C^{1-\rho}}{1-\rho}$, with coefficient of relative risk aversion $\rho = 2$. Take $p_H = 0.1$, $p_L = 0.05$, $W = 1$, $\ell = 0.2$, i.e. an accident such as a major medical problem entails a loss of 20% of wealth. Using $\mu_L + \mu_H = 1$ we get $\frac{dE\tilde{\pi}}{dC_H}(C_H^{(F)}, W, W - \ell) < 0 \Leftrightarrow \mu_L < 0.70$, i.e. it is sufficient that 30% of the population be of high risk for the monopolist to offer no insurance to the remaining 70% of the population.[10]

3 Government Provision of Insurance

In this section we study the effect of government entry to the insurance market. We assume that the government's ability to commit to its policy is greater than the monopolist's, possibly because a change in government policy requires legislation. Therefore we assign the government the role of the Stackelberg leader. The monopolist observes the government's move and then moves. The government is committed to its original move. We make the assumption that the monopolist will sell insurance contracts only if they make strictly positive profits, whereas the government will sell insurance as long as the resulting profits are non-negative.[11]

[10] If we set $p_H = 0.2$ the required percentage of high risk individuals becomes as low as 12%. When $p_H = 0.2$, $\ell = 0.2$, and $\rho = 3$ it is 15%.

[11] This is a simplifying assumption and is not essential for the analysis. Note that the assumption in (8) rules out the case in which the monopolist intentionally loses money on the high risk individuals.

Description of the Game

Players

The players are the monopolist and the government.

The buyers of insurance are numerous and do not play any strategic role in the game. Although the concern for their welfare and actions (purchase of insurance coverage) underlies the entry decision of the government, their formal role in the game is a technological one - their tastes impose restrictions on the actions of the players.

Payoffs

The monopolist maximizes (expected) profits.

The government cares, in a lexicographically decreasing order, about: (1) the expected utility levels of individuals and (2) the profits of the monopolist.[12]

This is a *partial ordering* of outcomes,[13] yet for our purposes it is sufficient, as the game is well defined, and the resulting equilibria are relatively simple and make economic sense.[14]

[12] For example, if in outcome I low risk individuals have the same expected utility level as in outcome II, but high risk individuals have an expected utility level which is higher than in II, then the government prefers I to II regardless of the level of the monopolist's profits.

[13] For example, if in outcome I the expected utility level of low risk individuals is higher than in outcome II, but for high risk individuals the opposite is true, then the outcomes cannot be ranked.

[14] The preferences of the player government should not be interpreted as a social welfare function. See the section on welfare.

Timing

date 1: The government decides whether to enter the insurance market, and offers one contract on the 45^o line, i.e. a full insurance contract.[15]

date 2: The monopolist, after having seen the contract offered by the government, offers any menu of contracts.

date 3: Every buyer of insurance chooses one contract. Previous contracts with the monopolist are rendered void, i.e. trading starts afresh from the endowment point $(W, W - \ell)$.

date 4: Nature moves, losses incurred by buyers of insurance are observed by all parties, contracts are executed, and consumption occurs.

Strategies

A strategy for the government consists of a choice of a full insurance contract anywhere on the 45^o line. In fact, prior to offering the contract the government decides whether to enter or not to enter. A decision not to enter is equivalent to the government offering a contract which no buyer of insurance would accept. Therefore we omit the explicit mentioning of the entry decision in the definition of the strategy.[16]

[15] As the government is restricted to offering full insurance contracts, limiting it to offering a single contract is without loss of generality. This is so because among any number of full insurance contracts there is always one which is preffered to all others by both low and high risk individuals.

[16] The rationale for constraining the government to offer a full insurance contract is that the primary motivation for government intervention was the fact that some individuals were not insured. It would seem natural to rule out the possibility, even on public opinion grounds, of the government itself offering partial coverage. In our model we rule out moral hazard. When the probabilities of having an accident are determined endogenously as a function of the degree of coverage, it is less appropriate to restrict the government to offering only full insurance contracts, as full insurance is not socially optimal. See Kaplow (1989).

For the reason presented in the introduction we constrain the government not to lose money. The profit or loss from a contract offered by the government depends on the response of the monopolist.

A strategy for the monopolist consists of a choice of a pair of (not necessarily distinct) contracts.[17] The monopolist can choose not to sell insurance. This is captured by a pair of contracts which no buyer of insurance will accept, so we need not mention this decision explicitly.

Equilibrium

The appropriate equilibrium concept for this game is that of *subgame-perfect equilibrium*. An equilibrium consists of (1) A full insurance contract, offered by the government, and (2) A pair of contracts offered by the monopolist, such that [A] Given the contract offered by the government, the monopolist maximizes profits, and [B] There is no other full insurance contract which the government can offer, that, given the best response of the monopolist and the decisions regarding purchase of insurance by individuals, does not lose money and yields an outcome preferred by the government.

Formal definition of equilibrium. Let

$$G = (C^{(G)}, C^{(G)}),$$
$$(M_L, M_H) = ((M_L^{NA}, M_L^A), (M_H^{NA}, M_H^A)) \tag{9}$$

be the contracts offered by the government and the monopolist respectively.

The menu of contracts from which buyers of insurance choose consists of the con-

[17] As low and high risk individuals are identical within their own type, the restriction to two contracts is without loss of generality.

tracts in (9) and the no-insurance contract $(W, W-\ell)$.
In order to simplify notation, let $V_i(M_j) = (1-p_i)U(M_j^{NA}) + p_i U(M_j^A)$, $i, j = L, H$.
Consider the following three programs:
[A1]

$$\max_{(M_L^{NA}, M_L^A),(M_H^{NA}, M_H^A)} \quad \begin{aligned} &\mu_L\{(1-p_L)[W - M_L^{NA}] - p_L[M_L^A - (W-\ell)]\} + \\ &\mu_H\{(1-p_H)[W - M_H^{NA}] - p_H[M_H^A - (W-\ell)]\} \end{aligned}$$

s.t.

$$V_L(M_L) \geq \max[U(C^{(G)}), \bar{U}_L] \tag{10}$$
$$V_H(M_H) \geq \max[U(C^{(G)}), \bar{U}_H] \tag{11}$$
$$V_L(M_L) \geq V_L(M_H) \tag{12}$$
$$V_H(M_H) \geq V_H(M_L). \tag{13}$$

A solution to [A1] is a profit maximizing menu of contracts such that both types purchase insurance from the monopolist.

[A2]

$$\max_{(M_L^{NA}, M_L^A)} \mu_L\{(1-p_L)[W - M_L^{NA}] - p_L[M_L^A - (W-\ell)]\}$$

s.t.

$$V_L(M_L) \geq \max[U(C^{(G)}), \bar{U}_L] \tag{10}$$
$$\max[U(C^{(G)}), \bar{U}_H] \geq V_H(M_L). \tag{14}$$

A solution to [A2] is a contract which maximizes profits and which will be accepted by the low risk individuals but not by the high risk individuals. Similarly for [A3]:

[A3]

$$\max_{(M_H^{NA}, M_H^A)} \mu_H\{(1-p_H)[W - M_H^{NA}] - p_H[M_H^A - (W - \ell)]\}$$

s.t.

$$V_H(M_H) \geq \max[U(C^{(G)}), \bar{U}_H] \tag{11}$$

$$\max[U(C^{(G)}), \bar{U}_L] \geq V_L(M_H). \tag{15}$$

Denote the maximal profits of [A1], [A2], and [A3] by Π_1, Π_2, and Π_3 respectively.

The contracts in (9) are a *subgame-perfect equilibrium* of the game if the following conditions are satisfied:

[A] The monopolist maximizes profits given the contract offered by the government. This means that if $\max[\Pi_1, \Pi_2, \Pi_3] = \Pi_1$ then the monopolist offers a pair of contracts which are a solution to [A1]; If $\max[\Pi_1, \Pi_2, \Pi_3] = \Pi_2$ then the monopolist offers a contract which is a solution to [A2], and similarly for [A3]. If $\max[\Pi_1, \Pi_2, \Pi_3] < 0$ then the monopolist offers contracts which no individual accepts.

[B] The government optimizes. This means that there is no other full insurance contract that the government can offer which, given the best response of the monopolist and the subsequent choice of contracts by buyers of insurance, does not lose money and yields an outcome which is preferred by the government.

We now turn to the solution of the game. We distiguish two cases, described in figures 2 and 3 respectively.

Case 1

Suppose both zero profits lines lie to the right of both indifference curves through the endowment point, as shown in figure 2. This case occurs when (1) The two types of individuals do not differ by much, i.e. $p_H - p_L$ is small, or (2) $U(\cdot)$ exhibits high risk aversion, i.e. both indifference curves are very flat at the endowment point. Formally, this case is characterized by the following condition:

$$C_L^{(J)} = U^{-1}(\bar{U}_L) < W - p_H \ell, \tag{16}$$

which says that the consumption of a low risk individual at point J, the certainty equivalent to his endowment point, is lower than consumption at point H. The equilibrium is characterized in

Proposition 1 *(a) The government serves the high risk individuals with the zero-profits contract* $(W - p_H\ell,\ W - p_H\ell)$*, which is point* H *in figure 2. (b) The monopolist serves the low risk individuals, at point* H*, with a contract which is infinitesimally more attractive for them than the contract offered by the government.*

Proof: The proof below is done with the aid of figure 2. A more formal proof is provided in the appendix.

Suppose the government offers point H. There is no contract which the high risk individuals prefer to H and that does not lose money. Thus, the best response of the monopolist is to attract only the low risk individuals, at H, where profits are maximal. The government will be left with the high risk individuals at point H, breaking even.[18]

[18]Recall that we have assumed that the monopolist sells insurance contracts only if they generate strictly positive profits. Without this assumption we would have the monopolist and the government splitting the market for high risk individuals, at point H.

We shall now show that offering any point other than H will either yield an outcome which is less preferred by the government, or will result in the government losing money.

Suppose the government offers a point such as X_2. By profit maximization, the monopolist will certainly not offer contracts such that both individual rationality constraints do not bind. Therefore the government prefers point H to the resulting outcome.

Suppose the government offers a point such as X_1. The best response of the monopolist is to attract all the buyers of insurance at point X_1. The government prefers point H to point X_1.

Suppose the monopolist offers a point such as X_3. The monopolist will respond by stealing away the low risk individuals at point X_3. The government will be left with the high risk individuals at point X_3, losing money.

Suppose the monopolist offers a point such as X_4. The monopolist will choose not to serve anyone, and the government will lose money. The proof is complete.

Case 2

Suppose the line $(\pi = 0)_H$ is less steep than $\bar{U}_L$ at point E, as shown in figure 3. This is the case where types are very different, i.e. $p_H - p_L$ is big. Formally, this case is characterized by the following condition:

$$C_L^{(J)} = U^{-1}(\bar{U}_L) > W - p_H\ell. \tag{17}$$

The equilibrium is characterized in

Proposition 2 *(a) The government serves the high risk individuals with the zero-profits contract* $(W - p_H\ell,\ W - p_H\ell)$*, which is point* H *in figure 3. (b) The mo-*

nopolist serves the low risk individuals, offering them the contract defined by

$(1-p_L)U(C_L^{NA})+p_LU(C_L^A)=\bar{U}_L$

$(1-p_H)U(C_L^{NA})+p_HU(C_L^A)=U(W-p_H\ell).$

This is point K in figure 3, a point on $\bar{U}_L$. It provides higher coverage than at the endowment point, but not full coverage.

Proof: The proof below is done with the aid of figure 3. A more formal proof is provided in the appendix.

Suppose the government offers point H. The best response of the monopolist is calculated as follows: First it is established, as in Siglitz (1977), that the high risk individuals will be offered a full insurance contract, say point X_6, and the low risk individuals will be offered the corresponding point on $\bar{U}_L$, which is point X'_6. Then, using our assumption in (8) and Lemma 1, we get that the monopolist will want to push the contract offered to the high risk individuals, X_6, along the 45$^{\text{o}}$ line, towards the origin (with point X'_6 sliding along $\bar{U_L}$ accordingly). Were it not for the contract offered by the government, the monopolist would offer points F and E, but as the government is offering point H, the monopolist will offer point K, serving the low risk individuals. The government will serve the high risk individuals at point H, breaking even.

If the government offers a point such as X_6, the monopolist, by the same reasoning as above, will attract the low risk individuals at X'_6. If the government offers X_7 the monopolist will attract the low risk individuals at X_7. In both cases the government will be left with the high risk individuals, losing money. Points such as X_5 and X_8 are ruled out in the same manner as in the proof of Proposition 1. This completes the proof.

Discussion

In case 2 the degree of coverage of low risk individuals has improved, but not all the way to full insurance. The intervention policy is less effective as compared to case 1, as the difference between types is big.

An interesting feature of the equilibria of cases 1 and 2 is the specialization of the monopolist in low risk buyers of insurance, and the shift of the high risk individuals from the monopolist to the government. This outcome conforms with a possible view regarding the "division of labor" between the government and the private sector, according to which the government should support the high risk population. The model resolves what may be regarded as a difficulty in the adverse selection explanation of the existence of uninsured individuals. This explanation predicts that *low* risk individuals will not purchase insurance, that is the low risk population is the one which needs government assistance, a result which may seem counter-intuitive. The shift of the high risk individuals from the monopolist to the government, when the latter enters, resolves this difficulty. The government is intentionally stealing from the monopolist its high risk clients. By doing so it is indirectly helping the low risk individuals whom the monopolist now finds it profitable to serve.

4 Welfare Analysis

As we have argued in the introduction, the pre - intervention equilibrium is *interim incentive efficient* in the sense of Holmström and Myerson (1983). That is, given that the government cannot distinguish between types, it is unable to reallocate consumption so as to make all agents (including the monopolist), better off. Yet, if we adopt the Hicks - Kaldor criterion, the equilibrium of the game, in both cases

1 and 2, is an improvement from the social standpoint: The buyers of insurance who have gained as a result of the government's intervention can compensate the monopolist and still be better off. This is seen as follows.

Case 1. Let t_H and t_L be defined by $U(C^{(H)} - t_H) = U(C^{(F)})$ and $U(C^{(H)} - t_L) = U(C^{(J)})$, where point J is defined as in (16) or (17).[19] Then, by (3) and the fact that profits from contracts with high risk individuals at point H, $E\tilde{\pi}_H^{(H)}$, are zero, we get $\mu_H t_H + \mu_L t_L - [\mu_H E\tilde{\pi}_H^{(F)} - \mu_L E\tilde{\pi}_L^{(H)}] = \mu_L E\tilde{\pi}_L^{(J)} > 0$.

Case 2. Similarly, $\mu_H t_H - [\mu_H E\tilde{\pi}_H^{(F)} - \mu_L E\tilde{\pi}_L^{(K)}] = \mu_L E\tilde{\pi}_L^{(K)} > 0$, where $E\tilde{\pi}_L^{(K)}$ is given by (2).

Interpreting these results we see that, in certainty equivalence terms, the monopolist has transferred all its profits from the high risk individuals, at point F, back to them. It is now making its profits from serving the low risk individuals. Therefore, the increase in social welfare is due entirely to the fact that the insurance coverage of the low risk individuals has increased: in Case 1 this new surplus is split between the monopolist and the low risk individuals, whereas in Case 2 the surplus goes to the monopolist in its entirety.

It is now evident that the preferences of the player government, in the game described above, are *not* to be understood as a social welfare function. Rather, they should be interpreted as the guidelines which the public firm must follow. If it does, then the outcome of the game will yield the described equilibrium, with the above welfare properties.

In that equilibrium (in both cases), the monopolist chooses to remain in the market, and is making strictly positive profits. This outcome is consistent with

[19] Note that this implies that $t_H = C^{(H)} - C^{(F)}$ and $t_L = C^{(H)} - C^{(J)}$.

the concept of *ex post participation* [see Green (1985)], as the monopolist is free to leave the market when the government enters. Thus, the coexistence of a privately owned firm, alongside a public firm, is endogenously obtained in the equilibrium of the model.

5 Taxation

Suppose the government decides to provide coverage to the uninsured, financing any losses through taxation. We show that this will either drive the monopolist out of the market or, if this is viewed as undesirable, will necessarily result in a game which is similar to the one presented above. Thus our methodology is appropriate for this form of intervention.

Taxation of buyers of insurance ex-ante and government provision of insurance. Suppose the government levies a lump-sum tax T from all buyers of insurance prior to the realization of uncertainty, shifting their endowment point from $(W, W - \ell)$, point E, to $(W - T, W - \ell - T)$, point E' - see figure 4a (where parameter values are as in case 1 of section 3). Consider the line $(\pi = 0)$, which is the break even line for contracts which are purchased by all the population when the endowment point is E. The government can then offer point M on this line, and use the tax revenues in order to break even. If T is chosen to be big enough, the slope of the line connecting points E' and M can be made bigger than the slope of the line $(\pi = 0)_L$, making it impossible for the monopolist to compete with the government.

However, this result is not always possible. Consider the case presented in figure 4b (parameter values as in case 2 of section 3). If the line $(\pi = 0)$ passes to the left of $\bar{U}_L$, as drawn in the figure, there is no contract which the government can offer that breaks even and is accepted by all individuals.

Moreover, driving the monopolist out of business may be viewed as undesirable. For example, when tax collection and the provision of insurance are carried out by two distinct government agencies, it is highly likely that the tax revenues will be used for other purposes.[20] Also, the presence of a private firm competing with the public firm may prevent the latter from becoming inefficient.

Suppose therefore that the government chooses not to drive the monopolist out of the market. We shall show that the taxation policy will result in a game. Suppose T is chosen to be small enough so that the slope of the line joining points E' and M is less steep than the slope of $(\pi = 0)_L$. If the government offers point M, the monopolist will react by attracting the low risk individuals at M. The government will be left with high risk individuals, losing money. But the government is still collecting T in taxes, so it can afford to lose up to this amount...Note that we are in the midst of a game similar to the one in section 3, where the strategy space of the government includes an additional dimension - the tax.

Taxation of the monopolist's profits and government provision of insurance. Suppose the government collects some fixed fraction of the monopolist's profits, and uses the revenues to subsidize more than actuarily fair insurance contracts, as above. However, the story does not end here. Any contract that is preferred by low risk individuals to their endowment point E, is preferred by the high risk individuals to their pre-intervention position F. Thus, any attempt by the government to offer the low risk individuals some coverage will result in *both types* purchasing insurance from the government. The monopolist's profits will be zero, so tax revenues will also be zero. But then the monopolist will surely react, say by stealing the more

[20]For example, the U.S. government has been recently criticized by members of Congress for using for current expenditures Social Security surpluses, earmarked for future payment to the baby boom generation.

profitable low risk individuals from the government, and so forth. Again, we have a game similar to the one in section 3.

6 Other Forms of Government Intervention

In this section we briefly illustrate two other forms of government intervention, pointing out some of the difficulties in implementing them.

Government relief. Suppose the government promises to pay a fixed amount R to anyone (low or high risk) who has incurred an accident and is not insured. Suppose the program is financed by taxing the monopolist. Buyers of insurance now perceive their endowment point to be $(W, W - \ell + R)$.[21] The equilibrium is presented in figure 4c: The perceived endowment point is E'; The monopolist serves the high risk individuals at point F', a full insurance contract, with pre-tax expected per capita profits equal to $W - p_H\ell - C^{(F')}$ (see (3)), which are lower than the pre-intervention profits; The tax revenues are used to pay the low risk individuals who have suffered a loss. Normalizing the population to 1, the government's balanced (on average) budget equation is $\alpha\mu_H[W - p_H\ell - C^{(F')}] = p_L\mu_L R$, where α is the tax rate, and $p_L\mu_L$ is the expected number of low risk individuals to have suffered a loss.

This policy succeeds in increasing expected utility for all buyers of insurance, but its scope for providing coverage to uninsured individuals is somewhat limited. This can be seen by noting that point $\bar{E}$ is an upper bound for points like E'. The major problem with this approach is that it earmarks a fraction of the population, the unfortunate low risk individuals who have suffered a loss, as recepients of

[21] This is a *perceived* endowment point, as ex-post it is not feasible in the aggregate. If the program is financed by taxing buyers of insurance the perceived endowment point is $(W - T, W - T - \ell + R)$.

government aid, as "welfare cases". This may be undesirable.

Regulation. Governments often regulate the insurance market. A plausible and rather common form of regulation is to forbid discrimination. Suppose that the government imposes on the monopolist to offer a single contract and serve all individuals in the market.[22] Consider Case 1 in figure 2. The monopolist will serve all individuals at point M, where the line ($\pi = 0$) is tangent to $\bar{U}_L$. Both types of individuals are on a lower indifference curve as compared to point H, and are not fully insured. Moreover, in Case 2 in figure 3, if the line ($\pi = 0$) passes below $\bar{U}_L$, then there is no contract which is accepted by both types and does not lose money.

7 Concluding Remarks

When a monopolistic insurance market suffers from an adverse selection problem, with low risk individuals not purchasing insurance, government entry to the market, as a competitor which sells insurance, may improve matters. It results in a higher degree of coverage for the low risk individuals, who had been uninsured, with no loss in coverage for the high risk individuals. Expected utility levels of all individuals are at least as high as in the pre-intervention situation, and the monopolist is still in business with strictly positive profits, although not as high as before.

An interesting feature of this result is the specialization of the monopolist in low risk individuals, and the shift of high risk individuals from the monopolist to the government. The government attracts the high risk individuals, and the low risk individuals are then served by the market.

We have also illustrated that when attempting to use the traditional tool of

[22] The simple constraint of one contract only will of course result in the monopolist continuing to serve only the high risk individuals at point F.

public finance, taxation, in this imperfect information environment, a similar game may develop.

Another feature of our approach is the coexistence of the government and private firms in the market, as competitors, possibly with different objective functions. This is often seen in health insurance markets, in financial markets, and in the provision of education.

Appendix

Proof of Proposition 1.

Step 1:

If $C^{(G)} = W - p_H\ell$ then

(a) $\max[U(C^{(G)}), \bar{U}_L] = \max[U(W - p_H\ell), \bar{U}_L] = U(W - p_H\ell)$.

(b) $\max[U(C^{(G)}), \bar{U}_H] = \max[U(W - p_H\ell), \bar{U}_H] = U(W - p_H\ell)$.

Proof of step 1: (a) follows from (16). (b) follows from $U(W - p_H\ell) = U[(1-p_H)W + p_H(W-\ell)] > (1-p_H)U(W) + p_H U(W-\ell) = \bar{U}_H$.

Step 2:

If the government offers $C^{(G)} = W - p_H\ell$

then the best response of the monopolist is

$(M_L^{NA}, M_L^{A}) = (W - p_H\ell, W - p_H\ell)$

Proof of step 2: We solve the programs in [A].

[A1]

Ignore for the moment constraints (12) and (13). The necessary conditions are then

$$-\mu_L(1-p_L) + \lambda_1(1-p_L)U'(M_L^{NA}) = 0 \tag{18}$$

$$-\mu_L p_L + \lambda_1 p_L U'(M_L^{A}) = 0 \tag{19}$$

$$-\mu_H(1-p_H) + \lambda_2(1-p_H)U'(M_H^{NA}) = 0 \tag{20}$$

$$-\mu_H p_H + \lambda_2 p_H U'(M_H^{A}) = 0 \tag{21}$$

$$\text{Complementary slackness for (10) and (11).} \tag{22}$$

Suppose that (10) does not bind. By (22) we have $\lambda_1 = 0$, contradicting (18). Analogously for (11). From (18) and (19) we have $M_L^{NA} = M_L^{A} = \bar{M}_L$, and from (20) and (21) we have $M_H^{NA} = M_H^{A} = \bar{M}_H$. As (10) and (11) bind we get $\bar{M}_L = \bar{M}_H = W - p_H\ell$.[23]

[23] Note that $M_i \in R^2$ is a contract, whereas $\bar{M}_i$ is a scalar.

As $[(W - p_H\ell, W - p_H\ell), (W - p_H\ell, W - p_H\ell)]$ satisfies (12) and (13), it is the solution of [A1], with $\Pi_1 = \mu_L(p_H - p_L)\ell$.

[A2]

Ignore for the moment (14). The necessary conditions are then

$$-\mu_L(1 - p_L) + \lambda(1 - p_L)U'(M_L^{NA}) = 0 \tag{23}$$

$$-\mu_L p_L + \lambda p_L U'(M_L^A) = 0 \tag{24}$$

$$\text{Complementary slackness for (10).} \tag{25}$$

Suppose that (10) does not bind. By (25) we have $\lambda = 0$, contradicting (23) and (24). From (23) and (24) we have $M_L^{NA} = M_L^A = \bar{M}_L$. As (10) binds we get $\bar{M}_L = W - p_H\ell$.
As $(W - p_H\ell, W - p_H\ell)$ satisfies (14), it is the solution of [A2], with $\Pi_2 = \mu_L(p_H - p_L)\ell$.

[A3]

Ignore for the moment (15). The necessary conditions are then

$$-\mu_H(1 - p_H) + \lambda(1 - p_H)U'(M_H^{NA}) = 0 \tag{26}$$

$$-\mu_L p_L + \lambda p_H U'(M_H^A) = 0 \tag{27}$$

$$\text{Complementary slackness for (11).} \tag{28}$$

Suppose that (11) does not bind. By (28) we have $\lambda = 0$, contradicting (26) and (27). From (26) and (27) we have $M_H^{NA} = M_H^A = \bar{M}_H$. As (11) binds we get $\bar{M}_H = W - p_H\ell$.
As $(W - p_H\ell, W - p_H\ell)$ satisfies (15), it is the solution of [A2], with $\Pi_3 = 0$.

As $\max[\Pi_1, \Pi_2, \Pi_3] = \Pi_1 = \Pi_2 > \Pi_3 = 0$, the proof of step 2 is complete.

We now know what is the best response of the monopolist to point H. We rule out all other contracts offered by the government as equilibrium contracts. We do so by showing that offering them will result either in negative profits or in an inferior outcome.

Step 3: $G \notin [J, H)$. Point J is defined by $C^{(J)} = U^{-1}(\bar{U}_L)$, i.e. it is the certainty equivalent to the endowment point for a low risk individual.

Proof of step 3: Suppose $G \in [J, H)$, say point X_1 in figure 2. The programs in [A] are analogous to those in step 2, with $\max[U(C^{(G)}), \bar{U}_L] = \max[U(C^{(G)}), \bar{U}_H] = U(C^{(X_1)})$. Thus, in the solutions of the programs we need only replace $(W - p_H\ell, W - p_H\ell)$ with $(M^{(X_1)}, M^{(X_1)})$. As at point X_1 contracts with both low and high risk individuals make strictly positive profits, we have $\Pi_1 > \Pi_2$ and $\Pi_1 > \Pi_3$.

The monopolist will therefore serve all buyers of insurance at point X_1. This is not an equilibrium as the government prefers the outcome obtained by offering point H, as in step 2 above.

Step 4: $G \notin [F, J)$.

Proof of step 4: Suppose $G \in [F, J)$, say point X_2 in figure 2. This time we have $\max[U(C^{(G)}), \bar{U}_H] = U(C^{(X_2)})$ but $\max[U(C^{(G)}), \bar{U}_L] = \bar{U}_L$, so the solutions to the programs in [A] may be quite different than the ones in steps 2 and 3. We do not need to characterize these solutions; it is sufficient to note the following trivial fact: The monopolist will not offer a menu of contracts which is strictly preferred by both types to the endowment point $(W, W - \ell)$, as such a contract does not maximize profits - offering ϵ less to at least one type of individuals in at least one contingency will yield higher profits. Therefore the government prefers the outcome obtained by offering point H, as in step 2.

Step 5: $G \notin (H, \infty)$.

Proof of step 5: Suppose $G \in (H, \infty)$. If the government offers a point like X_4 in figure 4 all three programs lose money, the monopolist stays out, and the government loses money. If the government offers a point like X_3, then $\Pi_3 < 0$. The solutions to [A2] and [A1] are analogous to those in step 2, with the appropriate change in the contract offered by the government (point X_3 rather than point H). In this case, though, in the solution to [A1], contracts with high risk individuals lose money (as X_3 lies to the right of $(\pi = 0)_H$). The monopolist will then serve only the low risk individuals, at X_3, and the government will be left with contracts that lose money.

Step 6: *Proof of proposition.*

In the above steps we have shown that in equilibrium the only contract the government can offer is the one in the proposition. In step 2 we have shown that the contracts which constitute the best response of the monopolist are as in the proposition. From steps 3, 4, and 5 it follows that these are the only contracts which satisfy [B] (the government optimizes) without violating the constraint that the government cannot lose money. This completes the proof of the proposition.

Proof of Proposition 2.

Step 1:

If $C^{(G)} = W - p_H\ell$ then

(a) $\max[U(C^{(G)}), \bar{U}_L] = \max[U(W - p_H\ell), \bar{U}_L] = \bar{U}_L$.

(b) $\max[U(C^{(G)}), \bar{U}_H] = \max[U(W - p_H\ell), \bar{U}_H] = U(W - p_H\ell)$.

Proof of step 1: (a) follows from(17). (b) follows from $U(W - p_H\ell) = U[(1-p_H)W + p_H(W-\ell)] > (1-p_H)U(W) + p_H U(W-\ell) = \bar{U}_H$.

Step 2:

If $C^{(G)} = W - p_H\ell$

then the best response of the monopolist is M_L defined by:

$(1-p_L)U(M_L^{NA}) + p_L U(M_L^{NA}) = \bar{U}_L$

and

$(1-p_H)U(M_L^{NA}) + p_H U(M_L^{NA} = U(W - p_H\ell)$.

Proof of step 2: We solve the programs in [A].

[A3]

Ignore for the moment (15). The necessary conditions are then

$$-\mu_H(1-p_H) + \lambda(1-p_H)U'(M_H^{NA}) = 0 \tag{29}$$

$$-\mu_H p_H + \lambda p_H U'(M_H^{A}) = 0 \tag{30}$$

$$\text{Complementary slackness for (11).} \tag{31}$$

Suppose that (11) does not bind. By (31) we have $\lambda = 0$, contradicting (29) and (30) . From (29) and (30) we have $M_H^{NA} = M_H^{A} = \bar{M}_H$. As (11) binds we get $\bar{M}_H = W - p_H\ell$. As $(W - p_H\ell, W - p_H\ell)$ satisfies (15), it is the solution of [A2], with $\Pi_3 = 0$.

[A2]

First we note that (10) binds, as if it does not bind M_L^{NA} and M_L^A can be decreased without violating either constraint, increasing profits. The necessary conditions are then

$$-\mu_L(1-p_L)+\lambda_1(1-p_L)U'(M_L^{NA})-\lambda_2(1-p_H)U'(M_L^{NA})=0 \tag{32}$$

$$-\mu_L p_L+\lambda_1 p_L U'(M_L^A)-\lambda_2 p_H U'(M_L^A)=0 \tag{33}$$

$$\text{Complementary slackness for (14).} \tag{34}$$

Suppose that (14) does not bind. Then from (32) and (33) we obtain $M_L^{NA}=M_L^A=\bar{M}_L$, and as (10) binds we have $\bar{M}_L=U^{-1}(\bar{U}_L)>W-p_H\ell$ (by (17)), contradicting (14). Thus, both constraints bind. Using step 1 the constraints become

$$\begin{aligned}(1-p_L)U(M_L^{NA})+p_L U(M_L^A)&=\bar{U}_L\\(1-p_H)U(M_L^{NA})+p_H U(M_L^A)&=U(W-p_H\ell).\end{aligned} \tag{35}$$

The only point which satisfies (35) is point K in figure 3, with $\Pi_2>0$.

[A1]

We first note that offering $(W-p_H\ell, W-p_\ell)$, point H, to the high risk individuals, and offering the contract defined by (35), point K, to the low risk individuals, satisfies the constraints of [A1]. Thus, $\Pi_1\geq\Pi_2$. We shall now show that this pair of contracts maximizes the profits of [A1], yielding $\Pi_1=\Pi_2$, i.e. the profits from the contracts sold to high risk individuals are zero. The monopolist will then serve the low risk individuals. This will establish step 2.

Ignore for the moment constraint (12). We first show that (10) must bind. If not, then M_L^{NA} and M_L^A can be decreased without violating any of the constraints, increasing profits. Thus (10) binds and $\bar{M}_L$ lies on $\bar{U}_L$.

Choose contracts (M_L, M_H) such that: (1) M_L lies on $\bar{U}_L$, and (2) M_H lies on an indifference curve of high risk individuals which passes through M_L and is higher than the indifference curve through point H. Formally,

$$\begin{aligned}
&(1-p_L)U(M_L^{NA}) + p_L U(M_L^A) = \bar{U}_L \\
&\text{and} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (36) \\
&(1-p_H)U(M_H^{NA}) + p_H U(M_H^A) = (1-p_H)U(M_L^{NA}) + p_H U(M_L^A) \geq U(W - p_H\ell).
\end{aligned}$$

By construction, constraints (10), (11), and (13) are satisfied. Assume now that M_L is fixed and satisfies (36), and that the monopolist can choose any M_H that satisfies (36). Then, as profits from contracts with low risk individuals are given, the contract M_H which maximizes profits must be a contract on the 45^o line. The second equation in (36) then becomes $U(\bar{M}_H) = (1-p_H)U(M_L^{NA}) + p_H U(M_L^A)$. Now (36) is identical to (4) and (5). By our assumption in (8), regarding the pre-entry equilibrium, using Lemma 1, total profits are maximized when the contracts in (36) are such that $\bar{M}_H$ is minimal. The smallest M_H which the monopolist can choose without violating (11) is $W - p_H\ell$.
It is easily verified that the contracts defined by (36) satisfy (12), and thus they are the solution to [A1]. As offering $\bar{M}_H = W - p_H\ell$ to the high risk individuals yields zero profits, we have $\Pi_1 = \Pi_2$ as we wanted. This completes the proof of step 2.

As in the proof to Proposition 1, we now proceed to rule out all other contracts offered by the government as equilibrium contracts.
Step 3: $G \notin [F, H)$.
Proof of step 3: Suppose $G \in [F, H)$, say point X_5 in figure 3. The programs in [A] are analogous to those in step 2, with $\max[U(C^{(G)}), \bar{U}_L] = \bar{U}_L$ and $\max[U(C^{(G)}), \bar{U}_H] = U(C^{(X_5)})$. Thus, in the solutions of the programs we need to replace $(W - p_H\ell, W - p_H\ell)$ with $(C^{(X_5)}, C^{(X_5)})$, and to note that the equations that previously defined point K now define point X'_5. As at point X_5 contracts with both low and high risk individuals make strictly positive profits, we have $\Pi_1 > \Pi_2$ and $\Pi_1 > \Pi_3$.

The monopolist will therefore serve all buyers of insurance at points X_5 and X'_5. This is not an equilibrium as the government prefers the outcome obtained by offering point H, as in step 2 above.
Step 4: $G \notin [H, J)$.

Proof of step 4: Suppose $G \in [H, J)$, a point like X_6 in figure 3. The solutions to the programs in [A] are analogous to those in step 2. We need only replace $(W - p_H\ell, W - p_H\ell)$ with $(C^{(X_6)}, C^{(X_6)})$. The solution to [A3] is the contract X_6 with $\Pi_3 < 0$, which the monopolist will not offer. The solution to [A2] is the contract X'_6 with $\Pi_2 > 0$. The solution to [A1] are the contracts X_6 and X'_6, with the former losing money, and thus $\Pi_2 > \Pi_1$. The monopolist will serve only the low risk individuals at point X'_6, leaving the government with the high risk individuals at point X_6. The government will then lose money.

Step 5: $G \notin (J, \infty)$.

Proof of step 5: Suppose $G \in (J, \infty)$. If the government offers a point like X_8 in figure 3 all three programs lose money, the monopolist stays out, and the government loses money. If the government offers a point like X_7, then $\Pi_3 < 0$. The solutions to [A2] and [A1] are analogous to those in step 2 of the proof of Proposition 1, with the appropriate change in the contract offered by the government (point X_7 rather than point H). In this case, though, in the solution to [A1], contracts with high risk individuals lose money (as X_7 lies to the right of $(\pi = 0)_H$). The monopolist will then serve only the low risk individuals, at X_7, and the government will be left with contracts that lose money.

Step 6: *Proof of proposition.*

In the above steps we have shown that in equilibrium the only contract the government can offer is the one in the proposition. In step 2 we have shown that the contracts which constitute the best response of the monopolist are as in the proposition. From steps 3, 4, and 5 it follows that these are the only contracts which satisfy [B] (the government optimizes) without violating the constraint that the government cannot lose money. This completes the proof of the proposition.

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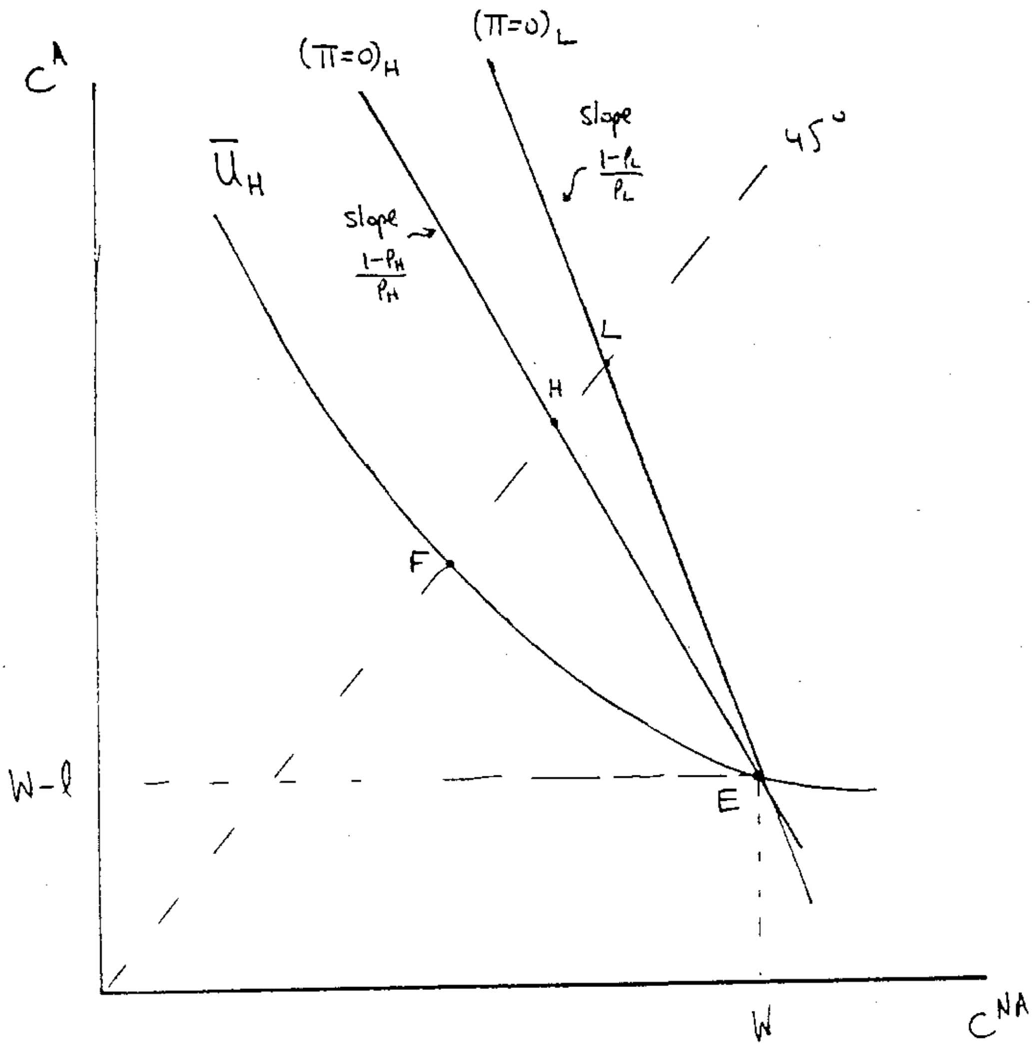

C^A
$(\Pi=0)_H$
$(\Pi=0)_L$
slope $\frac{1-p_L}{p_L}$
$45°$
$\bar{U}_H$
slope $\frac{1-p_H}{p_H}$
L
H
F
$W-\ell$
E
W
C^{NA}

Figure 1

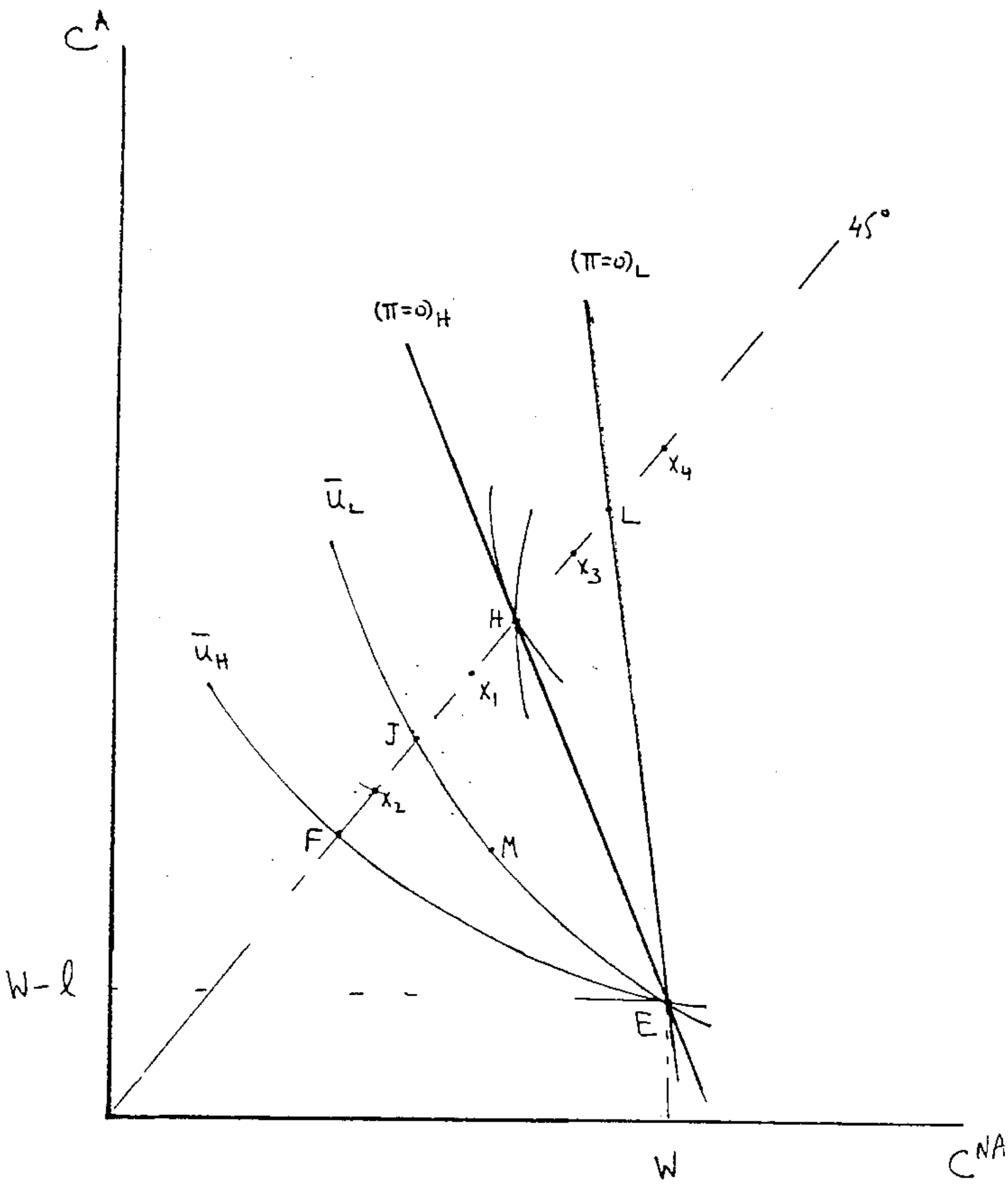

Figure 2

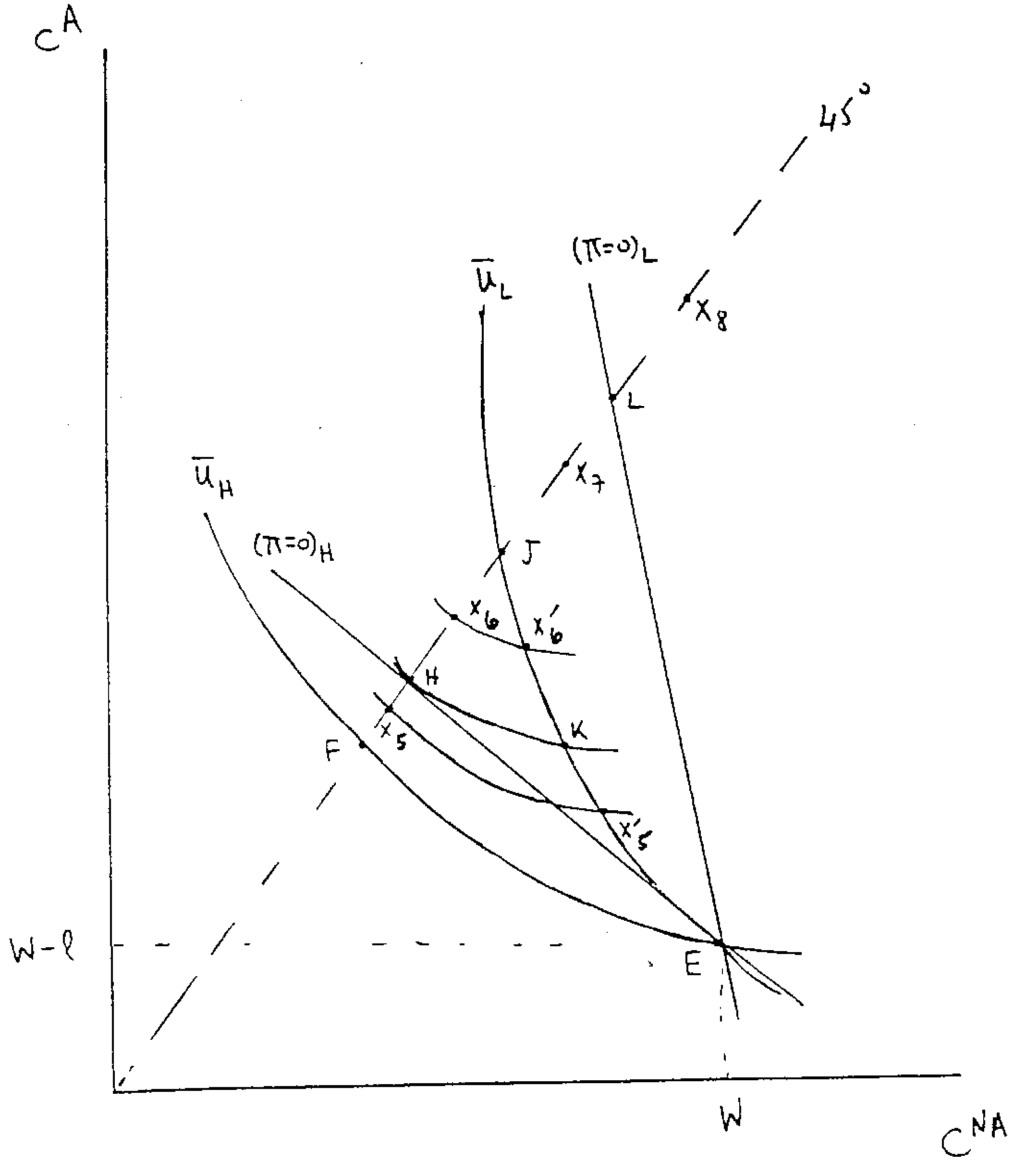

Figure 3

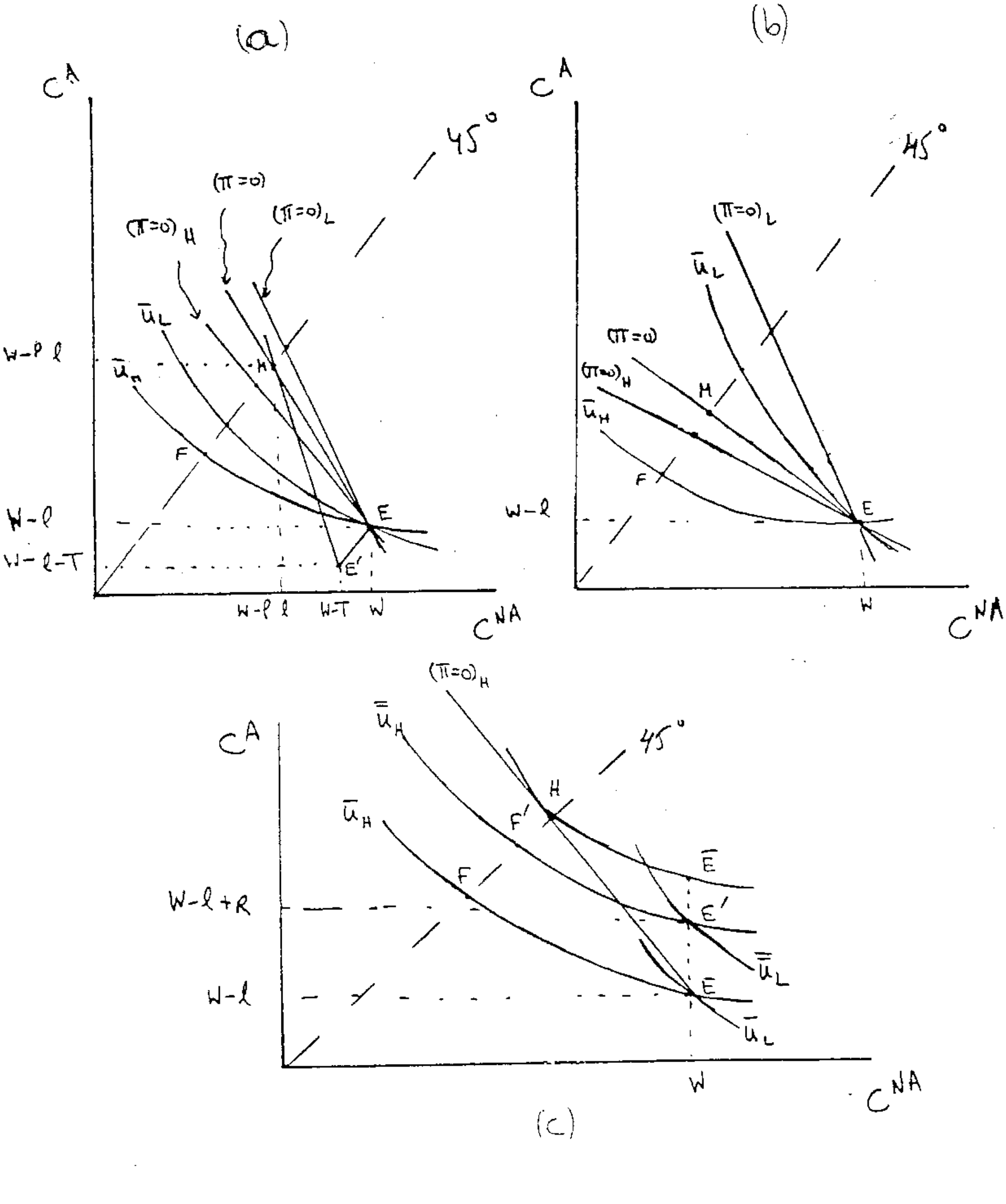
(a)
C^A
45°
$(\pi=0)$
$(\pi=0)_H$
$(\pi=0)_L$
$\bar{u}_L$
$\bar{u}_H$
$W-p\ell$
H
F
E
$W-\ell$
$W-\ell-T$
E′
$W-p\ell$
$W-T$
W
C^{NA}
(b)
C^A
45°
$(\pi=0)_L$
$\bar{u}_L$
$(\pi=0)$
$(\pi=0)_H$
H
$\bar{u}_H$
F
$W-\ell$
E
W
C^{NA}
$(\pi=0)_H$
C^A
$\bar{\bar{u}}_H$
45°
H
$\bar{u}_H$
F′
$\bar{E}$
F
$W-\ell+R$
E′
$\bar{\bar{u}}_L$
$W-\ell$
E
$\bar{u}_L$
W
C^{NA}
(c)

Figure 4

CHAPTER II
RISK SHARING AND COMPETITION AMONG DEPOSITORY INSTITUTIONS

1 Introduction

Large depository institutions can more easily diversify their portfolio of loans, as they are able to finance projects in more than one geographical region, and in various sectors of the economy. This makes large depository institutions less vulnerable to regional or sectoral shocks, enabling them to provide depositors with a relatively safe investment opportunity. A small number of large depository institutions, though, may enjoy market power, lowering the return offered to depositors. The welfare trade off between reduction in risk and enhanced market power, as depository institutions become larger but fewer, is the focus of this paper.

A central assumption of the paper is that markets are incomplete. I shall assume that depsoitors cannot fully diversify their portfolios (due to time and information constraints, and to the presence of transaction fees), and that risk sharing between depository institutions is possible only through merger. The logic behind this last assumption is as follows. Enforcing risk sharing agreements between depository institutions requires a great deal of cooperation and mutual monitoring, both when investments are made, and when their returns are realized. This facilitates coordination (such as price collusion or division of markets), and hence reduces competition. The assumption that risk sharing between depository institutions is possible only through merger is an extreme version of the assumption that risk sharing leads to greater market power.

The main result is that when there are enough independent risks in the economy, it is possible to achieve high diversification through mergers between depository institutions at a very small cost in terms of enhanced market power. When the number of independent risks in the economy is not large, the optimal market structure of depository institutions cannot be unambiguously detrermined. The trade off between diversification and market power, though, is still important. I provide a robust example where the number of independent risks is small, yet some degree of oligopoly is socially desirable.

A market for which the model is relevant is the long term money management market in the U.S. Depositors in this market are (mainly) pension funds, who deposit the savings toward retirement of their members with insurance companies, investment banks, commercial banks, and mutual funds. Many pension plans are defined contribution plans, where returns are random and are not insured by the government. The majority of pension plans are defined benefit plans, where returns are specified in advance, regardless of the performance of the pension fund's investments. These plans are insured by the Pension Fund Guarantee Corporation, up to a ceiling, and are therefore not completely riskless. Lakonishok, Shleifer, and Vishny (1991) report that the tax-exempt money management industry is not very concentrated, the largest ten firms holding a 22% market share. Higher concentration in this industry may result in more security for long term savers.

Another industry for which such an argument bears relevance is the banking industry in the U.S. In a recent study, the U.S. Treasury Department has recommended to allow full nationwide banking for bank holding companies, and interstate branching for banks.[1] According to the study, interstate branch banking will im-

[1] U.S. Treasury Department (1991), summary of recommendations, p.12. The legislation relevant to interstate banking is the McFadden Act of 1927, the Bank Holding Company Act of 1956, the Douglas amendment to this act, and the One-Bank Holding Company Act of 1970.

prove the "safety and soundness" of the system, and will render banks less prone to bankruptcies and insolvencies. A branch bank "serves as a mutual loss sharing arrangement under which losses to one part of a bank's operation are diffused across the entire organization. For example, the majority of bank failures in recent years occurred in the restricted branching states of the Southwest, notably Oklahoma and Texas." (p.XVII-9) The study points to evidence that during the 1920's and early 1930's the failure rate was inversely related to bank size, and that during the period 1921-1931 the bank failure rate, as a percentage of banks operating at the end of 1931, was considerably lower for banks with branches.[2]

Frankel and Montgomery (1991, p.282) make a similar point. In a comment therein, Mark Gertler points out that "A recent article in the *New York Times* suggests that, of the banks carrying nonperforming loans equal to 8 percent or more of total assets, 70 percent are concentrated in New England. In addition, banks in New England and Texas account for the vast majority of those with capital positions below the minimum regulatory requirement. These statistics suggest that easing restrictions on interstate banking may allow banks to better insulate themselves against regional disturbances and may help develop a more resilient national banking system." (p.305)

Eisenbis, Harris, and Lakonishok (1984) provide evidence that bank holding companies that are more diversified, both geographically and in the types of activities they perform, are perceived by shareholders as having higher value. Cherin and Melicher (1988) report that the returns on loans of banks with more branches exhibit lower unsystematic risk.[3]

The U.S. Treasury Department study also recommends to permit well capital-

[2] See pages XVII-8 and XVII-9 for more details and further references.

[3] See Nielsen, Seline, and Johnson (1988-9) for further discussion and references.

ized banks to have financial affiliates dealing with securities, mutual funds, and insurance.[4] This will further decrease the riskiness of banks' portfolios.[5]

A potential drawback of the suggested reforms is that they might result in a more concentrated and less competitive banking system. The U.S. Treasury Department study does not support this view. It points out that Canada has eight major banks, of which six operate nationwide, serving a population of 26.3 million; "If the U.S. had the same ratio of banks to population, it would have about seventy five banks, of which fifty six would operate nationwide" (p.XVII-17), which is quite a larger number. Moreover, it is argued, projecting the Canadian market structure to the U.S. may not be justified. California, for example, has allowed branch banking within the state since 1909. Yet it has 431 banks. According to the U.S. Treasury Department study, interstate banking will result in *more* competition.

Evidence from other industrialized countries points in the opposit direction, though. In countries where inter-regional banking is allowed the banking industry is relatively concentrated. Frankel and Montgomery (1991, figure 9) report that approximately 43% of bank assets in Japan are owned by the five largest banks. Approximately 64% of the assets are owned by the ten largest banks. Similar figures are reported for Germany. The French banking commission reports that in 1990 the five largest banks granted 44.3% of the credit and held 58.5% of the deposits in France.[6] The figures for the ten largest banks are 60.6% and 74.1%. Benjamin Friedman points out[7] that one third of bank assets in the U.S. are held by the fifteen largest banks, which suggests that the potential for nationwide domination by a few

[4]Chapter XVIII. The relevant legislation is the Glass-Steagall Act of 1933 and the Bank Holding Company Act of 1956.

[5]The study does not mention reduction in risk as an argument in favor of this proposal. I do not know the reason for this.

[6]Commission Bancaire, p.105.

[7]In a discussion of a paper by Zvi Bodie in Kopcke and Rosengren (1989), pp.131-5.

large banks exists.

It is no doubt hard to forecast whether allowing interstate banking and relaxing the Glass-Steagall restrictions will indeed entail concentration of market power in the hands of a small number of banks or bank holding companies. What this paper has to say in this context is that even if such concentration of market power does result, and since the U.S. is a big and well diversified economy, the gains from such a reform are likely to exceed the losses.

The model I construct is not a model of the U.S. banking system, nor of any other banking system. When we think of banking we tend to think in terms of fixed interest rate contracts, possibly insured by the government. In the model presented below interest rates paid by depository institutions are random variables, and there is no deposit insurance. There is no inherent reason for thinking of banking in terms of fixed interest rates. The model is therefore relevant for a banking system without deposit insurance, for the market for large uninsured deposits,[8] or as discussed above, for the long term money management industry in the U.S.

There are other considerations besides risk sharing and market power which are relevant for this issue. For example, the presence of economies of scale or economies of scope makes, other things equal, large depository institutions more desirable. On the other hand, large institutions might be less flexible in their decision making process, might be less aware of the particular needs of the various regions in the country, or might be biased towards investment and lending in big cities at the expense of rural regions.[9] I overlook these questions here. I also ignore important questions such as deposit insurance and moral hazard. My purpose is simply to highlight the welfare tradeoffs when risk sharing among depository institutions is

[8]In the U.S. the FDIC insures deposits up to a ceiling of $100,000.

[9]See Litan (1988) and Nielsen, Seline, and Johnson (1988-9) for further discussion.

achieved through mergers or mutual stock ownership, in a way which enhances market power.

The model is a simple general equilibrium model. As the focus of the paper is on risk sharing across regions and across sectors, a general equilibrium setting is appropriate because it prevents an exogenous "risk-neutral principal," such as a deposit insurance institution, from absorbing all the risk.

2 Model

Outline of the model. Consider a two period economy with a single good. There is a given number of depository institutions operating in the economy, which for the sake of conciseness I shall call banks. Each bank has access to a constant returns to scale technology which yields a random return. The returns of these technologies are i.i.d. random variables. The technologies can be thought of as the financing of projects in different geographical regions, or in different sectors of the economy such as real estate, agriculture, or energy, or a combination of both (e.g. real estate in the North East of the U.S.). A scenario where several banks use the same technology (i.e. finance similar projects in the same region) would be more realistic. In such a setting the returns of banks which use the same technologies would be highly correlated, whereas the returns of banks across technologies would be weakly correlated.[10] The assumption I make is an extreme case of this scenario. Returns in any single technology are perfectly correlated (as there is only one bank which uses it), whereas returns across technologies are stochastically independent. Since my focus is on inter-regional (or inter-sectoral) risk sharing, the assumption

[10] Ignoring idiosyncratic effects such as management quality or reputation.

seems appropriate.[11]

The banks in the economy are organized in banking groups of equal size. Each group pools the returns of the group members and offers a single savings contract to depositors. The assumption that banking groups are of equal size is made for reasons of tractability.[12]

Depositors are risk averse expected utility maximizers.[13] They live for two periods, and have the same first period endowment, but no second period endowment. They cannot store any amount of their first period endowment on their own account, but they can deposit any fraction of it with exactly one banking group. As the banks in the same banking group are fully coordinated (pooling resources and offering depositors the same savings contract) it does not matter with which particular bank within the group savings are deposited.

The no storage assumption implies that there is no truly riskless asset in the economy. This assumption and the assumption that depositing with more than one bank is not possible, capture the inability of depositors to fully diversify their savings portfolios (due to time and information constraints, and to the presence of transaction fees). I make the extreme assumption that no diversification of savings is possible.

In the second period the profits of the banking groups are distributed to depositors. One way to interpret this is to think of depositors as the share holders of the banking groups. Then, in the second period depositors receive dividends according

[11] It goes without saying that inter-regional (or inter-sectoral) risk sharing is no remedy for aggregate, systemic shocks. Therefore aggregate risk is omitted from the model.

[12] In this model large banking groups are better diversified, and would thus be at an advantage with respect to smaller groups. An interesting extension would be to introduce a cost to size, say the loss of flexibility in the decision making process, thus endogenizing group size (or the distribution of group sizes) and market concentration.

[13] Whether we regard depositors as individual savers or as pension funds representing the interests of a large group of depositors, risk aversion is an appropriate assumption.

to their share ownership in the banks. This is the approach taken in this section. The ownership structure of banks is exogenously given and there is no trade in shares. I shall assume that all depositors hold identical bank share portfolios, each owning a fraction of every bank in the economy. This ownership structure implies that in the capacity of share holders depositors are fully diversified. A different approach is explored in the next section.

Information between depositors and the banking group with which they deposit their savings is symmetric. A banking group can offer an interest rate schedule contingent on the average realization of the random technologies in which the banks in the group have invested. The interest rate schedule cannot be made contingent on the realizations of other technologies in the economy (due to costly state verification, for example). This assumption captures the incompleteness of the market for savings bonds. If this market were complete, then diversification would be achieved through trade in state contingent claims, and diversification through bank mergers into banking groups would be superfluous.[14]

In the first period the banking groups compete à la Cournot, using as the strategic variable the quantity of savings bonds to be issued. We can think of each banking group as selecting a scale of operation (the number of specialized personnel to hire, the computing capacity to install, and most important, the number of potential borrowers to screen and to maintain relations with). A given scale of operation

[14]The assumption of symmetric information between banks and their depositors is certainly worth relaxing. Whether a market structure with larger but fewer banking groups, i.e. less competition with more risk sharing, exacerbates or mitigates inefficiencies due to adverse selection, moral hazard, or costly state verification, is a question which deserves more research. I shall not address it here. The features of the savings contract which are essential for the model presented in this paper are (a) The savings contract involves some risk for the depositor, (b) This risk is related to the performance of the depository institution, and (c) The risk has a component which is diversifiable, given the other risks in the economy.

corresponds to a quantity of savings bonds brought to market.[15]

A methodological question which arises is how to define "the inverse demand for savings," as there may be several interest rate schedules which would induce depositors to purchase a given amount of savings bonds. I make the following assumptions. For each level of per capita savings, attention is restricted to the interest rate schedule which maximizes a banking group's expected profits, among all the interest rate schedules which induce these per capita savings.[16] If there is more than one such schedule, attention is restricted to a particular one.[17]

When the banking groups arrive to market in the first period, they are greeted by a market maker who calls an interest rate schedule which specifies, for every level of per capita savings in the economy, the gross interest rate to be paid in period 2 by a banking group to its depositors, conditional on the realized average return of the technologies in which the banking group has invested. The market maker selects the interest rate schedule according to the criterion in the previous paragraph. If there is more than one schedule which maximizes the banking groups' expected profits, the market maker selects one at random.[18]

The interpretation of these assumptions is that the set of financial assets in the economy is limited to those called by the market maker. For each level of per capita savings s, there is exactly one kind of asset in the economy, namely a savings bond

[15] We could also think of the scale of operation as representing "capacity." As competition between banking groups is modeled as a one stage game, the distinction is mainly semantic.

[16] As all depositors are identical, working in per capita terms is without loss of generality.

[17] In the next section I introduce an assumption which guarantees uniqueness of the profit maximizing interest rate schedule for each level of per capita savings.

[18] There are other plausible criteria for the selection of the contract called by the market maker. A more general formulation would be to restrict attention to interest rate schedules which maximize a weighted average of the banking group's expected profits and depositors' expected utility, subject to the same constraint on per capita savings. The weight might be determined through a bargaining procedure. The formulation I chose is a particular case of this formulation, with all the weight placed on the banking group's expected profits.

which promises to pay the interest rate schedule called by the market maker, and which *by construction* indeed generates per capita savings s. What is determined in the equilibrium of the Cournot game between the banking groups is the aggregate level of savings in the economy, which determines the per capita amount of savings. Thus, the equilibrium picks a particular financial asset from the family of assets called by the market maker.

With this setup welfare analysis is straightforward. As depositors own the banks, as they optimally choose to save the equilibrium amount of savings, and as equilibrium is symmetric across depositors by costruction of the model, the obvious welfare criterion is the expected utility of a depositor in equilibrium. The main question addressed below is the effect of the size and the number of banking groups on depositors' expected utility.

Notation. There are K banks, indexed $k = 1 \dots K$. Bank k has access to a constant returns to scale technology which yields a random gross return $\tilde{x}_k \in [0, \bar{x}]$, $\bar{x} \in \mathbf{R}_+ \cup \{\infty\}$. The $\tilde{x}_k$'s are i.i.d. random variables.

The K banks are organized in M groups of equal size, indexed by $m = 1 \dots M$, with $N = \frac{K}{M}$ banks per group. Choose K and M so that N is an integer. The average return per bank for banks belonging to group m of size N is the random variable $\tilde{x}_m^N = \frac{1}{N} \sum_{k \in m} \tilde{x}_k$. As the x_k's, $k = 1 \dots K$, are i.i.d. random variables, so are the $\tilde{x}_m^N$'s, $m = 1 \dots M$. Let $\tilde{x}^N = (\tilde{x}_1^N, \dots, \tilde{x}_M^N)$ and $\tilde{x}_{-m}^N = (\tilde{x}_1^N, \dots, \tilde{x}_{m-1}^N, \tilde{x}_{m+1}^N, \dots, \tilde{x}_M^N)$. Let $E(\cdot)$, $E_m(\cdot)$, and $E_{-m}(\cdot)$ be the expectation operators with respect to $\tilde{x}^N$, $\tilde{x}_m^N$, and $\tilde{x}_{-m}^N$.

There are I depositors in the economy. Although it is not necessary for any of the computations, it makes sense to assume that $I \gg K$. All depositors have a first period endowment of $\omega > 0$, and identical utility functions $u(c_1) + v(c_2)$, where c_1

and c_2 are first and second period consumption, and u and v are strictly concave. In the second period each depositor receives a dividend of $T(\tilde{x}^N)$, which is the average per capita profit of all the banking groups. Ex-ante, $T(\tilde{x}^N)$ is a random variable, whose distribution function is known to depositors.

The market maker calls the function $\rho(\tilde{x}^N_m, s)$, where s are savings per depositor. Given s, $\rho(x^N_m, s)$ is the gross interest rate to be paid in period 2 by banking group m, $m = 1 \ldots, M$, to a depositor, if the realized average return of the technologies in which the banks in the group have invested is x^N_m. For every s, the function satisfies $0 \leq \rho(x^N_m, s) \leq x^N_m$ for all x^N_m. Additional notation will be introduced as it is needed.

The inverse demand for savings. Consider the problem below, where banking group m, $m = 1 \ldots, M$, chooses an interest rate schedule which maximizes expected per capita profits, subject to the constraint that each depositor be willing to save s. The state by state feasibility constraints are a result of the assumed incompleteness of markets, namely of the inability of a banking group to raise money from external sources when its investments turn sour. In this economy "other sources" stands for the other banking groups, with which risk sharing contracts cannot be written by assumption.

Problem (∗):

$$\max_{r(\cdot)} E_m s[\tilde{x}^N_m - r(\tilde{x}^N_m)]$$

s.t.

$$-u'(\omega - s) + Ev'[r(\tilde{x}^N_m)s + T(\tilde{x}^N)]r(\tilde{x}^N_m) = 0 \quad ; \quad \mu \tag{1}$$

$$u(\omega - s) + Ev[r(\tilde{x}^N_m)s + T(\tilde{x}^N)] \geq u(\omega) + Ev[T(\tilde{x}^N)] \quad ; \quad \lambda \tag{2}$$

$$0 \leq r(x^N_m) \leq x^N_m \quad \forall x^N_m \in [0, \bar{x}] \quad ; \quad \gamma_{x^N_m}. \tag{3}$$

All economic agents take the dividend $T(\tilde{x}^N)$ as exogenously given. For depositors this assumption means that when they make their savings decision, they do not take into account the effect of this decision on bank profits. For the banking groups the assumption can be interpreted as reflecting the limited information each banking group has on the operations of other banking groups and on the composition of depositors' share portfolios. A banking group knows the total amount of dividend payments a depositor receives in period 2, but does not know the composition of the dividend payments, and does not take into account the effect of its own actions on them.

The market maker computes the necessary conditions for the maximization of *problem* $(*)$, as a banking group would compute them, namely taking $T(\tilde{x}^N)$ as exogenously given. Denote the solution to *problem* $(*)$ by $\rho(\tilde{x}_m^N, s)$. Depositors take the interest rate schedule $\rho(\tilde{x}_m^N, s)$ as given, and do not take into consideration the effect of their savings decision on it.

The Cournot game. The market maker calls the function $\rho(\tilde{x}_m^N, s)$, which the banking groups take as given, regarding it as the inverse per capita demand for savings bonds. Banking group m chooses S_m, the number of savings bonds to issue, by solving the problem

$$\max_{S_m} E_m S_m \left\{ \tilde{x}_m^N - \rho \left[\tilde{x}_m^N, \frac{1}{I}(S_m + \textstyle\sum_{m' \neq m} S_{m'}) \right] \right\}, \quad m = 1, \ldots, M, \tag{4}$$

where

$$S_m + \textstyle\sum_{m' \neq m} S_{m'} = sI. \tag{5}$$

Banking group m takes as given the number of savings bonds issued by the other groups, $S_{m'}$, $m' \neq m$. The first order conditions for an interior solution of (4) are[19]

$$E_m \tilde{x}_m^N = E_m \frac{\partial}{\partial S_m} S_m \rho \left[\tilde{x}_m^N, \frac{1}{I}(S_m + \textstyle\sum_{m' \neq m} S_{m'}) \right], \quad m = 1, \ldots, M. \tag{6}$$

The left hand side of (6) is the increase in expected revenue from selling an additional savings bond and investing the proceeds in the technologies to which banking group m has access. The right hand side is the expected cost of doing this.[20]

Each equation in (6) defines implicitly the best response correspondence for a banking group. Thus, all the banking groups will supply the same quantity of savings bonds, $S_m = \frac{Is}{M}$, $m = 1, \ldots, M$. Taking the derivative on the right hand side of (6), summing over $m = 1 \ldots M$, substituting for S_m, and rearranging, yields

$$E_m \tilde{x}_m^N = E_m \rho(\tilde{x}_m^N, s) \left[1 + \frac{1}{M} \epsilon_{\rho/s}(\tilde{x}_m^N, s) \right], \tag{7}$$

where $\epsilon_{\rho/s}(\tilde{x}_m^N, s) = \frac{\partial \rho(\tilde{x}_m^N, s)}{\partial s} \frac{s}{\rho(\tilde{x}_m^N, s)}$, and $\tilde{x}_m^N$ denotes the generic ex-ante return of a banking group of size m. Equation (7) determines (not necessarily uniquely) the equilibrium value of per capita savings $\hat{s}$.

The profits of banking group m are $\frac{\hat{s}I}{M}[\tilde{x}_m^N - \rho(\tilde{x}_m^N, \hat{s})]$. Therefore the dividend per depositor is

$$T(\tilde{x}^N) = \frac{\hat{s}}{M} \sum_{m=1}^{M} [\tilde{x}_m^N - \rho(\tilde{x}_m^N, \hat{s})]. \tag{8}$$

For the derivation of $\rho(\tilde{x}_m^N, s)$ and for a treatment of its differentiability in s, see the appendix.

[19] Differentiability of $\rho(\tilde{x}_m^N, s)$ is assumed. In the appendix, where $\rho(\tilde{x}_m^N, s)$ is derived, a (mild) condition for differentiability is provided.

[20] For second order conditions to be satisfied, it is sufficient that for every m, $E_m S_m \rho \left[\tilde{x}_m^N, \frac{1}{I}(S_m + \sum_{m' \neq m} S_{m'} \right]$ be strictly convex in S_m.

Remark. The function $\rho(\tilde{x}_m^N, s)$ which the market maker calls, defines a family of interest rate schedules, (or contracts or financial assets), one for each s. The equilibrium of the Cournot game determines $\hat{s}$ and hence the particular financial asset which is traded in the economy. Each depositor receives $\hat{s}$ units of this asset, paying with $\hat{s}$ units out of his endowment.

Analysis. The main result of the paper is now shown, namely that when the economy is large, it is possible to approximate the welfare level which would be achieved in a competitive economy with no uncertainty. This is done by organizing depository institutions in many large groups.

A large economy. I shall consider sequences of economies such that along every sequence the number of banking groups increases, but the size of the groups remains constant. Although it is not essential for the result, it makes sense to assume that the number of depositors also grows along the sequence. I shall assume that it grows so that the number of depositors per group (and hence per bank) remains constant.

Definition Consider the following sequence of economies. Let K_1 and M_1 satisfy $\frac{K_1}{M_1} = N$, where N is an integer, and let $I_1 \gg K_1$. Let $K_i = iK_1$, $M_i = iM_1$, and $I_i = iI_1$. Economy i is an i-replica of economy 1, with $\frac{K_i}{M_i} = N$ and $I_i \gg K_i$. Call this sequence of economies an *N sequence.*

Claim 1 *Consider an N sequence. If $E_m \epsilon_{\rho/s}(\tilde{x}_m^N, s)$ is bounded, then as $i \to \infty$, the expected profits of every banking group approach zero.*

Proof. As $i \to \infty$, $M_i \to \infty$. Then from (7) it follows that $E_m \tilde{x}_m^N \to E_m \rho^i(\tilde{x}_m^N, s)]. \|$

Along an *N sequence* each depositor saves $\hat{s}_i = \arg\max_s u(\omega - s) + Ev[\rho^i(\tilde{x}_m^N)s +$

$T^i(\tilde{x}^N)]$. The notation $\rho^i(x_m^N)$ is used (rather than $\rho^i(\tilde{x}_m^N, s)$) in order to emphasize that depositors, when choosing s, take the interest rate schedule as given. Let $U_i^N = u(\omega - \hat{s}_i) + Ev[\rho^i(x_m^N)\hat{s}_i + T^i(\tilde{x}^N)]$. By claim 1, along an N *sequence*, as $i \to \infty$, expected profits of all banking groups approach zero, and therefore so do dividend payments to the share holders. Thus, $\hat{s}_i$ approaches $\hat{s}^N = \arg\max_s u(\omega - s) + E_m v[\tilde{x}_m^N s]$, and U_i^N approaches $V^N = u(\omega - \hat{s}^N) + E_m v[\tilde{x}_m^N \hat{s}^N]$.

V^N is the welfare level in the competitive limit (i.e. when the number of competing banking groups approaches infinity), when group size is kept constant at N. In the limit, banking groups lose all their monopoly power with respect to depositors, as the outcome of the Cournot game constrains them to induce each depositor to save $\hat{s}^N$.

V^N is not the first best welfare level for the economy, as along an N *sequence* of economies risk sharing occurs only within banking groups of size N. When the size of banking groups increases, the welfare level in the competitive limit, V^N, also increases, approacing the welfare level which would be achieved in an economy with no uncertainty, where all the technologies yield the mean of $\tilde{x}_k$. This welfare level is denoted V^*. The following claim makes this precise.

Claim 2 *The sequence $\{V^N\}$ is strictly increasing in N, and converges to $V^* = u(\omega - \hat{s}^*) + v[E\tilde{x}_k \hat{s}^*]$, where $\hat{s}^* = \arg\max_s u(\omega - s) + v[E\tilde{x}_k s]$. Moreover, $V^* > U_i^N$ for any economy i along any N sequence.*

Proof. $\tilde{x}_m^{N+1}$ is a mean preserving contraction of $\tilde{x}_m^N$. As depositors' utility functions are strictly concave, we have $V^{N+1} = u(\omega - \hat{s}^{N+1}) + E_m v[\tilde{x}_m^{N+1}\hat{s}^{N+1}] > u(\omega - \hat{s}^N) + E_m v[\tilde{x}_m^{N+1}\hat{s}^N] > u(\omega - \hat{s}^N) + E_m v[\tilde{x}_m^N \hat{s}^N] = V^N$, which establishes that $\{V^N\}$ is strictly increasing.

As $U_i^N \to V^N$, there is i^N such that for all $M_i \geq M_{i^N}$, $|V^N - U_i^N| < \frac{\epsilon}{2}$. By the

Law of Large Numbers $\tilde{x}_m^N \to E\tilde{x}_m^N = E\tilde{x}_k$. From this fact and from the continuity of depositors' utility functions it follows that there is N^* such that for all $N \geq N^*$, $|V^* - V^N| < \frac{\epsilon}{2}$. Then for all M_i and N such that $M_i \geq M_{i^N}$ and $N \geq N^*$, $|V^* - U_i^N| \leq |V^* - V^N| + |V^N - U_i^N| < \epsilon$, for any $\epsilon > 0$. Hence $\{V^N\}$ converges to V^*.

Consider an arbitrary economy i along an arbitrary N *sequence*. $V^* = u(\omega - \hat{s}^*) + v[E\tilde{x}_k\hat{s}^*] > u(\omega - \hat{s}^i) + v[E\tilde{x}_k\hat{s}^i] = u(\omega - \hat{s}^i) + v\{E[\rho^i(\tilde{x}_m^N) + \frac{1}{M_i}\sum_{m=1}^{M_i}[\tilde{x}_m^N - \hat{\rho}(\tilde{x}_m^N)]]\hat{s}^i\} > u(\omega - \hat{s}^i) + Ev\{\rho^i(\tilde{x}_m^N) + [\frac{1}{M_i}\sum_{m=1}^{M_i}[\tilde{x}_m^N - \rho^i(\tilde{x}_m^N)]]\hat{s}^i\} = u(\omega - \hat{s}^i) + Ev[\rho^i(\tilde{x}_m^N)\hat{s}^i + T^i(\tilde{x}^N)] = U_i^N.\|$

The operational significance of claim 2 is summarized in

Proposition 1 *If the number of stochastically independent risks (technologies) K in the economy is very large, the optimal market structure is one with many large banking groups.*

Discussion. By organizing the banks in large groups, the risk sharing within groups which is achieved substitutes almost perfectly for risk sharing through trade in state contingent claims. A large number of such groups ensures that the market power of every group is negligible. The question, of course, is what does "large" mean. Take for example the banking industry in the U.S. There are approximately 12,500 commercial banks, 3000 savings and loan institutions, and 3000 credit unions. Suppose that these institutions were organized in 100 groups of 185 banks per group. If each group were well diversified, which for a group of 185 banks in a country such as the U.S. is a feasible requirement, V^* would be approximated quite closely. There might be some non-competitive behavior on the part of the groups, but with 99 other groups around, the market power of a single group would be small.

A small economy. When the number of independent risks in the economy is not large, V^* cannot be approximated. The tradeoff between diversification and market power is still an important consideration. Consider some finite K. The problem a planner faces is to choose the optimal group size N (or equivalently, the optimal number of groups $M = \frac{K}{N}$).

Problem $(*)$ is formulated for a given group size. When N is a choice variable its solution can be written as $\bar{\rho}(N, s)$. Given $\bar{\rho}(N, s)$, the Cournot game yields $\bar{s}(N)$ and hence in equilibrium we have $\bar{\rho}(N, \bar{s}(N))$. The per capita dividend as a function of N is $\bar{T}(N) = \frac{s(\bar{N})}{(K/N)} \sum_{m=1}^{K/N} [\frac{1}{N} \sum_{k \in m} \tilde{x}_k - \bar{\rho}(N, \bar{s}(N))]$. Substitute these magnitudes into a depositor's expected utility function and maximize over N, subject to the constraint $1 \leq N \leq K$ (and the constraint that K/N is an integer). This is a well defined problem which must have a solution. Unfortunately, it is not a tractable problem. In the next section a simplifying assumption is introduced which enables me to provide an example where the optimal group size is larger than 1, i.e. the optimal number of banking groups competing with each other is lower than the maximal number possible.

3 A Simplifying Assumption

Suppose that the profits of each banking group are distributed only to the depositors who deposited with the group. There are two manners to interpret this assumption. *Interpretation a*: Every depositor owns shares of exactly one bank. Depositors have a marginal preference for depositing with the bank they own. As this preference is only marginal, depositors shop around for the best savings contract. By the symmetry of the model, in equilibrium all the banking groups offer the same savings contract, so every depositor ends up depositing with the bank he owns. *Interpretation b*: The profits banks make are not distributed as dividends. Rather, they are invested in the communities where the banks are situated (i.e. where the banks *raise* money. Note that banks may *lend* money elsewhere). Depositors have a marginal preference for depositing with the bank in their community. As this preference is only marginal, depositors shop around for the best savings contract. By the symmetry of the model, in equilibrium all the banking groups offer the same savings contract, so every depositor ends up depositing with the bank in his community. Thus, when a banking group does well, the inhabitants of the regions where the banks in the group are situated benefit. These benefits are indirect (e.g. better paying jobs, more infra-structure). In the model the benefits are captured by the profits of the banking groups accruing to their respective depositors.

With this assumption in hand, the (ex-ante random) dividend per depositor is

$$T(\tilde{x}_m^N) = \hat{s}[\tilde{x}_m^N - \rho(\tilde{x}_m^N, \hat{s})], \tag{9}$$

and the second period consumption of a depositor is

$$\begin{aligned} c_2 &= \hat{s}\rho(\tilde{x}_m^N, \hat{s}) + T(\tilde{x}_m^N) \\ &= \hat{s}\rho(\tilde{x}_m^N, \hat{s}) + \hat{s}[\tilde{x}_m^N - \rho(\tilde{x}_m^N, \hat{s})] \\ &= \hat{s}\tilde{x}_m^N. \end{aligned} \tag{10}$$

This fact simplifies the calculations substantially.[21]

Note that when deciding how much to save, depositors regard the interest rate schedule and the dividend as exogenously given. As long as $\rho(x_m^N, s) < x_m^N$ on a set having non-zero probability (i.e. as long as banking groups have some degree

[21]Equation (22) in the appendix then becomes a closed form expression for $\rho(x_m^N, s)$, which is defined uniquely and is differentiable in s, with no need for the qualification in (23).

of market power), this results in a sub-optimal choice of savings, and hence in a dead-weight loss.

Logarithmic utility. If $u(c_1) + v(c_2) = \log c_1 + \delta \log c_2$, the solution to *problem (*)* is[22]

$$\rho(x_m^N, s) = x_m^N - \frac{\delta(\omega - s) - s}{\delta(\omega - s) E_m \tilde{x}_m^N} (x_m^N)^2. \tag{11}$$

The first order conditions for expected profit maximization by banking group m in the Cournot game are (see (4) and (6))

$$-S_m \frac{1}{I} \frac{\omega E_m (\tilde{x}_m^N)^2}{\delta(\omega - s)^2 E_m \tilde{x}_m^N} + \frac{\delta(\omega - s) - s}{\delta(\omega - s) E_m \tilde{x}_m^N} = 0, \quad m = 1 \ldots M. \tag{12}$$

Second order conditions are readily verified. Summing over $m = 1 \ldots M$, and rearranging yields

$$-\frac{s\omega}{\omega - s} + M[\delta(\omega - s) - s] = 0. \tag{13}$$

Denoting the (unique) solution of (13) by $s(N)$ (where $N = \frac{K}{M}$ and K is given) and recalling (10), a depositor's expected utility in equilibrium is

$$\log[\omega - s(N)] + \delta E_m \log[s(N) \tilde{x}_m^N]. \tag{14}$$

An example. Let $\omega = \delta = 1$, and let $K > 0$ be an even number. Then equation (13) becomes a quadratic equation in s. For $M > 0$ the equation has two real roots, one strictly larger than one (which is economically irrelevant as $\omega = 1$), and the other strictly smaller than one, denoted $s(N)$. Let $\tilde{x}_k$ be equal to α, $0 < \alpha < 1$, with probability p, $0 < p < 1$, and to 1 with probability $(1 - p)$.

Suppose $N = 1$ and $M = K$. Then $\tilde{x}_m^N = \tilde{x}_k$ for $m = 1, \ldots, K$. The expected utility

[22] Using equation (22) in the appendix.

of a depositor in equilibrium is $U^1 = \log[1 - s(1)] + p \log[s(1)\alpha] + (1-p)\log s(1) = \log[1 - s(1)] + \log s(1) + p \log \alpha$.

Suppose $N = 2$ and $M = K/2$. Then $\tilde{x}_m^N$ is equal to α with probability p^2, to $\frac{\alpha+1}{2}$ with probability $2p(1-p)$, and to 1 with probability $(1-p)^2$, for $m = 1, \ldots, K/2$. The expected utility of a depositor in equilibrium is $U^2 = \log[1-s(2)]+p^2\log[s(2)\alpha]+2p(1-p)\log[s(2)\frac{\alpha+1}{2}] + (1-p)^2 \log s(2) = \log[1-s(2)] + \log s(2) + p^2 \log \alpha + 2p(1-p)\log[s(2)\frac{\alpha+1}{2}]$.

Then $U^2 - U^1 = Q + p(1-p)\log\frac{(\alpha+1)^2}{4\alpha}$, where $Q = \log[1-s(2)] + \log s(2) - \{\log[1-s(1)] + \log s(1)\}$. As $\lim_{\alpha\to 0}[\log\frac{(\alpha+1)^2}{4\alpha}] = \infty$, we have that for any $K > 0$ and $p > 0$ there is $\alpha^* > 0$ such that for all $\alpha \geq \alpha^*$, $U^2 > U^1$.

Note that the constant Q does not depend on α, which renders the analysis very simple. This seems peculiar at first sight, as Q depends on savings, which are chosen optimally by depositors given the interest rate schedule and the dividend, which depend on the distribution of $\tilde{x}_m^N$. The non-dependence of Q on α is most likely due to the logarithmic utility. Although $\rho(x_m^N, s)$ does depend on α (equation (11)), in the derivation of equation (13), α vanishes.

The above remark notwithstanding, the example is robust to slight perturbations of the parameters, as the inequality $U^2 > U^1$ is strict. In particular, it is possible to slightly relax the assumption that profits of a banking group are distributed only to the depositors who deposited with the group.

The morale of the example is summarized in

Proposition 2 *When the number of independent risks in the economy is not large, a certain degree of market power for banking groups may be socially desirable, as the gains from risk sharing between the groups may exceed the dead-weight loss* *associated with the less than perfectly competitive behavior of the groups.*

4 Concluding Remarks

The above analysis points to an important factor that regulators should consider when dealing with financial markets. Some market power in the financial sector enhances the stability of the financial system, and may alleviate pressure from regulatory agencies. In a large economy the social cost of allowing financial intermediaries to merge can be made negligible. In a small economy it has to be carefully weighed against the benefits.

As a final note, the Japanese government encourages the formation of temporary cartels in depressed sectors as part of its industrial policy (Weinstein (1992)), as cartels have a better chance of surviving the hard times. The model presented in this paper may help rationalize this phenomenon.

Appendix

Derivation of the inverse per capita demand for savings $\rho(\tilde{x}_m^N, s)$:

Step 1. Denoting the density function of $\tilde{x}_m^N$ by $f_N(\tilde{x}_m^N)$, *problem* $(*)$ can be written as

$$\max_{r(\cdot)} \int s[\tilde{x}_m^N - r(\tilde{x}_m^N)] f_N(\tilde{x}_m^N) d\tilde{x}_m^N$$

s.t.

$$-u'(\omega - s) + \int \{E_{-m} v'[r(\tilde{x}_m^N)s + T(\tilde{x}^N)]\} r(\tilde{x}_m^N) f_N(\tilde{x}_m^N) d\tilde{x}_m^N = 0 \quad ; \quad \mu \tag{15}$$

$$u(\omega - s) + \int E_{-m}\{v[r(\tilde{x}_m^N)s + T(\tilde{x}^N)] - [u(\omega) + v[T(\tilde{x}^N)]]\} f_N(\tilde{x}_m^N) d\tilde{x}_m^N \geq 0 \quad ; \quad \lambda \tag{16}$$

$$0 \leq r(x_m^N) \leq x_m^N \quad \forall x_m^N \in [0, \bar{x}] \quad ; \quad \gamma_{x_m^N}, \tag{17}$$

where $T(\tilde{x}^N)$ is given by (8).

The Kuhn-Tucker conditions for pointwise maximization are

$$\begin{aligned} -sf_N(x_m^N) + \mu E_{-m}\{v'[r(x_m^N)s + T(x_m^N, \tilde{x}_{-m}^N)] \\ + v''[r(\tilde{x}_m^N) + T(x_m^N, \tilde{x}_{-m}^N)] r(x_m^N) s\} f_N(x_m^N) \\ + \lambda E_{-m} v'[r(x_m^N)s + T(x_m^N, \tilde{x}_{-m}^N)] s f_N(x_m^N) - \gamma_{x_m^N} \leq 0, \\ r(x_m^N) \geq 0, \quad \forall x_m^N \in [0, \bar{x}] \end{aligned} \tag{18}$$

$$\gamma_{x_m^N} \geq 0, \quad r(x_m^N) \leq x_m^N, \quad \forall x_m^N \in [0, \bar{x}] \tag{19}$$

$$-u'(\omega - s) + \int \{E_{-m} v'[r(\tilde{x}_m^N)s + T(\tilde{x}^N)]\} r(\tilde{x}_m^N) f_N(\tilde{x}_m^N) d\tilde{x}_m^N = 0, \tag{20}$$

$$\lambda \geq 0, \quad u(\omega - s) + \int E_{-m}\{v[r(\tilde{x}_m^N)s + T(\tilde{x}^N)] - [u(\omega) + v[T(\tilde{x}^N)]]\} f_N(\tilde{x}_m^N) d\tilde{x}_m^N \geq 0, \tag{21}$$

with complementary slackness for (18), (19), and (21).

Step 2. At a solution to *problem (∗)* constraint (16) does not bind, and therefore $\lambda = 0$. This can be seen as follows. The second derivative with respect to s of a depositor's expected utility function is $u''(\omega - s) + E_m E_{-m} v''[r(\tilde{x}_m^N)s + T(\tilde{x}^N)][r(\tilde{x}_m^N)]^2 < 0$. Thus, any $s > 0$ which satisfies the first order condition (15) is a unique global maximum of the depositor's problem, and therefore his individual rationality constraint does not bind.

Step 3. Using (8) we note that $r(x_m^N)s + T(x_m^N) = r(x_m^N)s + \frac{s}{M}\sum_{m'=1}^{M}[x_{m'}^N - r(x_{m'}^N)] = r(x_m^N)s + \frac{s}{M}[x_m^N - r(x_m^N)] + \frac{s}{M}\sum_{m' \neq m}[x_{m'}^N - r(x_{m'}^N)] = [\frac{M-1}{M}r(x_m^N) + \frac{1}{M}x_m^N]s + T(x_{-m}^N)$, where $T(x_{-m}^N)$ is the average per capita profit of all other banking groups. For notational convenience, define $H[r(x_m^N), x_{-m}^N] = [\frac{M-1}{M}r(x_m^N) + \frac{1}{M}x_m^N]s + T(x_{-m}^N)$.

Step 4. Let $0 < r(x_m^N) < x_m^N$ for some x_m^N. Then condition (18) holds with equality and $\gamma_{x_m^N} = 0$. Rearranging, we obtain $r(x_m^N) = \frac{s - \mu E_{-m} v'[H[r(x_m^N), \tilde{x}_{-m}^N]]}{\mu s E_{-m} v''[H[r(x_m^N), \tilde{x}_{-m}^N]]}$. Substituting into (20), solving for μ, and substituting back into the expression for $r(x_m^N)$, we get

$$\rho(x_m^N, s) = \frac{su'(\omega - s) + B - AE_{-m}v'[H[\rho(x_m^N, s), \tilde{x}_{-m}^N]]}{sAE_{-m}v''[H[\rho(x_m^N, s), \tilde{x}_{-m}^N]]} \tag{22}$$

where

$$A = \int \frac{E_{-m}v'[H[\rho(\tilde{x}_m^N, s), \tilde{x}_{-m}^N]]}{E_{-m}v''[H[\rho(\tilde{x}_m^N, s), \tilde{x}_{-m}^N]]} f_N(\tilde{x}_m^N) d\tilde{x}_m^N,$$

and

$$B = \int \frac{[E_{-m}v'[H[\rho(\tilde{x}_m^N, s), \tilde{x}_{-m}^N]]^2}{E_{-m}v''[H[\rho(\tilde{x}_m^N, s), \tilde{x}_{-m}^N]]} f_N(\tilde{x}_m^N) d\tilde{x}_m^N.$$

Equation (22) defines $\rho(x_m^N, s)$ implicitly as A and B depend on $\rho(x_m^N, s)$.

Step 5. Pointwise profit maximization implies that for a given x_m^N, either $\rho(x_m^N, s) = 0$ or $\rho(x_m^N, s) = x_m^N$, but not both. Suppose that $\gamma_{\xi_{Nm}} > 0$ for some ξ_{Nm}. If $f_N(\cdot)$ is continuous at ξ_{Nm}, then by continuity of the Kuhn-Tucker conditions, $\gamma_{x_m^N} > 0$ in a neighborhood of ξ_{Nm}, and therefore $\rho(x_m^N, s) = x_m^N$ in a neighborhood of ξ_{Nm}. In this neighborhood $\rho(\cdot, s)$ is differentiable with respect to s, with derivative equal to zero. A similar argument holds for neighborhoods where $\rho(x_m^N, s) = 0$.

Step 6. Consider x_m^N such that $0 < \rho(x_m^N, s) < x_m^N$. Applying the implicit function theorem to equation (22), we find that $\rho(\cdot, s)$ is differentiable in s at (x_m^N, s) as long as

$$1 - \frac{\partial}{\partial \rho} \left\{ \frac{su'(\omega - s) + B - AE_{-m}v'[H[\rho(x_m^N, s), \tilde{x}_{-m}^N]]}{sAE_{-m}v''[H[\rho(x_m^N, s), \tilde{x}_{-m}^N]]} \right\} \neq 0, \tag{23}$$

which holds a.e. (at least generically in the parameters of the model). Thus, recalling step 5, the function $\rho(\cdot, s)$ is differentiable in s a.e.

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CHAPTER III
DISCLOSURE REGULATIONS AND THE DECISION TO ISSUE SECURITIES ON THE STOCK MARKET

1 Introduction

Firms wishing to issue securities, debt or equity, on the stock market, are required to disclose a substantial amount of private information.[1] The primary purpose of the disclosure requirements is to protect small investors from fraud, and to provide them with the information they need in order to make their own rational investment decisions.[2] The information disclosed by a firm during the process of a public offering will inevitably become available to third parties, in particular to the firm's competitors.[3] A natural question is whether this leakage of information might affect the decision of firms whether to issue securities on the stock market.[4] In some situations, a firm may want its competitors to learn any "good news" it has. If it needs to raise money, it can use the stock market as a vehicle to transmit

[1] In the U.S., before issuing a security on one of the stock exchanges, a firm is required to submit a registration statement to the Securities Exchange Commission (SEC). It includes "information about the proposed financing, and the firm's history, existing business, and plans for the future. The SEC studies this document and sends the company a "deficiency memorandum" requesting any changes. Finally an amended statement is filed with the SEC." (Brealey and Myers (1988), pp.329-330). An abridged version of the registration statement, known as a prospectus, has to be distributed to all purchasers of the security and to all those who were offered to purchase the security through the mail. The prospectus includes the exact amount raised, and the planned use of the proceeds.

[2] See Benston (1973) and Kripke (1979).

[3] Rice (1990) notes that disclosure of information may have implications for labor relations inside the firm, and for the relations of the firm with regulators and the tax authorities.

[4] Rice (1990) provides evidence from South Korea, where in late 1975 the government began to enforce disclosure regulations. This had a negative effect on the stock market which lost 2.4% of its value, suggesting that investors expected profitability to decline as a result of the increased disclosure.

its private information in a credible way. In other situations, the disclosure of particularly "good news" may trigger an undesirable reaction on the part of a firm's competitors, rendering the stock market less attractive for the firm (examples are provided below). Although I shall restrict attention to the latter case, it should be kept in mind that the analysis for the two cases is completely symmetric.[5]

By issuing securities privately, a firm can considerably reduce the amount of information which is disclosed to third parties.[6] The firm can borrow directly from institutional lenders such as insurance companies and pension funds, or from banks. Equity can also be placed privately, with institutional investors, or with venture capital firms. Thus, ignoring the numerous other factors which may affect the decision whether to issue securities publicly or privately, and focusing on the problem of revelation of private information, when there are disclosure regulations, a firm with particularly "good news" might prefer to issue securities privately in order to conceal the news from its competitors. Unfortunately, matters are not as simple as this. When disclosure is mandatory, a firm which chooses not to issue securities publicly is presumed to have something to hide. As will be shown below, all firms will prefer to issue securities publicly, for fear of being suspected of hiding information which is better than they in fact know it to be.

This conclusion is in the spirit of results by Grossman and Hart (1980), Grossman (1981), and Milgrom (1981). Milgrom proposes the term "games of persuasion," in which an agent with private information makes a report to a decisionmaker in an attempt to influence his decision. The agent cannot lie, say because of ex-post verifiabilty, but can make his report vague. If communication is costless, and the

[5]A brief discussion of the symmetric case is provided in a remark at the end of section 2.

[6]"There are no hard and fast definitions of a private placement, but the SEC has insisted that the security should be sold to no more than a dozen or so knowledgeable investors." (Brealey and Myers (1988), p.341).

decision maker can detect any withholding of information, then in equilibrium he assumes that information is withheld only if it is very unfavorable. In response, the agent will fully disclose what he knows.

It seems therefore, that disclosure regulations do not deter firms from issuing securities on the stock market. Rather, they create an incentive for them to do so. Thus, what needs to be explained is why do some firms, despite the presence of disclosure regulations, finance their investments entirely through private placements of debt or equity. The standard explanation is that transaction costs (administrative and underwriting costs) are substantially lower for private placements than for public placements. Moreover, these costs are mainly fixed.[7] The transaction costs differential in and of itself certainly provides an explanation. In the context of disclosure regulations, though, the presence of this cost differential has a further significance. When a firm issues securities privately but not publicly, it may be either because it is hiding private information which, if revealed, could be very harmful to the firm, or because it has, say, average information but finds it too costly to issue securities publicly.[8]

In the equilibria of the model presented below, firms with "good news," those who are most harmed if their private information were revealed to competitors, issue securities privately. The rest of the firms can be thought of as signaling their "bad news" by incurring the extra cost of issuing securities on the stock market.

Consider the set of firms who issue securities privately. The firms in this group

[7]See Brealey and Myers (1988), pp.340-342. James (1987) reports that firms using private placements of debt and bank loans are smaller on average than firms using public offerings of debt. Blackwell and Kidwell (1988) report similar findings. They add that public offerings are larger than private ones.

[8]This logic follows Verrecchia (1983) who showed that the introduction of fixed disclosure-related costs to a Hart-Grossman-Milgrom type model results in less than full disclosure, as withheld information can no longer be unambiguously interpreted as an attempt to hide information.

with news of quality above the group average are genuinely hiding their information. The other firms in this group would be better off if their competitors knew their true information, but the cost differential they have to incur in order to disclose it - by raising money publicly rather than privately - exceeds the benefit.

This argument relies crucially on the assumption that a firm which raises money privately, cannot credibly disclose to a third party private information, even if it desires to do so, as there is a clear incentive on its part to misrepresent the private information. Making contractual agreements regarding the truthfulness of disclosed private information is first, very costly, as in many cases ex-post verifiability of private information is hard, and second, is in violation of anti-trust laws. A firm cannot overcome this difficulty by making a unilateral promise to compensate anyone who was harmed by false information. In order for such a promise to be enforceable in court, "the plaintiff must prove more than just that someone promised him something; he must show there was a deal of some sort," (Posner (1986), p.86), which, again, amounts to admitting the existence of collusion. Moreover, the offering of securities is not an event which occurs periodically. Therefore reputational considerations cannot be invoked to sustain truthtelling. The SEC, on the other hand, enjoys considerable economies of scale in the process of verifying and enforcing truthtelling. This makes information disclosed to the SEC credible. For simplicity I assume that a firm can credibly disclose all its private information by issuing securities publicly, and cannot disclose it privately.

One implication of the model is that firms which rely solely on bank loans or venture capital, are firms who might have sensitive information, "good news" in the case we are focusing on. This highlights one special role of banks and venture capital in financial markets - the provision of debt and equity financing, respectively, *confidentially.*[9]

The accepted view regarding the special role of banks and venture capital in financial markets emphasizes their specialization in the monitoring of risky projects.[10] Recently, Diamond (1991) has pursued this view of "banks as monitors" to ask when would a firm borrow from a bank rather than issue bonds on the stock market. He studies a repeated game where firms can build a reputation for being credit worthy by repaying their loans. One result he obtains is that firms with a clean record of defaults borrow on the stock market. The other firms are forced to borrow from banks, who monitor their projects in order to reduce the probability of default.

According to this approach, firms who are perceived as being highly risky get rejected by investors in the stock market, and are forced to subject themselves to tighter control by private lenders and investors, such as banks and venture capital firms. By contrast, in the model presented in this paper, *it is the stock market which is rejected by firms* - those possessing sensitive private information, as the stock market imposes on them public disclosure of their private information.

The Diamond (1991) result and the result of this paper are complementary, as they apply to different situations. In the Diamond model firms would like to conceal information - "bad news" regarding the probability of default - from the lender. Thus, escaping from the stock market to private lenders is of no avail, as a private lender cares as much as the potential lenders on the stock market about the firm's private information. Moreover, private lenders can monitor the firm better than the stock market. Thus, concealing any information from private lenders is even harder. In the model presented in this paper firms want to conceal information - "bad news" or "good news," according to the case - from a third party, not from

[9]The role of banks as providers of confidentiality was pointed out by Campbell (1979).
[10]Diamond (1984) for banks; Barry et. al. (1990) for venture capital.

the lender or the investor. A private lender, or investor, can provide a safe haven for such a firm, whereas the stock market - when disclosure is mandatory - cannot.

The result of this paper is relevant for the literature on the timing of initial public offerings.[11] The point this paper makes in this context, is that firms who would lose a lot from the disclosure of their private information, will tend to delay the issuance of securities on the stock market.

The model, as presented in section 2, deals with debt financing only. Bankruptcy is ruled out by assumption, say because the firm in question has illiquid assets which can be used, in extremis, to repay loans. Fraud, i.e. taking the lenders' money and not producing, is also assumed to be impossible, say because the penalty for doing so and the probability of getting caught are sufficiently high. Thus, there is no strategic informational tension between the borrowing firm and lenders. The only tension is between the firm and its competitors. Under these assumptions the model's conclusions apply to equity financing as well, as long as the firm retains some positive fraction of the equity.

Section 3 deals with welfare issues. The primary purpose of disclosure regulations is to protect small investors from fraud. This factor is probably important enough to overshadow any other welfare consideration. Nevertheless, it is instructive to analyze other factors affecting the desirability of disclosure regulations. First, I ask whether the protection of investors comes at the expense of firms. I maintain the no fraud and no bankruptcy assumptions. Then, ex-post, when the private information is already known to the issuing firms, those firms with information whose quality is below average would prefer a regime of mandatory disclosure. The opposite is, of course, true for firms with information whose quality is above aver-

[11] See for example Lee, Shleifer, and Thaler (1991), who argue that initial public offerings are more prevalent when investors in the stock market are optimistic.

age. Then I turn to ex-ante analysis, asking whether mandatory disclosure would be desirable to the issuing firms before obtaining their private information. I provide conditions under which, for a sufficiently small cost differential, a regime of mandatory disclosure is beneficial ex-ante to an issuing firm. Finally, I show that competitors benefit from disclosure regulations ex-ante as well as ex-post (regardless of the cost differential, as they do not bear the costs of issuing securities by other firms).

Section 4 deals with an adverse effect that disclosure regulations may have on production. Some of the firms who choose to produce when there are no disclosure regulations, might decide not to produce when such regulations are imposed, due to the predicted reaction of competitors. Section 5 concludes.

2 Model

Description of the model. There are two firms, A and B. Firm B is about to undertake a project which requires an outlay of size $c > 0$. Having no funds, it needs to borrow the entire amount. Before borrowing takes place, firm B observes the realization θ of the random variable $\tilde{\theta}$. We shall refer to such a firm as being of type θ. This variable affects both firms' profits. Firm A does not observe θ, and has a prior, atomless, distribution of $\tilde{\theta}$ on $[\underline{\theta}, \bar{\theta}] \subset \mathbf{R}_+$, with strictly increasing c.d.f. $F(\tilde{\theta})$, which is known to firm B.

Firm B can borrow c privately or publicly. A decision by firm B of type θ to borrow privately is denoted $b(\theta) = PR$, where PR stands for "privately." If firm B borrows publicly, it is required to disclose θ. It is assumed that lying is not possible, as θ is ex-post verifiable by the SEC. Firm A observes firm B's action $b(\theta)$. Thus, if firm B decides to borrow publicly, firm A learns θ. A decision by firm B of type

θ to borrow publicly is denoted $b(\theta) = (PU, \theta)$, where PU stands for "publicly."

If firm B decides to borrow privately, firm A forms a conjecture regarding the set of types to which firm B could belong. Firm A's beliefs are summarized by the map $C(b)$:

(a) $C(PU, \theta) = \{\theta\}$,

(b) $C(PR) = \{\theta \in [\underline{\theta}, \bar{\theta}] | b(\theta) = PR\}$, non-empty,

where $C(PR)$ is conjectured by firm A. Denote firm A's expectation of firm B's type, as a function of firm B's observed action, by

$$\hat{\theta}(b) = E[\tilde{\theta} | \tilde{\theta} \in C(b)], \tag{1}$$

i.e. $\hat{\theta}(PU, \theta) = \theta$ and $\hat{\theta}(PR) = E[\tilde{\theta} | \tilde{\theta} \in C(PR)]$. Equipped with these beliefs, firm A chooses action $a(\hat{\theta}) \in \mathbf{R}_+$, which affects both firms' profits.

Let the profit function of firm B of type θ, net of the initial outlay c, but not of the fixed cost of borrowing, be

$$B(a, \theta), \tag{2}$$

satisfying

$$B_1 < 0, \quad B_2 > 0, \tag{3}$$

$$B_{12} \geq 0. \tag{4}$$

It is assumed that (1) $B(a, \theta) \geq 0$ for all $a \in \mathbf{R}_+$ and $\theta \in [\underline{\theta}, \bar{\theta}]$, and (2) $B[a(\bar{\theta}), \bar{\theta}] \geq B[a(\underline{\theta}), \underline{\theta}]$. The first assumption can be interpreted as a no bankruptcy assumption. It implies that firm B will always want to produce (revenue always exceeds the outlay c). The assumption will be relaxed in section 4. The second assumption is merely technical, and is not necessary for any of the results. It is also assumed that fraud (borrowing money but not producing) is impossible.

Let firm A's profit function be

$$A(a, \tilde{\theta}) = \tilde{\theta} f(a) - c(a), \tag{5}$$

satisfying

$$A_{11} < 0, \tag{6}$$

$$A_{12} > 0. \tag{7}$$

Firm A maximizes expected profits, so we can write its objective function as

$$A(a, \hat{\theta}) = \hat{\theta} f(a) - c(a), \tag{8}$$

where $\hat{\theta}$ is as defined in (1). As $a'(\hat{\theta}) = -\frac{A_{12}}{A_{11}}$, we have

$$a'(\hat{\theta}) > 0. \tag{9}$$

Both profit functions are continuous and twice continuously differentiable. Summarizing, firm B observes θ, and then chooses $b(\theta)$. Firm A observes b, updates its prior regarding firm B's type, and chooses a. Then firm B's production takes place, and θ becomes publicly known. Finally, profits are realized.

The following examples should clarify the economic interpretation of this setup.

Example 1. Firm B is a company seeking investment opportunities in a new market, and is conducting market research regarding profitability, measured by θ. Firm A, another company in the industry, is also planning to enter the same market, but is not conducting any market research. Firm A chooses a scale of operations level in the new market, a, at a cost $c(a)$, which generates revenue $\tilde{\theta} f(a)$, where f and c satisfy $f > 0$, $f' > 0$, $c > 0$, $c' > 0$, and $f'' \leq 0$, $c'' \geq 0$, with at least one strict inequality.

The higher is $\hat{\theta}$ - the expected profitability as perceived by firm A, the larger will be the scale of operations, a, it will choose. Firm B's profits decrease with a. Thus, firm B would like to reveal low realizations of θ - "bad news," and to conceal "good news."

Example 2. Firm B is an innovator which has just completed the development of a new product of quality θ. The product is a substitute for firm A's product, who fights back with intensity a, at a cost $c(a)$. We can think of a as marketing efforts in order to attract new customers, customer service improvements in order to prevent current customers from switching to the new product, feature enhancements, and so forth. When firm A chooses intensity a, and B's product quality is θ, the damage to firm A is $\theta f(a)$, where $f < 0$. Also, $f' > 0$, $c > 0$, $c' > 0$, and $f'' \leq 0$ and $c'' \geq 0$, with at least one strict inequality. The maximization of firm A's profit function is in fact the minimization of a loss function. As in example 1, firm B would like to conceal "good news" about its product quality, in order to minimize firm A's reaction.

In the above examples $A_{12} = f'(a) > 0$, meaning that a higher expected value of $\tilde{\theta}$ increases the marginal benefit to firm A of taking action a. This assumption will be maintained throughout the paper. In the next example, though, $A_{12} < 0$. The analysis for this case would be completely symmetric. See the remark at the end of section 2.

Example 3. Firm B is an innovator, as in example 2. The inverse demand for firm

A's product is $K - \tilde{\theta} - a$, where a is quantity produced, and K some positive constant, $K - \bar{\theta} > 0$. Let marginal cost be zero. Setting $f(a) = -a$ and $c(a) = a^2 - Ka$ (not to be interpreted as cost) yields the profit function in (5), with $A_{12} < 0$ and $a'(\hat{\theta}) = -\frac{1}{2}$.

In this example firm A accommodates when it believes that firm B's product is of high quality, say because it prefers to divert resources to other markets rather than fight. Thus, firm B would like to reveal "good news" and to conceal "bad news."

The interpretation of the condition $B_{12} \geq 0$ is that firm A's action becomes less harmful at the margin (recall that $B_1 < 0$) the "higher" the type of firm B. This condition is in fact stronger than is necessary for proving existence of equilibrium. We can allow $B_{12} < 0$, as long as it is small in absolute value. This will become apparent in the proof of Proposition 2 below.

Definition of equilibrium. An equilibrium consists of (a) A decision rule $b(\theta)$ for every type of firm B, whether to borrow publicly or privately, (b) A belief $C(b)$ by firm A regarding firm B's type, for $b = (PU, \theta)$ and $b = PR$, and (c) An action choice rule for firm A, $a(\hat{\theta})$, as a function of its beliefs regarding firm B's type, such that:

(I) For every b, $a[\hat{\theta}(b)]$ solves $\max_a A[a, \hat{\theta}(b)]$.

(II) For every $\theta \in [\underline{\theta}, \bar{\theta}]$, $b(\theta)$ maximizes $B[a[\hat{\theta}(b)], \theta]$ less the fixed cost of borrowing.

(III) $C(PU, \theta) = \{\theta\}$ and $C(PR) = b^{-1}(PR)$, i.e. firm A's conjecture is correct.

This definition simply says that both firms optimize given their information, and that firm A's beliefs about firm B's type are confirmed by firm B's action.

Let the process of borrowing publicly be at least as costly as the process of borrowing privately. Denote the difference by $\Delta \geq 0$. For simplicity, normalize to zero the fixed cost of borrowing privately.

Equilibrium. The following proposition establishes that when the two processes are equally costly, firm B of any type (except for possibly the type with the most sensitive information) will prefer to borrow publicly, for fear of being perceived as having information that is more sensitive than it in fact knows it to be.

Proposition 1 *If* $\Delta = 0$ *then there is a unique equilibrium where:*

$(a)\quad b(\theta) = (PU, \theta) \quad \textit{for} \quad \theta \in [\underline{\theta}, \bar{\theta}],$

$(b)\quad C(PU, \theta) = \{\theta\},$

$\qquad C(PR) = \{\bar{\theta}\}.$

See figure 1.

Remark. $b(\theta)$ fully reveals θ.

Remark. In fact, as firm B of type $\bar{\theta}$ is indifferent between borrowing privately or publicly, we could allow it to randomize. This does not change the nature of the equilibrium.

The proof is based on the following intuition. The set of types who borrow privately cannot contain more than one type. If this were not the case, then any type in this set with information whose quality is below the average quality of the set would resent being treated as the average type, and would prefer to borrow publicly, inducing a weaker action on the part of firm A. The type which borrows privately is $\bar{\theta}$. Suppose not, namely that some $\theta < \bar{\theta}$ is the type which borrows privately. Then type $\bar{\theta}$ would prefer to borrow privately in order to induce a weaker action on the part of firm A.

Proof of Proposition 1.

Step 1. The set $C(PR)$ is a singleton. Suppose not. Then there is some $\theta \in C(PR)$ such that $\theta < \hat{\theta}(PR)$, and thus $a[\hat{\theta}(PU, \theta)] = a(\theta) < a[\hat{\theta}(PR)]$. As $B_1 < 0$, we have $B[a[\hat{\theta}(PU, \theta)], \theta] > B[a[\hat{\theta}(PR)], \theta]$, i.e. $b(\theta) = PR$ is not optimal for type θ, namely $\theta \notin b^{-1}(PR) = C(PR)$, which is a contradiction.

Step 2. $C(PR) = \{\bar{\theta}\}$. Suppose $C(PR) = \{\theta\}$, $\theta < \bar{\theta}$. Then, for some $\theta' \in (\theta, \bar{\theta}]$, $a[\hat{\theta}(PU, \theta')] = a(\theta') > a(\theta) = a[\hat{\theta}(PR)]$, and therefore $B[a[\hat{\theta}(PU, \theta')], \theta'] < B[a[\hat{\theta}(PR)], \theta']$. Thus, $b(\theta') = (PU, \theta')$ is not optimal for θ', contradicting step 1.

Step 3. For firm B of type $\theta \in [\underline{\theta}, \bar{\theta}]$, it is optimal to choose action $b(\theta) = (PU, \theta)$, as $a[\hat{\theta}(PU, \theta)] = a(\theta) \leq a(\bar{\theta}) = a[\hat{\theta}(PR)]$, and therefore $B[a[\hat{\theta}(PU, \theta)], \theta] \geq B[a[\hat{\theta}(PR)], \theta]$.‖

We turn to the case $\Delta > 0$. Let

$$\bar{\Delta} = B[a(\underline{\theta}), \underline{\theta}] - B[a(E\tilde{\theta}), \underline{\theta}] > 0. \tag{10}$$

In order to ensure that the cost differential is not prohibitive, attention will be restricted to values of Δ such that $\Delta < \bar{\Delta}$. The next proposition establishes that when borrowing publicly is strictly more costly than borrowing privately, but not prohibitively costly, an equilibrium exists. The equilibrium must satisfy the following property: Types with sensitive information, above some cutoff value of θ, borrow privately; The other types borrow publicly.

Proposition 2 *For $\Delta \in (0, \bar{\Delta})$ an equilibrium exists, and must satisfy:*

$$(a) \quad \begin{array}{rcll} b(\theta) & = & (PU, \theta) & \textit{for} \quad \theta \in [\underline{\theta}, \theta_E), \\ b(\theta) & = & PR & \textit{for} \quad \theta \in [\theta_E, \bar{\theta}], \end{array}$$

where θ_E is some element of $(\underline{\theta}, \bar{\theta})$.

(b) $C(PU,\theta) = \{\theta\}$,

$C(PR) = [\theta_E, \bar{\theta}]$.

See figure 2. The following intuition plays a key role in the proof. Let the profit function for a firm which borrows publicly intersect the profit function for a firm which borrows privately at some θ. Consider a small increase in θ. This tends to raise profits. If the firm borrows publicly, it must disclose its type, so a higher θ entails a stronger action on the part of firm A, an effect which works against the increase in profits. If the firm borrows privately, firm A's action is fixed. Thus, the profit function for a firm which borrows privately is steeper wherever the two profit functions intersect.

Proof of Proposition 2.

Step 1. Consider the equation

$$B[a(\theta),\theta] - \Delta - B[a[E(\tilde{\theta}|\tilde{\theta} \in [\theta,\bar{\theta}])],\theta] = 0. \tag{11}$$

When $\theta = \underline{\theta}$ the left hand side is $B[a(\underline{\theta}),\underline{\theta}] - \Delta - B[a(E\tilde{\theta}),\underline{\theta}] = \bar{\Delta} - \Delta > 0$. When $\theta = \bar{\theta}$ the left hand side is $-\Delta < 0$. By continuity, there is $\theta_E \in (\underline{\theta},\bar{\theta})$ such that (11) holds. This step establishes that if firm A conjectures that the set of types who borrow privately is of the form $[\theta_E,\bar{\theta}]$, then the two profit functions must intersect at $\theta = \theta_E$.

Step 2. Let $B[a(\theta),\theta] - \Delta$ and $B[a[E(\tilde{\theta}|\tilde{\theta} \in [\theta,\bar{\theta}])],\theta]$ intersect at $\theta_E \in (\underline{\theta},\bar{\theta})$. Then

$$
\begin{aligned}
& \tfrac{d}{d\theta}\{B[a(\theta),\theta]-\Delta\}_{|\theta=\theta_E} \\
= \;& B_1[a(\theta_E),\theta_E]a'(\theta_E)+B_2[a(\theta_E),\theta_E] \\
< \;& B_2[a(\theta_E),\theta_E] && (B_1<0, a'>0) \\
\leq \;& B_2[a[E(\tilde{\theta}|\tilde{\theta}\in[\theta_E,\bar{\theta}])],\theta_E] && (\theta_E<[E(\tilde{\theta}|\tilde{\theta}\in[\theta_E,\bar{\theta}])], a'>0, B_{12}\geq 0) \\
= \;& \tfrac{d}{d\theta}B[a[E(\tilde{\theta}|\tilde{\theta}\in[\theta_E,\bar{\theta}])],\theta]_{|\theta=\theta_E}.
\end{aligned}
$$

This step establishes that if firm A conjectures that the set of types who borrow privately is of the form $[\theta_E,\bar{\theta}]$, then whenever the two profit functions of firm B intersect, the profit function when borrowing publicly has lower slope, confirming firm A's beliefs.[12]

Step 3. We now rule out equilibria where the set of types who borrow privately is not of the form $[\theta_E,\bar{\theta}]$. Suppose it is not. Then the two profit functions of firm B intersect at some $\theta^*\in(\underline{\theta},\bar{\theta})$, where the profit function when borrowing publicly is steeper.

Let θ_M be the mean of the set of types who borrow privately. By an argument analogous to the one in step 2, we obtain that whenever the two profit functions intersect to the left of θ_M, the profit function when borrowing publicly has lower slope. Thus $\theta^*\geq\theta_M$.

Consider some $\theta\geq\theta_M$. Then $a(\theta)\geq a(\theta_M)$, which implies $B[a(\theta),\theta]\leq B[a(\theta_M),\theta]$, and therefore $B[a(\theta),\theta]-\Delta<B[a(\theta_M),\theta]$. This means that the profit function when borrowing publicly lies below the profit function when borrowing privately, for all $\theta\geq\theta_M$, contradicting the assertion that the two profit functions intersect at $\theta^*\geq\theta_M$ with the profit function when borrowing publicly being steeper at this point.$\|$

[12] Note that this step would still go through if $B_{12}<0$, but small in absolute value.

Remark. The slope of the profit function for a firm which borrows publicly may be negative. For the time being we are assuming that profits are always positive, and that $B[a(\bar{\theta}), \bar{\theta}] \geq B[a(\underline{\theta}), \underline{\theta}]$. Thus, even if the profit function has negative slope, it does not cross the θ axis. The case of negative profits will be addressed in section 4. Note also that $B[a(\bar{\theta}), \bar{\theta}] - \Delta \geq B[a(\underline{\theta}), \underline{\theta}] - \Delta > B[a(\underline{\theta}), \underline{\theta}] - \bar{\Delta} = B[a(E\tilde{\theta}), \underline{\theta}] \geq 0$.

The firms who borrow privately, and are of type higher than $E\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}]$, are hiding their private information, as they are perceived as being of type $E\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}]$, but are in fact of higher type. The other firms who borrow privately, those of type lower than $E\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}]$, would prefer firm A to know their true type, but the gain from revealing it - by borrowing publicly - would be less than the cost differential Δ. For the firm of type θ_E the gain is exactly Δ. This is where the assumption that firms who borrow privately cannot credibly disclose their private information, or can do so only at a very high cost, is crucial. The firms who borrow publicly can be thought of as signaling to firm A that they are of low type. The cost of signaling is Δ.

Uniqueness. The cut-off value in the equilibrium characterized in Proposition 2 is not in general unique. The following additional assumptions guarantee uniqueness:

$$B_{11} \geq 0, \tag{12}$$

$$a''(\hat{\theta}) \leq 0, \tag{13}$$

$$\frac{d}{d\theta} E(\tilde{\theta}|\tilde{\theta} \in [\theta, \bar{\theta}]) < 1 \quad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}]. \tag{14}$$

The first assumption says that the marginal ability of firm A's action to hurt firm B is decreasing (recall that $B_1 < 0$). The second assumption is in fact an assumption on third derivatives of firm A's profit function. The last assumption is satisfied, for example, by the uniform distribution, in which case $\frac{d}{d\theta}E(\tilde{\theta}|\tilde{\theta} \in [\theta, \bar{\theta}]) = \frac{1}{2}$.

In equilibrium, the set of types who borrow privately, $[\theta_E, \bar{\theta}]$, is characterized by θ_E, which must satisfy $B[a(\theta_E), \theta_E] - \Delta = B[a[E(\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}])], \theta_E]$. Totally differentiating both sides of this equation with respect to θ_E, we get

$$\begin{aligned}
\frac{d}{d\theta_E}LHS &= B_1[a(\theta_E), \theta_E]a'(\theta_E) + B_2[a(\theta_E), \theta_E] \\
&< B_1[a[E(\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}])], \theta_E]a'(E(\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}]))\frac{d}{d\theta_E}E(\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}]) \\
&\quad + B_2[a[E(\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}])], \theta_E] \\
&= \frac{d}{d\theta_E}RHS,
\end{aligned}$$

which implies that there can be at most one value of θ_E which satisfies the equality.

Comparative statics. Total differentiation of $B[a(\theta_E), \theta_E] - \Delta = B[a[E(\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}])], \theta_E]$ with respect to Δ, using assumptions (12), (13), and (14), yields sign $\frac{d}{d\Delta}\theta_E =$ sign $\frac{d}{d\Delta}E(\tilde{\theta}|\tilde{\theta} \in [\theta_E, \bar{\theta}]) = \ominus$. In figure 2, this is a transition from point X to point Y.

$\mathbf{\Delta \rightarrow 0}$. By continuity of equation (11), the equilibria described in Proposition 2 converge, as $\Delta \rightarrow 0$, to the equilibrium described in Proposition 1.

Remark. If $A_{12} < 0$, implying that $a'(\theta) < 0$, and if $B_{12} \leq 0$ (or $B_{12} > 0$ but small), then as firm A's behavior is accommodating, firm B wants to reveal "good news" and to conceal "bad news." In equilibrium, firm B of type $\theta \in [\underline{\theta}, \theta_E]$ borrows privately. The remaining types borrow publicly, signaling their "good news." The analysis is symmetric.

3 Welfare

The first question addressed in this section is whether the protection of investors through disclosure regulations comes at the expense of firms. In this section the no fraud and no bankruptcy assumptions are maintained. The methodology will be to compare firms' profits (or expected profits) across two regimes - with and without disclosure regulations, when $\Delta = 0$. All the results continue to hold for $\Delta > 0$, but small.

Consider a regime without disclosure regulations, and let $\Delta = 0$. Then firm B is indifferent between borrowing privately and publicly. I will assume that it chooses between the two financing options at random, with equal probability. Then its profit function is $B[a(E\tilde{\theta}), \theta]$.[13] In a regime with disclosure regulations, by Proposition 1, all types of firm B borrow publicly, with profit function $B[a(\theta), \theta]$.

For all θ, $B[a(\theta), \theta]$ has lower slope than $B[a(E\tilde{\theta}), \theta]$. Also, the two functions intersect at $\theta = E\tilde{\theta}$. See figure 3. From the figure it is clear that ex-post, i.e. when firm B knows θ, types higher than $E\tilde{\theta}$ prefer a regime of no mandatory disclosure, whereas types lower than $E\tilde{\theta}$ prefer a regime of mandatory disclosure. As Δ increases, this cutoff value of θ decreases.

We turn to firm A. With no disclosure regulations firm A's profits are $A[a(E\tilde{\theta}), \theta]$. When there are disclosure regulations, as firm B always borrows publicly, firm A's choice of action is based on knowledge of θ, with profits $A[a(\theta), \theta]$. Since firm A chooses action a optimally, $A[a(\theta), \theta] \geq A[a(E\tilde{\theta}), \theta]$. Thus, firm A prefers, ex-post (i.e. when firm B knows θ), a regime of mandatory disclosure. Firm A does not bear firm B's costs of borrowing. Therefore, this result is true for any value of Δ.

[13]When $\Delta > 0$ all firms borrow privately, as it is less costly than borrowing publicly. Then $C(PR) = [\underline{\theta}, \bar{\theta}]$, with $\hat{\theta} = E\tilde{\theta}$, so the profit function is $B[a(E\tilde{\theta}), \theta]$.

Next I ask is whether firms A and B would favor a regime of mandatory disclosure ex-ante, i.e. prior to firm B knowing its type. Firm A would favor such a regime, as it prefers to be able to condition its choice of action a on more precise information. Technically, this can be seen by noting that firm A's profit function, after having optimally chosen action a, $\tilde{\theta} f[a(\tilde{\theta})] - c[a(\tilde{\theta})]$, is convex in $\tilde{\theta}$.[14] Then, $EA[a(\tilde{\theta}), \tilde{\theta}] \geq A[a(E\tilde{\theta}), E\tilde{\theta}] = (E\tilde{\theta}) f[a(\tilde{\theta})] - c[a(\tilde{\theta})] = E\{\tilde{\theta} f[a(\tilde{\theta})] - c[a(\tilde{\theta})]\} EA[a(E\tilde{\theta}), \tilde{\theta}]$.

In order to assert firm B's position regarding the institution of mandatory disclosure, we need assumptions (12), (13), (with one of these inequalities or the inequality in (4) strict), and the following additional assumption:

$$B_{22} \leq 0. \tag{15}$$

If, moreover, B_{22} is small in absolute value, we have that the second derivative of $B[a(\theta), \theta]$ with respect to θ, $B_{11}(a')^2 + B_1 a'' + 2B_{12}a' + B_{22} > 0$. Then $EB[a(\tilde{\theta}), \tilde{\theta}] > B[a(E\tilde{\theta}), E\tilde{\theta}] \geq EB[a(E\tilde{\theta}), \tilde{\theta}]$, i.e. firm B prefers, ex-ante, a regime of mandatory disclosure.

These results are summarized in

Proposition 3 *Ex-post, i.e. when firm B knows θ: (a) Firm A prefers a regime of mandatory disclosure, regardless of the value of Δ. (b) For $\Delta = 0$, firm B of type higher (resp. lower) than $E\tilde{\theta}$ prefers (resp. does not prefer) a regime of no mandatory disclosure. (c) As Δ increases, this critical value decreases. Ex-ante: (d) Firm A prefers a regime of mandatory disclosure, regardless of the value of Δ. (e) With assumptions (12), (13) (with one of these inequalities or the inequality in*

[14] The proof is analogous to the proof that the indirect profit function of a firm producing a single good, is convex in the good's price.

(4) strict), (15), and B_{22} small in absolute value, for sufficiently small Δ, firm B prefers a regime of mandatory disclosure.

Remark. $B(a, \theta) = g(\theta) - h(a)$ with g and h strictly increasing, concave, with g not too concave (e.g. $g(\theta) = \theta$), satisfies all the assumptions made so far.

4 The Effect on Production: A Linear-Uniform Example

In this section I drop the assumption that the profit function of firm B is always positive. An example is studied, in which this profit function may become negative. The essential features of the example are as follows. When there are no disclosure regulations, it is always an equilibrium for all types of firm B to produce. When disclosure is mandatory and Δ is small, high θ types prefer not to produce due to the predicted reaction of firm A. When Δ exceeds a certain threshold, an additional equilibrium appears, in which all firms produce, some borrowing publicly, the others privately. The example is followed by a discussion of the intuition for this equilibrium, and for the role played by Δ in sustaining it. A brief welfare analysis of the example concludes the section.

The example is constructed in the spirit of example 2, in section 2, where firm B is an innovator, whose product is of quality θ. Firm A does not know θ. It regards it as the random variable $\tilde{\theta} \sim U[0, 1]$. If firm B of type θ decides to produce, a decision denoted by IN, the damage caused to firm A is $(a - k)\theta$, where a is firm A's action, and $k > 0$ is a constant. If firm B of type θ decides not to produce, a decision denoted by OUT, no damage is caused to firm A.

The cost to firm A of choosing action a is $\frac{a^2}{2}$. Firm A chooses a *after* having observed firm B's entry decision. Thus, if OUT is observed, the optimal action is $a = 0$, and firm A suffers no loss. If IN is observed, firm A chooses a so as to minimize the expected loss $(a-k)\hat{\theta} - \frac{a^2}{2}$, yielding $a = \hat{\theta}$. Let $k > 1$, so that when a is chosen optimally, $(a-k)\theta < 0$, i.e. entry by firm B is always harmful to firm A. The optimal choice of a is a minimization of loss on the part of firm A, as a reaction to firm B's decision to enter. See example 2 in section 2 for possible interpretations.

If firm B of type θ chooses OUT, its profits are zero. If it chooses IN, its profits are $\alpha + \theta - \beta a$, where $\alpha, \beta > 0$ are constants. As $a = \hat{\theta}$, these profits are equal to $\alpha + \theta - \beta\hat{\theta}$.

Let α and β satisfy the following assumptions: (a) $\alpha < \beta - 1$, (b) $\alpha > \frac{\beta}{2}$. The set of (β, α) which satisfy these assumptions is a cone in the strictly positive quadrant, to the north-east of the point $\beta = 2$, $\alpha = 1$. Hence: (c) $\beta > 2$, and (d) $\alpha > 1$.

If firm B of type θ chooses IN and PU, we have $\hat{\theta} = \theta$, so its profits are $\alpha - (\beta - 1)\theta$, which are strictly decreasing in θ. This profit function intersects the θ axis from above at $\theta^* = \frac{\alpha}{\beta - 1} \in (0, 1)$. See figure 4.

Thus, when disclosure is mandatory and $\Delta = 0$, firms of types $\theta \in (\theta^*, 1]$ prefer remaining OUT to choosing IN and PU. I shall now show that in equilibrium these firms indeed choose OUT (rather than IN and PR), and that the firms of type $\theta \in [0, \theta^*]$ choose IN and PU.

Let firm A conjecture that if firm B borrows privately, it is of type $\theta \in C \subset (\theta^*, 1]$. Thus, whenever firm A observes IN and PR, it chooses action $a = \hat{\theta}$, where $\hat{\theta}$ is the mean of C. For any conjecture C by firm A such that $\hat{\theta} \in (\frac{\alpha+1}{\beta}, 1]$, the profits of firm B (of any type) when borrowing privately are $\alpha + \theta - \beta\hat{\theta} < \alpha + 1 - \beta\frac{\alpha+1}{\beta} = 0$. Recall that profits are defined to be net of the outlay c. Thus, revenue falls short of the loan c, so no private lender will agree to lend. It follows that firm B of type $\theta \in (\theta^*, 1]$ will not produce, whereas firm B of type $\theta \in [0, \theta^*]$ will produce and borrow publicly.[15] Denote this as an equilibrium of type I.

When $\Delta > 0$ the type I equilibrium is still valid, with one difference: The profit schedule when borrowing publicly cuts the θ axis to the left of θ^*, with fewer firms choosing to produce.

When Δ exceeds some threshold (to be derived below) an additional equilibrium appears, in which all types of firm B choose to produce, some borrowing publicly, and others privately. See figure 5. From the figure it is evident that this equilibrium must satisfy the following condition: $\alpha+\theta_E-\beta\theta_E-\Delta=\alpha+\theta_E-\beta\frac{\theta_E+1}{2}\geq 0$, where $\theta_E \in [0,1]$. Solving the equation, we get $\theta_E = 1-\frac{2\Delta}{\beta}$. In order to have $\theta_E \geq 0$, we need $\Delta \leq \frac{\beta}{2} \equiv \bar{\Delta}$. For profits to be positive at $\theta = \theta_E$ (and hence also at all other values of θ), we need $\Delta \geq \frac{\beta}{\beta-2}[(\beta-1)-\alpha] \equiv \Delta_L > 0$. Note that $\Delta_L < \bar{\Delta}$. Thus, for all $\Delta \in [\Delta_L, \bar{\Delta}]$ there is an equilibrium where all types of firm B choose to produce. Denote this as an equilibrium of type II.

It remains to show that when disclosure is not enforced by the SEC, it is an equilibrium for all types of firm B to choose to produce. This step is important because it makes evident that the decision to remain OUT in the type I equilibrium is a consequence of disclosure regulations. When disclosure is not enforced by the SEC, the choice between PU and PR conveys no information to firm A, so we can ignore it. If, whenever firm A observes IN, it conjectures that $\theta \in [0,1]$, then it is indeed optimal for all types of firm B to choose IN, as $\alpha+\theta-\beta a = \alpha+\theta-\beta E\tilde{\theta} = \alpha+\theta-\frac{\beta}{2} \geq \alpha-\frac{\beta}{2} > 0$ (see assumption (b) above).[16]

[15] The condition $\hat{\theta} \in (\frac{\alpha+1}{\beta}, 1]$ is also necessary for equilibrium, i.e. the above equilibrium is unique. Suppose $\hat{\theta} \leq \frac{\alpha+1}{\beta}$. Then $\alpha+\theta^{**}-\beta\hat{\theta}=0$ for some $\theta^{**} \in [0,1]$. If $\theta^{**} > \theta^*$, then $C = [\theta^{**}, 1]$. Then the two profit functions intersect at $\hat{\theta} < \theta^{**}$, but then $\hat{\theta}$ cannot be the mean of C. If $\theta^{**} \leq \theta^*$, then $C = [\hat{\theta}, 1]$ (as the two profit functions intersect at $\hat{\theta}$ where profits are positive), but then $\hat{\theta}$ cannot be the mean of C.

[16] This equilibrium is not unique. There is another equilibrium where types $\theta \in [0, \theta^*)$ choose OUT, and the rest choose IN, where θ^* is determined by $\alpha+\theta^*-\beta\frac{\theta^*+1}{2}=0$. In both equilibria it

Discussion. The appearance of equilibria where all types of firm B decide to produce when Δ is sufficiently big, requires explanation. The intuition is as follows. When Δ is small, all but the very high θ types are willing to incur the cost of signaling their true type by borrowing publicly. Thus, firm A's information is quite precise. Whenever it observes IN and PR it takes a rather vigorous action aimed at the very high θ group, which then prefers to remain OUT. As Δ increases, the set of types who prefer to borrow privately becomes larger, and its mean decreases. Firm A's action, which is tailored for the average type who might choose IN and PR, becomes weaker. When Δ is sufficiently big, firm A's action is sufficiently weak to accommodate entry by all types of firm B.

Welfare. An interesting question is whether type II equilibria, where all types of firm B produce, are socially desirable. I shall show below that firm A and firm B prefer ex-ante (i.e. before firm B learns its type) a regime where $\Delta = 0$ and the prevailing equilibrium is of type I, to a regime where $\Delta = \Delta_L$ and the prevailing equilibrium is of type II. The benefit to society from a type II equilibrium is derived from the certainty that firm B will choose to produce. Type II equilibria are socially desirable if this benefit is sufficiently big to offset the loss to firms A and B.

Let $\Delta = \Delta_L$. The two types of equilibria are depicted in figure 6a, where $0 < \theta_E = \frac{2\alpha-\beta}{\beta-2} < \frac{\alpha}{\beta-1} = \theta^* < 1$. In the type I equilibrium types $\theta \in [0, \theta^*]$ choose IN and PU; types $\theta \in (\theta^*, 1]$ choose OUT. In the type II equilibrium types $\theta \in [0, \theta_E)$ choose IN and PU; types $\theta \in [\theta_E, 1]$ choose IN and PR.

Firm A. Firm A's loss in the two of equilibria, as a function of firm B's type,

cannot occur that low θ types choose IN whereas high θ types choose OUT. This can occur only in equilibria with disclosure regulations.

are shown in figure 6b. The dotted line represents firm A's loss in the type I equilibrium. For $\theta \in [0, \theta^*]$ firm B chooses IN and PU, firm A chooses $a = \theta$, with loss $(a-k)\theta - \frac{a^2}{2} = \frac{\theta^2}{2} - k\theta$. For $\theta \in (\theta^*, 1]$ firm B chooses OUT, firm A chooses $a = 0$, with loss equal to zero.

The solid line represents firm A's loss in the type II equilibrium. For $\theta \in [0, \theta_E]$ this line coincides with the dotted line, as in both equilibria firm B chooses IN and PU revealing its type, inducing action $a = \theta$ on the part of firm A. For $\theta \in (\theta_E, 1]$ firm B chooses IN and PR, firm A chooses $a = \frac{\theta_E+1}{2}$, with loss $(a-k)\theta - \frac{a^2}{2} = -\frac{1}{2}\left(\frac{\theta_E+1}{2}\right)^2 + \left(\frac{\theta_E+1}{2} - k\right)\theta$. When $\theta = \frac{\theta_E+1}{2}$ firm A's action is optimal given firm B's type. For any other value of θ firm A's action is not optimal, yielding a higher loss. Thus, the solid line passes below the dotted line, and is tangent to it at $\theta = \frac{\theta_E+1}{2}$.

From the figure it is evident that firm A's expected profits (ex-ante) are strictly greater in the type I equilibrium. The difference is a result of two factors. The first is that in the type II equilibrium all types of firm B produce, whereas in the type I equilibrium high θ types do not. The second is that for $\theta \in [\theta_E, \theta^*]$ firm A chooses its action on the basis of less precise information.

Firm B. Consider figure 6a. Firm B's profits in the type I equilibrium are given by the area of the triangle R (with the segment $[0, \theta^*]$ as its base), and are equal to $\pi_B^I = \frac{1}{2}\alpha\theta^*$. Firm B's profits in the type II equilibrium are given by the sum of the areas of the triangles S and T (with the segments $[0, \theta_E]$ and $[\theta_E, 1]$ as bases), and are equal to $\pi_B^{II} = \frac{1}{2}(\alpha - \Delta_L)\theta_E + \frac{1}{2}(1-\theta_E)(\alpha + 1 - \beta\frac{\theta_E+1}{2})$.

Denote the difference in profits across the two types of equilibria by $D = \pi_B^{II} - \pi_B^I$. Substituting for θ^*, θ_E, and Δ_L, and rearranging, we get $D = \frac{3\beta-2}{4}\left(\frac{2\alpha-\beta}{\beta-2}\right)^2 + \frac{1}{2}[\alpha + 1 - \frac{\beta}{2} - (\alpha+1)\left(\frac{2\alpha-\beta}{\beta-2}\right)] - \frac{\alpha^2}{2(\beta-1)}$.

In the next paragraph I show that $D < 0$, i.e. firm B prefers ex-ante the type I equilibrium, even though it involves a positive probability of not producing.

Recall that $\frac{\beta}{2} < \alpha < \beta - 1$ and $\beta > 2$ (see assumptions (a), (b), and (c) above). Fix some $\beta > 2$. Then $D_{|\alpha=\frac{\beta}{2}} = \frac{-\beta^2+4\beta-4}{8(\beta-1)} < 0$, $D_{|\alpha=\beta-1} = 0$, $\left(\frac{d}{d\alpha}D\right)_{|\alpha=\frac{\beta}{2}} = -\frac{2}{\beta-2} - \frac{\beta}{2(\beta-)} < 0$, $\left(\frac{d}{d\alpha}D\right)_{|\alpha=\beta-1} = \frac{\beta}{\beta-2} > 0$, and $\frac{d^2}{d\alpha^2}D = 3\beta^2 - 4 > 0$. Hence $D < 0$.

Summarizing, both firms prefer the type I equilibrium ($\Delta = 0$). If firm B's innovation is sufficiently important to consumers or to other producers so as to offset the loss to firms A and B, then the type II equilibrium (with $\Delta = \Delta_L$) is socially desirable.

5 Concluding Remarks

The central role played by Δ in sustaining the equilibrium in the above example (and in the equilibrium in section 2) - where all types produce, some borrowing publicly and others privately, renders one curious about the nature of Δ.

The straightforward interpretation of Δ is that it reflects the extra administrative costs involved in a public offering of securities. Δ can also be viewed as the compensation underwriters require for the risk involved in initial public offerings, as in most cases underwriters guarantee the issuing firm a fixed price for the securities, before having sold them to the public. It may also be that Δ represents a monopoly rent, as the underwriting industry is not perfectly competitive (see Hayes, Spence, and Van Praag Marks (1983), chapter 2). Finally, a SEC filing for a public offering obliges a firm to make no material changes of any kind for at least sixty days. This loss of flexibility is sometimes quite costly.[17]

[17] *FORTUNE*, February 10, 1992, pp.122-123.

One consequence of the model is that a universal stock market, where all securities are traded, and nowhere else, might have some drawbacks. The logic is similar to the one in Diamond (1991). There, in the absence of banks, risky projects might not be undertaken as a consequence of the inability of the stock market to efficiently monitor these projects. The presence of banks, who specialize in monotoring, solves the problem. Here, when there are disclosure regulations, some profitable projects might not be undertaken as a result of the information which might leak to third parties. A less perfect (i.e. with transaction costs differentials) and more differentiated financial system, as the one described in this paper, may result in these projects being undertaken.

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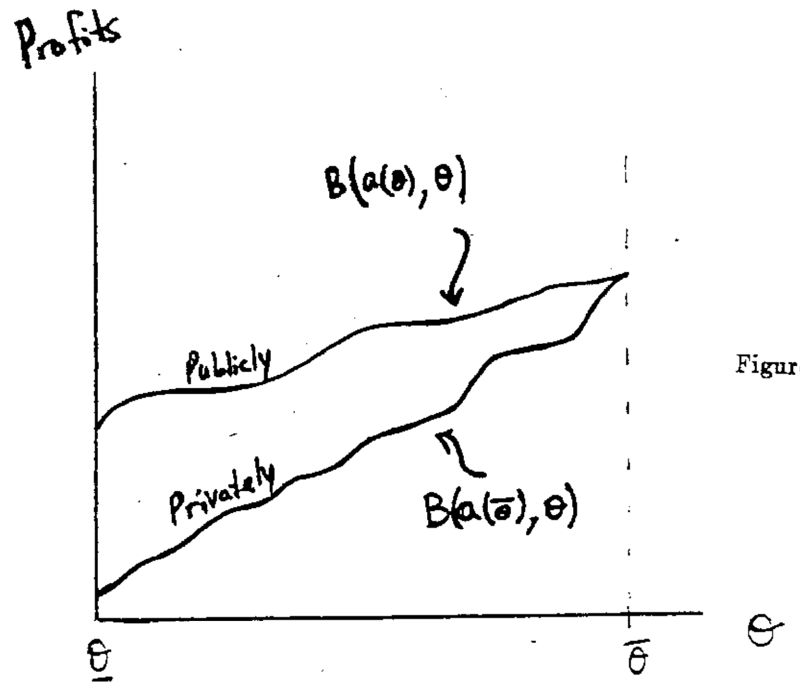


Figure 1

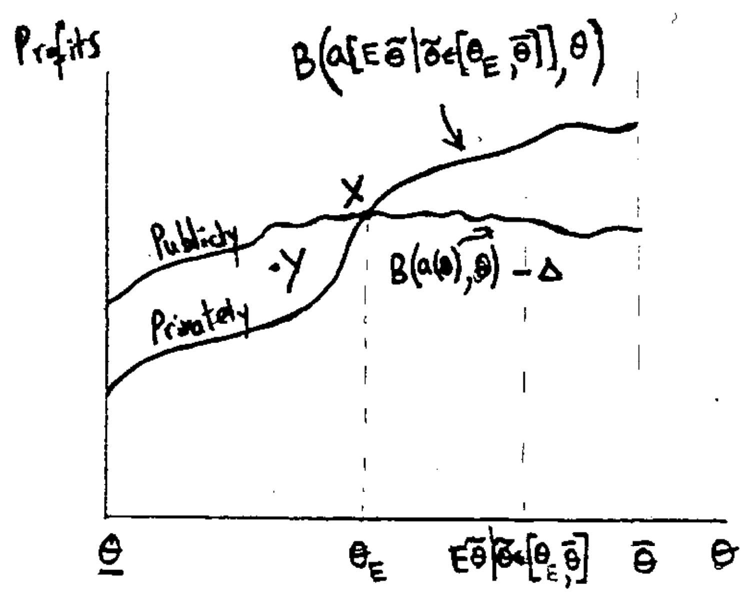


Figure 2

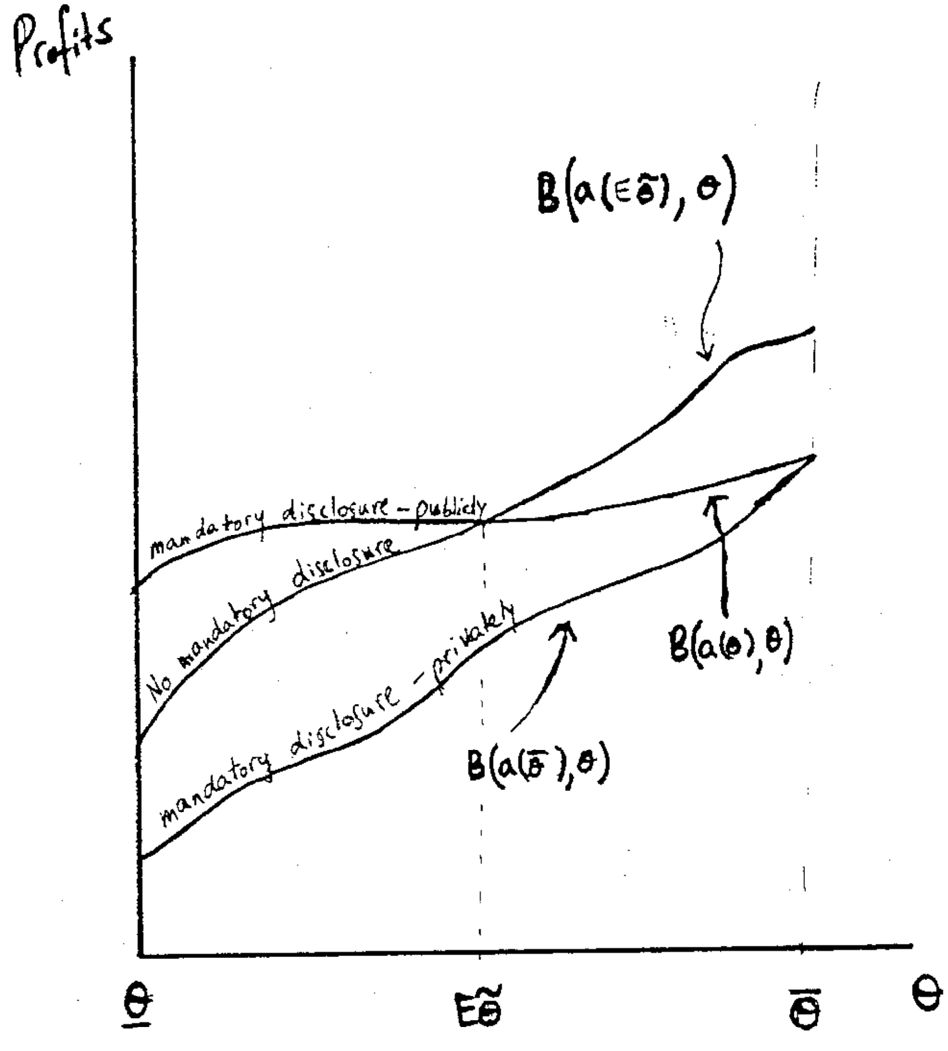
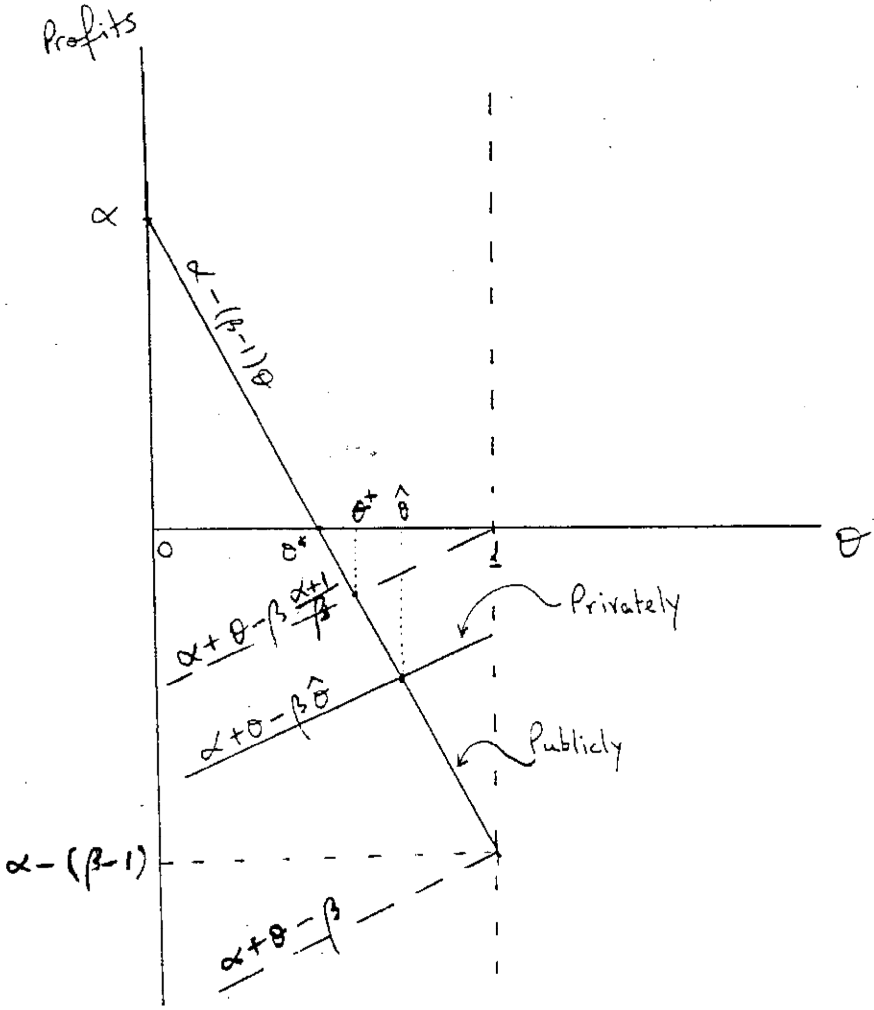
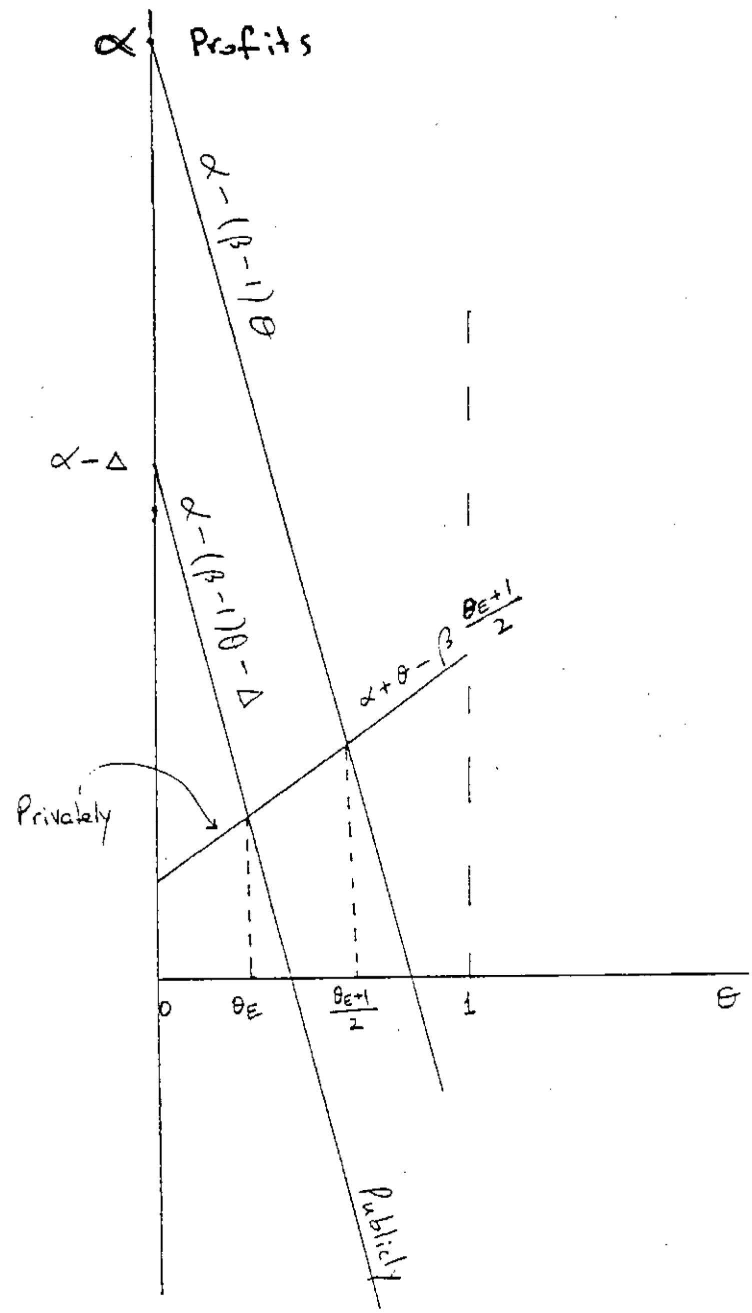

Profits
B(a(Eθ̃), θ)
mandatory disclosure - publicly
No mandatory disclosure
mandatory disclosure - privately
B(a(θ), θ)
B(a(θ̃), θ)
θ
Eθ̃
θ̄
θ

Figure 3



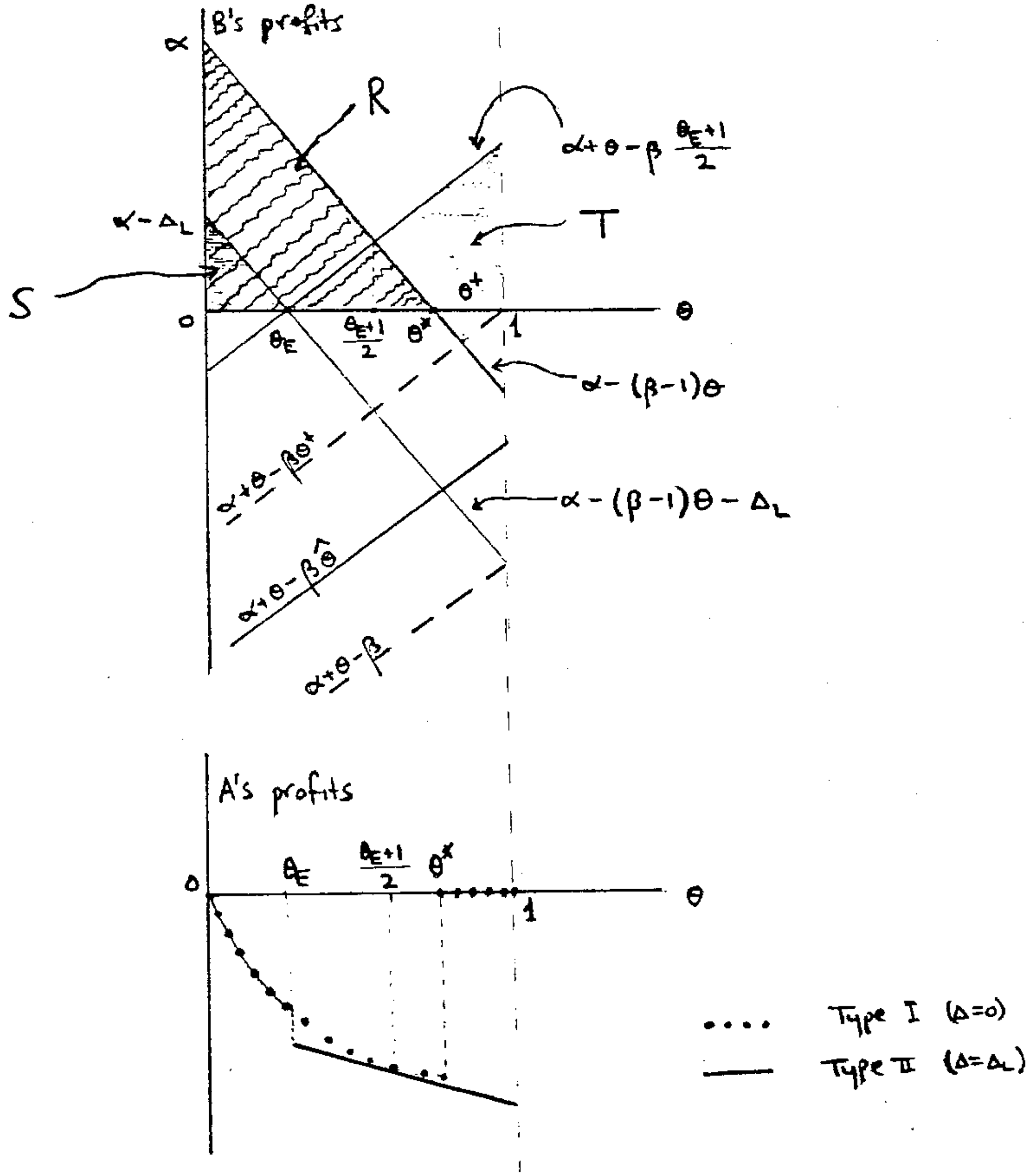
Profits
α
$\alpha-(\beta-1)\theta$
θ^+
$\hat{\theta}$
θ
0
θ^*
1
$\alpha+\theta-\beta\frac{\alpha+1}{\beta}$
Privately
$\alpha+\theta-\beta\hat{\theta}$
Publicly
$\alpha-(\beta-1)$
$\alpha+\theta-\beta$

Figure 4



α
Profits
$\alpha - (\beta - 1)\theta$
$\alpha - \Delta$
$\alpha - (\beta - 1)\theta - \Delta$
$\alpha + \theta - \beta \frac{\theta_E + 1}{2}$
Privately
Publicly
0
θ_E
$\frac{\theta_E + 1}{2}$
1
θ

Figure 5



B's profits
α
R
α + θ − β (θ_E+1)/2
α − Δ_L
T
S
θ+
0
θ
θ_E
(θ_E+1)/2
θ*
1
α − (β−1)θ
α+θ−βθ+
α − (β−1)θ − Δ_L
α+θ−βθ̂
α+θ−β
A's profits
0
θ_E
(θ_E+1)/2
θ*
θ
1
Type I (Δ=0)
Type II (Δ=Δ_L)

Figure 6

CHAPTER IV
INFORMATION REVELATION IN A MARKET WITH PAIRWISE MEETINGS:
THE ONE SIDED INFORMATION CASE[1]

1 Introduction

Wolinsky [5] shows that in a decentralized market with asymmetric information, where transactions are concluded in pairwise meetings of agents, and prices are not called[2], the process of trade does not fully reveal the private information of informed agents to the uninformed, even when the market becomes approximately frictionless, that is when agents become almost infinitely patient. Paraphrasing Wolinsky, a non-negligible fraction of those who are uninformed when they enter the market, end up transacting at a price which they would not want to transact at, had they known the true state of the world. Such a price, which Wolinsky calls the wrong price, could not have arisen in a Walrasian economy with complete and symmetric information, nor in a fully revealing rational expectations equilibrium. In this sense, Wolinsky's decentralized economy does not resemble a centralized one even if the cost of staying in the market and learning becomes negligible.

Impatience is captured by a constant discount factor δ, so instead of saying "as agents become almost infinitely patient" we shall simply say "as $\delta \rightarrow 1$."

The information structure in Wolinsky's model is two sided - there are uninformed agents among sellers and buyers. Three forces are at work in his model. Force CL (Cost of Learning): As $\delta \rightarrow 1$ it is less costly for the uninformed to

[1] Joint work with Roberto Serrano.
[2] See Osborne and Rubinstein [4] for a survey of this literature.

remain in the market and learn from meetings with other agents which may be informed. Force N (Noise): As $\delta \to 1$ the informative content of the pairwise meetings decreases because there are more uninformed agents on both sides of the market trying to learn. Force MI (Misrepresentation of Information): As $\delta \to 1$ the informative content of the pairwise meetings decreases because it costs less for informed agents to try and extract surplus from the uninformed (e.g. a seller who knows that the quality and production cost of the good are low, will ask for a high price in the hope that the buyer he faces is uninformed and will agree.) Force CL drives the economy towards the right price. Forces N and MI drive it towards the wrong price.

It turns out that although force MI becomes stronger as $\delta \to 1$ at the individual level, in the aggregate it becomes negligible because as $\delta \to 1$ the agents behind force MI become a negligible proportion of the market. Thus, as $\delta \to 1$ what may prevent agents from learning is force N. Wolinsky shows that in at least one state of the world force N overcomes force CL, and trade occurs at the wrong price.

Gale [1] conjectures that Wolinsky's result depends crucially on the two sided information structure. He provides a one sided information model in which all sellers are informed whereas all buyers are not, and which departs from Wolinsky's model in several other ways.[3] Our work confirms Gale's conjecture, but it departs from Wolinsky's model only in that all sellers are informed. We assume that the sellers of a good know the true state of the world (say the quality of the good), but some of the buyers do not.[4]

In the one sided information model force N is eliminated. However, the presence of force MI renders the model non-trivial. When δ increases, sellers, who are all

[3] We provide more details at the end of section 3.

[4] The reverse yields an analogous model. For example, if the state of the world is the probability of discovering oil in a certain region, the buyers of drilling rights - the oil companies - may know this probability with more precision than the government selling these rights.

informed, tend to stay longer in the market, but only for the reason in force MI. Uninformed buyers tend to stay longer in the market as well and learn - force CL. The interplay between these two forces determines whether trade will occur at a right or at a wrong price.

In section 2 we study the one sided information model and characterize the set of equilibria. In section 3 we study the behavior of the equilibria as the market becomes approximately frictionless. Our results are quite different from Wolinsky's result for two sided information. We find that for any one sided information economy there is an equilibrium where trade occurs at the right price. Moreover, there is an open set of economies where in all the equilibria trade occurs at the right price. In section 4 we present the essence of Wolinsky's model and some discussion.

2 The One Sided Information Model

Time runs discretely from $-\infty$ to ∞. All periods are identical. In the beginning of each period M sellers and M buyers enter the market. Sellers want to sell one unit of an indivisible good, and buyers want to buy one unit of it. Each period sellers and buyers are randomly matched. Each meeting results in an agreement or a disagreement. Those who agree transact and exit the market, and those who disagree stay in the market to be matched anew. Thus, the number of buyers in the market is always the same as the number of sellers. All agents discount future payoffs by a constant factor $\delta \in (0, 1)$.

There are two states of the world. In state H buyers have a high valuation u_H for the good, and the cost of the good to the sellers c_H is also high. In state L the valuation u_L and the cost c_L are low.

When matched, agents make simultaneous announcements. Each agent can send one of two possible messages: h or l. If both agents say h they trade at price p^{hh}. If both say l they trade at price p^{ll}. If the buyer says h and the seller l they trade at price p^{hl}. Finally, if the seller says h and the buyer l there is disagreement. The payoff of perpetual disagreement is zero. It is convenient to refer to messages as "tough" (h for the seller and l for the buyer) and "soft" (l for the seller and h for the buyer). It is assumed that

$$c_L < p^{ll} < u_L < p^{hl} < c_H < p^{hh} < u_H. \tag{1}$$

In state H the price p^{hh} is defined as the right price, as at this price there are gains from trade for buyer and seller. The other two prices are defined to be wrong because the seller loses when transacting at these prices. Similarly, in state L the price p^{ll} is right and the other prices are wrong.

All sellers are informed, i.e. know the true state of the world. A fraction $x_B \in (0,1)$ of buyers who enter the market each period are also informed. The rest have a common prior belief $\alpha_H \in (0,1)$ which is the probability that the state of the world is H. The case $x_B = 0$ will be treated separately.

Note that disagreement occurs only if both buyer and seller play "tough." Thus, if an agent plays "soft" in some period, he will surely transact and leave the market. It follows that the only relevant decision variable for an agent is the number of periods during which he will play "tough." Let n_{SH} be this number for a seller who knows that the true state is H. Similarly for n_{SL}, n_{BH}, and n_{BL}. Let n_B be this number for an uninformed buyer. The true state of the world is either H or L, but an uninformed buyer must take into account what a seller would do if he knew that the true state is H, or knew that the true state is L.

Uninformed buyers extract information from the announcement of their (informed) trading partners. By playing "soft" an uninformed buyer ensures trade, taking the risk of a transaction at a disadvantageous price, and forgoing the opportunity to learn. By playing "tough" he ensures that if trade occurs, it will be at a price which is advantageous for him, and if trade does not occur he will learn, updating α_H. Thus, a strategy n_B can be interpreted as a decision to keep sampling sellers for n_B periods, as long as exit has not yet occurred. The cost of sampling an additional seller is captured by δ.

Let S_H^h be the proportion of the total number of sellers in the market who in state H say h. Similarly for S_L^h, B_H^l, and B_L^l. Note that these are the proportions of agents who play "tough." Agents know the distribution of announcements amongst their trading partners in each state of the world. Uninformed buyers, though, cannot observe the prevailing distribution (S_L^h or S_H^h). Otherwise they would be able to infer the state. Let K_H and K_L be the total number of sellers (and therefore of buyers) in the market in state H and in state L. The market is said to be in steady state when these six numbers are constant through time. The analysis is performed in stationary steady state only (From now on we shall simply say steady state).

Let $V_B(n; \alpha_H, S_H^h, S_L^h)$ be the expected payoff of strategy n to an uninformed buyer who believes with probability α_H that the state of the world is H. Let $V_{SH}(n; B_H^l)$ be the expected payoff of strategy n to a seller in state H. Similarly for $V_{SL}(n; B_L^l)$, $V_{BH}(n; S_H^h)$, and $V_{BL}(n; S_L^h)$. Note that uninformed buyers take into account the steady state proportions of "tough" sellers in both states of the world, whereas informed agents take into account the proportions of "tough" trading partners only in the state of which they are informed.

Claim 1 *(a) $n_{SH} = \infty$, (b) $n_{BL} = \infty$, (c) $n_{BH} = 0$, (d) In steady state $n_B < \infty$.*

Proof. (a) A seller who knows that the state is H understands that declaring l in some period will entail immediate trade at a sure loss. (b) Similarly for a buyer who knows that the state is L. (c) A buyer who knows that the state is H understands that sellers always declare h. Thus it is optimal for such a buyer to play "soft" right away, as playing "tough" for any number of periods will simply delay the payoff $u_H - p^{hh}$. (d) Suppose $n_B = \infty$. Then as $n_{SH} = \infty$, in state H uninformed buyers never trade. Also, as $n_{BH} = 0$, the number of buyers who leave the market each period in state H is precisely $x_B M < M$, which cannot happen in a steady state.$\|$

The market is said to be in equilibrium if each agent maximizes his expected payoff and the market is in steady state. An equilibrium is therefore fully described by eleven numbers: n_B, n_{SH}, n_{SL}, n_{BH}, n_{BL}, S_H^h, S_L^h, B_H^l, B_L^l, K_H and K_L. Using claim 1, the following eleven conditions characterize the equilibrium.

$$M = K_H(1 - S_H^h B_H^l). \tag{2}$$

$$M = K_L(1 - S_L^h B_L^l). \tag{3}$$

$$\begin{aligned} K_L(1 - B_L^l) &= M[x_B(S_L^h)^{n_{BL}} + (1 - x_B)(S_L^h)^{n_B}]. && (4) \\ K_H(1 - S_H^h) &= M(B_H^l)^{n_{SH}}. && (5) \\ K_L(1 - S_L^h) &= M(B_L^l)^{n_{SL}}. && (6) \end{aligned}$$

$$B_H^l = \frac{(1 - x_B)n_B}{(1 - x_B)(n_B + 1) + x_B}. \tag{7}$$

$$\begin{aligned} n_{SL} &\in \arg\max_n V_{SL}(n; B_L^l). && (8) \\ n_B &\in \arg\max_n V_B(n; \alpha_H, S_H^h, S_L^h). && (9) \end{aligned}$$

$$n_{SH} = \infty. \tag{10}$$

$$n_{BH} = 0. \tag{11}$$

$$n_{BL} = \infty. \tag{12}$$

Equations (2) and (3) are the steady state conditions for the market size in the two states of the world. Consider (2). The left hand side is the number of entering sellers (or buyers), and the right hand side is the number of sellers leaving the market, which are all the sellers in the market except for those who disagreed. The term $S_H^h B_H^l$ is the proportion of agents who disagreed, namely sellers who played "tough" and met buyers who also played "tough." Analogously for (3).

Consider (4). The left hand side is the number of buyers in state L who play "soft," i.e. those who say h. The first term on the right hand side, $Mx_B(S_L^h)^{n_{BL}}$, is the number of informed buyers who had planned to play "tough" for n_{BL} periods, and met "tough" buyers every time (the probability of this event is $(S_L^h)^{n_{BL}}$). They have now switched, as planned, to saying h. The second term, $M(1-x_B)(S_L^h)^{n_B}$, is analogous for the uninformed buyers. The right hand side is therefore the number of buyers who have switched this period to saying h. This is also the total number of buyers who are saying h because any buyer who said h in the previous period has for sure transacted and left the market. Similarly for equations (5) and (6), where the right hand side includes only one term, as there are no uninformed sellers.

Equations (4), (5), and (6) are the stationarity conditions for the proportions of "tough" buyers in state L, and of "tough" sellers in the two states. Note that the condition for buyers in state H, $K_H(1 - B_H^l) = M[x_B(S_H^h)^{n_{BH}} + (1 - x_B)(S_H^h)^{n_B}]$, is missing. The reason is as follows. $n_{SH} = \infty$ means that $S_H^h = 1$, as all sellers are informed. Together with $n_B < \infty$ this entails $(S_H^h)^{n_B} = 1$. Also, $n_{BH} = 0$. Therefore this equation is identical to (2). The explanation is simple enough. As sellers are all “tough,” trade - and therefore exit - occurs whenever a buyer switches to “soft.” Thus, the number of buyers who play “soft” in a given period is also the number of buyers who trade and exit the market in this period.

Equation (7) completes the system. It is derived as follows. Buyers who play “tough” do not trade, as sellers are also playing “tough.” Therefore, all the uninformed buyers who entered the market $n_B - 1$ periods ago or later are still in the market and are playing “tough.” Their number is $M(1 - x_B)n_B$. The uninformed buyers who entered the market n_B periods ago are also in the market but are playing “soft.” Their number is $M(1 - x_B)$. Uninformed buyers who entered prior to that have already left the market. Thus, the total number of uninformed buyers in the market is $M(1-x_B)+M(1-x_B)n_B$. Informed buyers remain in the market for exactly one period. Therefore the total number of buyers in the market is $M(1 - x_B) + M(1 - x_B)n_B + Mx_B$, and the fraction of “tough” buyers is as in (7).

We turn to the determination of n_{SL}. A seller in state L faces the following trade-off, described in figure 1. If he plays “soft” he will trade at price p^{ll} or at price p^{hl}. If he plays “tough” he will either make a very successful trade at price p^{hh}, or will disagree with his trading partner and remain in the market. Being informed, his prior belief regarding the state of the world does not change as a result of the announcement of his trading partner. Therefore, if he remains in the market an additional period the problem he faces is identical to the one he faced in the preceding period.

The stationarity of the seller’s problem implies that it can be solved by comparing the expected gain from playing “tough” for one period and then switching to “soft,” to the expected gain from playing “soft” right away. The difference in expected gain is

$$
\begin{aligned}
\Delta V_{SL} &= V_{SL}(1; B_L^l) - V_{SL}(0; B_L^l) \\
&= (1 - B_L^l)(p^{hh} - c_L) + B_L^l \delta[(1 - B_L^l)(p^{hl} - c_L) + B_L^l(p^{ll} - c_L)] \\
&\quad - B_L^l[(1 - B_L^l)(p^{hl} - c_L) + B_L^l(p^{ll} - c_L)] \\
&= \delta(p^{ll} - p^{hl})(B_L^l)^2 + [\delta(p^{hl} - c_L) + p^{hl} - p^{hh} + c_L - p^{ll}]B_L^l \\
&\quad + p^{hh} - p^{hl}.
\end{aligned} \tag{13}
$$

This establishes

Claim 2 *If* $\Delta V_{SL} > 0$ *then* $n_{SL} = \infty$. *If* $\Delta V_{SL} < 0$ *then* $n_{SL} = 0$. *If* $\Delta V_{SL} = 0$ *then* $n_{SL} \in \{0, \ldots, \infty\}$.

The uninformed buyer's problem is not stationary, for if he encounters a "tough" seller who says h, he revises α_H. The event tree following a decision to play "tough" for n periods by a buyer with prior α_H is described in figure 2. The expected gain from such a decision, recalling that $S_H^h = 1$, is

$$
\begin{aligned}
V_B(n; \alpha_H, 1, S_L^h) &= \alpha_H \delta^n (u_H - p^{hh}) \\
&\quad + (1 - \alpha_H)(S_L^h)^n \delta^n [S_L^h(u_L - p^{hh}) + (1 - S_L^h)(u_L - p^{hl})] \\
&\quad + (1 - \alpha_H) \textstyle\sum_{t=0}^{n-1} (S_L^h)^t (1 - S_L^h) \delta^t (u_L - p^{ll}).
\end{aligned} \tag{14}
$$

The first term is the discounted expected gain if the state is H. Playing "tough" for n periods results in disagreement, as sellers are also playing "tough." Switching to "soft" in period $n+1$ entails trade at p^{hh}. The other two terms describe the payoff if the state is L. The first of these terms is the expected loss if the buyer is unlucky enough to encounter "tough" sellers n times, an event which happens with probability $(S_L^h)^n$. In period $n+1$, switching to "soft" results in trade with a "tough" or with a "soft" seller. In both cases the buyer loses. The last term is the discounted expected gain from finding a "soft" seller in one of the n periods in which the buyer is playing "tough." n_B maximizes the expression in (14).

Beliefs of an uninformed buyer evolve along the tree in figure 2 as follows. If the buyer hears h from the seller (points A_1 and B_1), α_H is updated to

$$\alpha_{H1} = \frac{\alpha_H}{\alpha_H + (1-\alpha_H) S_L^h}, \tag{15}$$

and so forth recursively (points A_2 and B_2 etc.).

The possible best responses n_B for an uninformed buyer are established in

Claim 3 *(a) If $S_L^h = 1$ then*

$$n_B \begin{cases} = & 0 \\ \in & \{0, \ldots, \infty\} \\ = & \infty \end{cases} \quad as \quad \alpha_H \begin{cases} > \\ = \\ < \end{cases} \frac{p^{hh}-u_L}{u_H-u_L},$$

i.e. the prior α_H determines whether V_B is strictly increasing in n, strictly decreasing, or flat.

(b) If $S_L^h < 1$ then either there is a unique $n_B \in \{0, \ldots, \infty\}$ which maximizes V_B, or there are two consecutive integers n_B, $n_B + 1$, which maximize V_B.

Proof. Note first that the third term in (14) is equal to $(1-\alpha_H)(1-S_L^h)(u_L - p^{ll})\frac{1-(S_L^h\delta)^n}{1-S_L^h\delta}$. (a) If $S_L^h = 1$ then, as the payoff of perpetual disagreement is zero, $V_B = 0$ for $n_B = \infty$. For $n_B < \infty$, $V_B = \delta^n[\alpha_H(u_H - p^{hh}) + (1-\alpha_H)(u_L - p^{hh})]$, which is strictly positive and strictly decreasing if $\frac{p^{hh}-u_L}{u_H-u_L} > 0$ (hence $n_B = 0$), strictly negative if $\frac{p^{hh}-u_L}{u_H-u_L} < 0$ (hence $n_B = \infty$ with $V_B = 0$), and equal to zero if $\frac{p^{hh}-u_L}{u_H-u_L} = 0$ (hence $n_B \in \{0, \ldots, \infty\}$).

(b) Consider the following continuous and differentiable function of $x \in \mathbf{R}_+$:

$$
\begin{aligned}
V_B(x;\alpha_H,1,S_L^h) &= \alpha_H\delta^x(u_H-p^{hh}) \\
&\quad+(1-\alpha_H)(S_L^h)^x\delta^x[S_L^h(u_L-p^{hh})+(1-S_L^h)(u_L-p^{hl})] \\
&\quad+(1-\alpha_H)(1-S_L^h)(u_L-p^{ll})\tfrac{1-(S_L^h\delta)^x}{1-S_L^h\delta}.
\end{aligned}
\tag{16}
$$

Its first derivative is

$$
\begin{aligned}
\tfrac{\partial}{\partial x}V_B(x;\alpha_H,1,S_L^h) &= \alpha_H\delta^x(\log\delta)(u_H-p^{hh}) \\
&\quad+(1-\alpha_H)[(S_L^h)^x\delta^x\log(S_L^h\delta)][S_L^h(u_L-p^{hh})+(1-S_L^h)(u_L-p^{hl})] \\
&\quad-(1-\alpha_H)\tfrac{1-S_L^h}{1-S_L^h\delta}(u_L-p^{ll})(S_L^h\delta)^x\log(S_L^h\delta).
\end{aligned}
\tag{17}
$$

One of the following is true: (I) $\frac{\partial}{\partial x}V_B(x)>0$ for all x, (II) $\frac{\partial}{\partial x}V_B(x)<0$ for all x, or (III) $\frac{\partial}{\partial x}V_B(x^*)=0$ for some x^*. If (I) is true, then $V_B(n)$ is strictly increasing in n, and $n_B=\infty$. Similarly, if (II) is true, then $n_B=0$. If (III) is true, then note that $\frac{\partial^2}{\partial x^2}V_B(x^*)<0$, implying that x^* is a unique global maximum of $V_B(x)$. Thus, $V_B(n)$ is maximized at n_B, at n_B+1, or at both, where n_B is the integer part of x^*.$\|$

The system of equations which characterizes the equilibrium must be modified whenever best responses are not singletons. Suppose that uninformed buyers are indifferent between n_B and n_B+1. Define $g_B\in[0,1]$ as the fraction of uninformed buyers who adopt strategy n_B, while the remaining uninformed buyers adopt strategy n_B+1. Then equations (4) and (7) become

$$K_L(1-B_L^l)=M\{x_B(S_L^h)^{n_{BL}}+(1-x_B)[g_B(S_L^h)^{n_B}+(1-g_B)(S_L^h)^{(n_B+1)}]\}. \tag{18}$$

$$B_H^l=\frac{(1-x_B)[g_Bn_B+(1-g_B)(n_B+1)]}{(1-x_B)[g_Bn_B+(1-g_B)(n_B+1)+1]+x_B}. \tag{19}$$

Suppose that uninformed buyers are indifferent between any $n_B \in \{0, \ldots, \infty\}$. Define $r_B \in [0,1]$ as the fraction of uninformed buyers who adopt strategy $n_B = 0$, while the remaining uninformed buyers adopt strategy $n_B = \infty$. Then equations (4) and (7) become[5]

$$K_L(1 - B_L^l) = M[x_B(S_L^h)^{n_{BL}} + (1 - x_B)r_B]. \tag{20}$$

$$B_H^l = (1 - x_B)(1 - r_B). \tag{21}$$

If sellers in state L are indifferent between any $n_{SL} \in \{0, \ldots, \infty\}$, we define $r_S \in [0,1]$ in an analogous manner, replacing equation (6) by

$$K_L(1 - S_L^h) = Mr_S. \tag{22}$$

Thus far, we have specified the equations which determine the equilibrium, and have established agents' best responses. Next, we turn to the issue of existence. We begin by computing two classes of "boundary" equilibria. We use them to establish existence of additional, "interior" ones, fully characterizing the set of equilibria of the model. The extreme one sided information case, $x_B = 0$, will be studied at the end of the section.

Claim 4 *In equilibrium $S_L^h < 1$.*

Proof. Suppose $S_L^h = 1$. Then only "soft" buyers trade. As $n_{BL} = \infty$ (claim 1), informed buyers never trade. Therefore the number of buyers who leave the market each period is at most $(1 - x_B)M < M$, which cannot happen in a steady state.‖

[5] As uninformed buyers are indifferent between any $n_B \in \{0, \ldots, \infty\}$, there may be equilibria where these agents choose strategies other than $n_B = 0$ and $n_B = \infty$. Any such equilibrium is equivalent to an equilibrium where the only strategies which are chosen are $n_B = 0$ and $n_B = \infty$, for some value of $r_B \in [0,1]$. See Wolinsky [5], p. 10, footnote 2.

Claim 5 *In equilibrium* $\Delta V_{SL} \leq 0$.

Proof. If $\Delta V_{SL} > 0$ then $n_{SL} = \infty$, implying that $S_L^h = 1$, contradicting claim 4.‖

Claim 6 *If*

$$\alpha_H < \frac{p^{hl} - p^{ll}}{(1-\delta)(u_H - p^{hh}) + p^{hl} - p^{ll}}, \tag{23}$$

there is an equilibrium where $\Delta V_{SL} < 0$*. In this equilibrium* $n_{SL} = 0$ *and* $n_B = 1$*.*

Proof.
Step 1. Suppose $\Delta V_{SL} < 0$, implying that $n_{SL} = 0$ and $S_L^h = 0$. Then for $n \geq 1$, $V_B(n; \alpha_H, 1, 0) = \alpha_H \delta^n (u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{ll})$, and for $n = 0$, $V_B(n; \alpha_H, 1, 0) = \alpha_H (u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{hl})$. Note that for $n \geq 1$, $V_B(n; \alpha_H, 1, 0)$ is strictly decreasing in n. The intuition is that the sellers' behavior is fully revealing. Thus, after one period of disagreement a buyer learns that the state is H and therefore that sellers are "tough." Insisting on l for more periods only delays the payoff $u_H - p^{hh}$.

The trade-off which the buyer faces is between $n_B = 0$, i.e. playing "soft" right away, and $n_B = 1$, i.e. hoping to trade at p^{ll} in the first period, or to learn that the state is H, switch to h, and trade at p^{hh} in the second period.

Step 2. If $V_B(1; \alpha_H, 1, 0) > V_B(0; \alpha_H, 1, 0)$, which is equivalent to (23), $n_B = 1$ is sustained. As $S_L^h = 0$, all buyers exit after one period in the market. Thus the uninformed buyers do not get a chance to switch from l to h, and therefore (as $n_{BL} = \infty$) $B_L^l = 1$, yielding $\Delta V_{SL} = (\delta - 1)(p^{ll} - c_L) < 0$, which verifies our initial assumption.‖

The right hand side of (23) increases with δ. The intuition is that when buyers become more patient, their inclination to play "tough" at the risk of staying in the market for one period and learn rises, so they will do so despite a stronger belief that the state is H. When $\delta \to 1$, the right hand side of (23) approaches 1. Thus, for any $\alpha_H \in (0,1)$, there is a $\hat{\hat{\delta}} < 1$ such that for all $\delta \geq \hat{\hat{\delta}}$ an equilibrium of this type exists.

Denote this equilibrium by E1. In this equilibrium trade occurs at the right price, and sellers are truth-telling, i.e. force MI is not active. Recall that force N is not present in the one sided information model. It is therefore not surprising that trade occurs at the right price, and that as $\delta \to 1$ agents become more inclined to learn - force CL. More precisely, uninformed buyers play "tough" for a larger region of the prior α_H.

We introduce the following notation. Let $B(\delta)$ be the positive root of the equation $\Delta V_{SL} = 0$. We shall make extensive use of the fact that $\lim_{\delta \to 1} B(\delta) = 1$, which follows from continuity of $B(\delta)$ and $B(1) = 1$ (see (13)).

Claim 7 *If*

$$\alpha_H > \frac{p^{hl} - p^{ll}}{(1-\delta)(u_H - p^{hh}) + p^{hl} - p^{ll}}, \tag{24}$$

and $x_B > B(\delta)$, there is an equilibrium where $\Delta V_{SL} < 0$. In this equilibrium $n_{SL} = 0$ and $n_B = 0$.

Proof.
Step 1. Identical to step 1 in the proof of claim 6.
Step 2. If $V_B(1; \alpha_H, 1, 0) < V_B(0; \alpha_H, 1, 0)$, which is equivalent to (24), $n_B = 0$ is sustained.
As $S_L^h = 0$, all buyers exit after one period in the market. The informed buyers exit after one period of "tough" play, whereas the uninformed exit after one period of "soft" play. Thus $B_L^l = x_B$. If $x_B > B(\delta)$, then $\Delta V_{SL} < 0$.$\|$

Denote this equilibrium by E2. In this equilibrium sellers are truth-telling, as in equilibrium E1. In state L informed buyers trade at the right price, but uninformed buyers, believing that the probability of the state being H is big, play "soft" and pay p^{hl} for the good, a price at which they would not have agreed to trade had they known that the state is L. The restriction on x_B guarantees that there are enough "tough" buyers around, to discourage sellers from misrepresenting their information.

Note that for sufficiently big δ condition (24) in claim 7 and the condition $x_B > B(\delta)$ do not hold, so when $\delta \to 1$ this type of equilibrium ceases to exist.

Remark. When (24) holds but $x_B = B(\delta)$, we have $\Delta V_{SL} = 0$. Therefore $n_{SL} = S_L^h = 0$ can be sustained in equilibrium. For large enough δ this equilibrium vanishes as well.

Remark. When $\alpha_H = \frac{p^{hl}-p^{ll}}{(1-\delta)(u_H-p^{hh})+p^{hl}-p^{ll}}$ there is a "mixed" equilibrium where a fraction $g_B \in (0,1)$ of the uninformed buyers choose $n_B = 0$, and the rest of the uninformed buyers choose $n_B = 1$. As we want $\Delta V_{SL} < 0$, we must have $B_L^l = x_B + (1-g_B)(1-x_B) > B(\delta)$, or $x_B > \frac{B(\delta)-(1-g_B)}{g_B}$.

Equilibria E1 and E2 are "boundary" equilibria, in the sense that $S_L^h = 0$. These are the only equilibria in which $\Delta V_{SL} < 0$. The next claim establishes the existence of additional, "interior," equilibria which we denote by E3.

Claim 8 *If $x_B < B(\delta)$ then there are equilibria where $\Delta V_{SL} = 0$.*

Proof. Consider the equilibrium system when best responses are singletons. As $n_{SH} = \infty$, $S_H^h = 1$. As $B_H^l < 1$ (see (7)), (5) becomes an identity. By simple substitutions, using $S_L^h < 1$ (claim 4) and $n_{BL} = \infty$ (claim 1), the system can be reduced to equations (2), (7), and the equations

$$1-(1-S_L^hB_L^l)(B_L^l)^{n_{SL}(B_L^l)} = S_L^h, \tag{25}$$

$$1-(1-S_L^h B_L^l)(1-x_B)(S_L^h)^{n_B(S_L^h,\alpha_H)} = B_L^l, \tag{26}$$

where $n_{SL}(B_L^l)$ and $n_B(S_L^h, \alpha_H)$ are agents' best responses.

When uninformed buyers are indifferent between n_B and n_B+1, we use equations (18) and (19), and equation (26) is replaced by

$$1-(1-S_L^h B_L^l)(1-x_B)[g_B(S_L^h)^{n_B} + (1-g_B)(S_L^h)^{(n_B+1)}] = B_L^l. \tag{27}$$

When uninformed buyers are indifferent between any $n_B \in \{0,\ldots,\infty\}$, we use equations (20) and (21), and equation (26) is replaced by

$$1-(1-S_L^h B_L^l)(1-x_B)r_B = B_L^l. \tag{28}$$

When sellers are indifferent between any $n_{SL} \in \{0,\ldots,\infty\}$, we use equation (22), and equation (25) is replaced by

$$1-(1-S_L^h B_L^l)r_S = S_L^h. \tag{29}$$

The fractions g_B, r_B, and r_S can take any value in the interval $[0,1]$.

Equations (25) and (26) (and their respective replacements) define a correspondence from $[0,1]^2$ to itself. It suffices to focus on the fixed points $(\hat{B_L^l}, \hat{S_L^h})$ of this correspondence. Then B_H^l will be determined by (7), (19), or (21), which in turn will determine K_H using (2).

Fix $x_B \in (0,1)$, and choose $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$, $x_B < B(\delta)$. This can be done because $\lim_{\delta\to 1} B(\delta) = 1$. We are looking for fixed points of a map $F : X \times I \to X$, where $X = [0,1]^2$, and $I = [0,1]$ is the set of parameter values α_H. Clearly, X is compact and convex. It is straightforward to see that $n_{SL}(B_L^l)$ is u.h.c. Using the implicit correspondence theorem[6], $n_B(S_L^h, \alpha_H)$ is shown to be u.h.c. as well. Both correspondences are convex valued as best responses are singletons or intervals. Hence, the correspondence defined by the system is u.h.c. and convex valued.

We invoke a theorem (Mas-Colell [2], theorem 3, an extension of a theorem of F. Browder) which states that[7] the graph of fixed points of a correspondence like F contains a connected set which projects onto the set I. When $x_B < B(\delta)$ equilibrium E2 does not exist. Equilibrium E1 is a fixed point of F, with $(\hat{B_L^l}, \hat{S_L^h}) = (1, 0)$ and α_H satisfying (23). As the graph of E1 equilibria does not project onto the domain of α_H, the graph of fixed points must contain E3 equilibria. Moreover, as in an E3 equilibrium $B_L^l = B(\delta) < 1$, the graph of E3 equilibria must itself project onto the domain of α_H. Hence, for any $x_B \in (0,1)$ there is a $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$ an E3 equilibrium exists for any $\alpha_H \in (0,1)$.$\|$

Extreme one sided information: $\mathbf{x_B = 0}$. This case is special in two important ways. First, there is no E2 equilibrium for any value of δ. Technically speaking this happens because when $x_B = 0$ the condition $x_B > B(\delta)$ in claim 7 cannot be met. Another way to see this is by noting that if $n_B = 0$, then $B_L^l = 0$ and $\Delta V_{SL} = p^{hh} - p^{hl} > 0$, so $n_{SL} = 0$ cannot be sustained. Intuitively, the E2 equilibrium disappears due to the disappearance of informed buyers, who in state L always play "tough." If sellers are sufficiently impatient, the presence of these buyers may deter sellers from misrepresenting their information, but with no such buyers around, truth-telling is not a best response for the sellers.

[6] See e.g. Mas-Colell [3], p.49.

[7] The theorem is stated for X open, but can be extended to closed sets as follows. Let $F : X \times I \rightarrow X$, $X \subset R^n$, X closed and convex. Define $G : R^n \times I \rightarrow X$ such that $G(x, \alpha_H) = F(y, \alpha_H)$ where $y \in X$ is the foot of x in X. Mas-Colell's theorem applies to G, and F and G have the same graph of fixed points.

The second special feature of this case is the appearance of another equilibrium, denoted E4, where trade occurs at the wrong price. In this equilibrium we have $\Delta V_{SL} > 0$ and thus $n_{SL} = \infty$, implying that $S_L^h = 1$. Therefore $n_B < \infty$, otherwise there would be no trade and no steady state equilibrium. Then $V_B(n; \alpha_H, 1, 1) = \delta^n[\alpha_H(u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{hh})]$. As we want $n_B < \infty$, it must be that $[\alpha_H(u_H - p^{hh}) + (1 - \alpha_H)(u_L - p^{hh})] \geq 0$, or

$$\alpha_H \geq \frac{p^{hh} - u_L}{u_H - u_L}. \tag{30}$$

When this inequality is strict, $n_B = 0$ implying that $B_L^l = 0$. Then indeed we have $\Delta V_{SL} = p^{hh} - c_L > 0$: When all buyers are "soft" it is optimal for sellers to play "tough." Given that sellers are always "tough," buyers enter the market playing "soft." Trade occurs immediately at the wrong price. What drives this equilibrium is the fact that buyers attribute a high probability to the state being H. When the state is in fact L these beliefs are "very wrong." Sellers take advantage of this, misrepresenting their information (force MI).[8] Note that in this equilibrium there is no learning, as sellers behave in the same way in both states.[9]

Except for these two differences the limiting case is analogous to the general one sided information model (namely, there are E1 equilibria for α_H satisfying (23), and E3 equilibria for δ big enough.)

[8]For example, when the differences between adjacent magnitudes in (1) are all equal, (30) becomes $\alpha_H \geq 3/4$.

[9]When the inequality in (30) is not strict there are more equilibria of this type, where n_B is not zero. They are computed by solving the quadratic inequality $\Delta V_{SL} > 0$. For each value of B_L^l satisfying this inequality there is a corresponding value of n_B determined by (7). In these equilibria buyers enter the market playing "tough" for several periods despite the fact that sellers play "tough" in both states. The reason is that a buyer's expected payoff from any transaction is exactly zero. This expectation does not change through time as in this equilibrium the pairwise meetings convey no information. The buyer is completely indifferent between receiving zero now or later. Trade occurs at the wrong price with a delay.

3 The Revelation of Information

The question we now ask is whether trade at the wrong price is possible even when the market becomes approximately frictionless. As in state H sellers are always "tough," trade occurs at the right price p^{hh}. Also, as informed buyers in state L are always "tough," they trade at the right price p^{ll}. Thus, the only transactions which might occur at the wrong price are those involving uninformed buyers in state L. Let the fraction of transacting uninformed buyers who in state L end up trading at the wrong price be

$$f_B = \frac{K_L(1 - B_L^l)}{M(1 - x_B)}. \tag{31}$$

In the following propositions we characterize the behavior of equilibrium sequences for which limits exist as $\delta \to 1$. The propositions hold for $x_B \in [0, 1)$.

Proposition 1 *Generically in the parameters of the model, along any sequence of equilibria for which limits exist, as $\delta \to 1$, the following statements are equivalent:*

(I) $\lim_{\delta \to 1} f_B = 0$.

(II) $\lim_{\delta \to 1} S_L^h < 1$.

(III) $\lim_{\delta \to 1} n_{SL} < \infty$.

Proof.

Step 1. Along any sequence of E1 equilibria $S_L^h = 0$, $n_{SL} = 0$, and $f_B = 0$. Along any sequence of E4 equilibria $S_L^h = 1$, $n_{SL} = \infty$, and $f_B = 1$. Note that for δ sufficiently big E2 equilibria do not exist.

Step 2. We turn to sequences of E3 equilibria. Although sellers are indifferent between any $n_{SL} \in \{0, \ldots, \infty\}$, we shall assume throughout the proof that they all adopt the same n_{SL}. Without this assumption the proof is essentially the same, with

$r_S \in [0,1]$ replacing $(B_L^l)^{n_{SL}}$. It may also happen that buyers are indifferent between any $n_B \in \{0, \ldots, \infty\}$ or between n_B and $n_B + 1$. In the former case replace $(S_L^h)^{n_B}$ by $r_B \in [0,1]$. In the latter case replace $(S_L^h)^{n_B}$ by $g_B(S_L^h)^{n_B} + (1 - g_B)(S_L^h)^{n_B+1}$. We now derive some facts which will be used in the following steps.

(a) $f_B = \frac{1-B_L^l}{(1-x_B)(1-B_L^l S_L^h)}$. This is obtained by substituting for $\frac{K_L}{M}$ in (31) using (3).

(b) $f_B = (S_L^h)^{n_B}$. Noting that $n_{BL} = \infty$ and that in equilibrium $S_L^h < 1$, this follows from (4) and (31).

(c) $(1 - x_B)f_B = 1 - (B_L^l)^{n_{SL}+1}$. Multiply both sides by M. The left hand side is the number of uninformed buyers who trade at the wrong price. This must be equal to the number of sellers who trade at the wrong price, namely all the sellers except those who switched to "soft" after having met "tough" buyers for n_{SL} periods, and then met a "tough" buyer one more time, trading at the right price p^{ll}.

(d) $S_L^h = \frac{1-(B_L^l)^{n_{SL}}}{1-(B_L^l)^{n_{SL}+1}}$. Write (3) as $M = K_L[(1 - B_L^l) + B_L^l(1 - S_L^h)]$. Eliminate $\frac{M}{K_L}$ using (6) and solve for S_L^h.

Step 3. Along a sequence of E3 equilibria, whenever $(1 - x_B)\frac{\alpha_H}{1-\alpha_H}(u_H - p^{hh}) \neq p^{ll} - c_L$, (I) implies (II). We establish this by showing that $\lim_{\delta\to1} S_L^h = 1$ implies $\lim_{\delta\to1} f_B \neq 0$.

Suppose $\lim_{\delta\to1} S_L^h = 1$. From fact (a), noting that $\lim_{\delta\to1} B_L^l = 1$, and using the assumption $\lim_{\delta\to1} S_L^h = 1$, we can use l'Hôpital's rule to obtain $\lim_{\delta\to1} f_B = \frac{\lim_{\delta\to1} B'}{(1-x_B)(\lim_{\delta\to1} S' + \lim_{\delta\to1} B')}$, where B' and S' are the derivatives of B_L^l and S_L^h with respect to δ along a sequence of E3 equilibria.

From $\Delta V_{SL} = 0$ (see (13)) we obtain $\lim_{\delta\to1} B' = -\lim_{\delta\to1} \frac{\partial\Delta V_{SL}/\partial\delta}{\partial\Delta V_{SL}/\partial B_L^l} = \frac{p^{ll}-c_L}{p^{hh}-p^{ll}} > 0$.

Consider the equation $\frac{\partial}{\partial x}V_B(x; \alpha_H, 1, S_L^h) = 0$ (see (17)). Substitute f_B for $(S_L^h)^{n_B}$ (fact (b)) and solve for f_B. Taking limits, recalling the assumption $\lim_{\delta\to1} S_L^h = 1$, and using l'Hôpital's rule to compute $\lim_{\delta\to 1}\frac{\log\delta}{\log(\delta S_L^h)}$ and $\lim_{\delta\to 1}\frac{1-S_L^h}{1-\delta S_L^h}$, we obtain an equation involving $\lim_{\delta\to 1} f_B$ and $\lim_{\delta\to 1} S'$. Using the expressions for $\lim_{\delta\to 1} f_B$ and $\lim_{\delta\to 1} B'$ derived above, we obtain

$$\lim_{\delta\to 1} f_B = \frac{(1-x_B)\frac{\alpha_H}{1-\alpha_H}(u_H - p^{hh}) - (p^{ll} - c_L)}{(1-x_B)[(p^{hh} - u_L) - (p^{ll} - c_L)]}, \tag{32}$$

which establishes step 3.

Step 4. Along a sequence of E3 equilibria, where $\delta \to 1$, (II) implies (III). Consider the expression in fact (d). $\lim_{\delta\to 1} S_L^h < 1$ implies that $\lim_{\delta\to 1}(B_L^l)^{n_{SL}} = 1$. Otherwise, as $\lim_{\delta\to 1} B_L^l = 1$, $\lim_{\delta\to 1} S_L^h = 1$.
Dividing the numerator and the denominator of the right hand side by $1 - B_L^l$ and taking limits we get $\lim_{\delta\to 1} S_L^h = \lim_{\delta\to 1}\frac{1+B_L^l+(B_L^l)^2+\cdots+(B_L^l)^{n_{SL}-1}}{1+B_L^l+(B_L^l)^2+\cdots+(B_L^l)^{n_{SL}}} = \lim_{\delta\to 1}\frac{n_{SL}}{n_{SL}+1}$, which establishes step 4.

Step 5. Along a sequence of E3 equilibria, where $\delta \to 1$, (III) implies (I). As $\lim_{\delta\to 1} B_L^l = 1$ and $\lim_{\delta\to 1} n_{SL} < \infty$, we have $\lim_{\delta\to 1}(B_L^l)^{n_{SL}} = 1$. Using fact (c) the result follows.‖

Corollary *Generically in the parameters of the model, along any sequence of E3 equilibria for which limits exist, as $\delta \to 1$, either $\lim_{\delta\to 1} n_B = \infty$ or, when $\lim_{\delta\to 1} S_L^h = 0$, $\lim_{\delta\to 1} n_B = 1$.*

Proof. First consider the case $\lim_{\delta\to 1} f_B = 0$. Then by the proposition above $\lim_{\delta\to 1} S_L^h < 1$. If $\lim_{\delta\to 1} S_L^h > 0$ then from fact (b) in the proposition it must be that $\lim_{\delta\to 1} n_B = \infty$. If $\lim_{\delta\to 1} S_L^h = 0$ then the sequence of functions V_B (which are continuous) approaches a function which is maximixed at 0, at 1, or at some value between 0 and 1. If it is maximized at zero then $n_B \to 0$ and hence $B_L^l \to 0$. But the sequence of ΔV_{SL}'s approaches a strictly positive number, whereas in an

E3 equilibrium $\Delta V_{SL} = 0$. If the limiting V_B is maximized strictly between 0 and 1, then it must be that in the limit $\alpha_H = \frac{p^{hl}-p^{ll}}{(1-\delta)(u_H-p^{hh})+p^{hl}-p^{ll}}$ (see the second remark following claim 7), which cannot be as the right hand side approaches 1 as $\delta \to 1$. If $\lim_{\delta\to 1} f_B \neq 0$, then by the proposition above $\lim_{\delta\to 1} S_L^h = 1$. Suppose $\lim_{\delta\to 1} n_B < \infty$. From fact (b) in the proposition it must be that $\lim_{\delta\to 1} f_B = 1$. But by (32), generically in the parameters of the model $\lim_{\delta\to 1} f_B \neq 1$.

Remark. Whenever trade occurs at the wrong price the fraction of wrong price trades is a well determined number given by (32).

Proposition 2 *(a) For any cost, valuation, and price configuration satisfying (1), there is an open region of the parameters α_H and x_B for which, along any sequence of equilibria such that limits exist, as $\delta \to 1$, trade always occurs at the right price.*

(b) For any cost, valuation, and price configuration satisfying (1), and any $\alpha_H \in (0,1)$ and $x_B \in [0,1)$, there exist a sequence of E1 equilibria and a sequence of E3 equilibria along which, as $\delta \to 1$, trade occurs at the right price.

Proof. (a) Consider $x_B \in (0,1)$. The shaded areas in figures 3a and 3b are those for which the parameters of the model are such that the expression in (32) is strictly negative or strictly greater than one. By definition of f_B this is not possible. Therefore, in the shaded regions sequences of E3 equilibria such that $\lim_{\delta\to 1} S_L^h = 1$ are not possible, and thus along any sequence of equilibria such that limits exist, as $\delta \to 1$, $\lim_{\delta\to 1} S_L^h < 1$. By proposition 1, $\lim_{\delta\to 1} f_B = 0$.
For later use, denote by $\underline{\alpha}(x_B)$ and $\overline{\alpha}(x_B)$ the values of α_H on the lower and upper boundaries of the non-shaded region, for a given x_B.
When $x_B = 0$ the constraint $\alpha_H < \frac{p^{hh}-u_L}{u_H-u_L}$ must be added in order to rule out the E4 equilibrium. Then, for any $\alpha_H \in \left(0, \min\left(\frac{p^{hh}-u_L}{u_H-u_L}, \frac{p^{ll}-c_L}{(u_H-p^{hh})+(p^{ll}-c_L)}\right)\right)$ trade always occurs at the right price.

(b) For any cost, valuation, and price configuration satisfying (1), and any $\alpha_H \in (0,1)$ and $x_B \in [0,1)$, there exists a $\hat{\delta} < 1$ such that for all $\delta \geq \hat{\delta}$ equation (23) holds, and therefore an E1 equilibrium exists.

We turn to sequences of E3 equilibria. Let x^* satisfy $\frac{\partial}{\partial x} V_B(x^*) = 0$ (see (17)). Rearranging this equation and equation (26), replacing S_L^h by S and B_L^l by $B(\delta)$, the root of $\Delta V_{SL} = 0$ (see (13)), and setting $f = S^{x^*}$, we obtain

$$\frac{\alpha_H (\log \delta)(p^{hh} - u_H)}{(1-\alpha_H)\log(S\delta)\{[S(u_L - p^{hh}) + (1-S)(u_L - p^{hl})] - \frac{1-S}{1-S\delta}(u_L - p^{ll})\}} = f, \quad (33)$$

$$\frac{f(1-x_B) - (1-B(\delta))}{B(\delta) f (1-x_B)} = S. \quad (34)$$

These equations define a map from $X \times I$ to X, where $X = (0,1] \times [0,1]$, with $f \in (0,1]$ and $S \in [0,1]$, and $I = [0,1]$ is the domain of α_H. Extend the map by assigning to the vector $(0, S)$ the ordered pair whose second component is the interval $[0,1]$. The first component, f, is calculated using (33), which for $\alpha_H \in (0,1)$ is strictly positive. Hence, for $\alpha_H \in (0,1)$ the extension does not generate any spurious fixed points. As the extended map is u.h.c. and convex valued, we can apply Mas-Colell's theorem, to obtain the existence of fixed points $(\hat{f}, \hat{S})$ whose graph contains a connected set which projects onto the domain of α_H.

Let $(\hat{f}, \hat{S})$ be a fixed point of the above system and recall that x^* maximizes $V_B(x; \hat{S})$. Suppose w.l.o.g. that n_B, the integer value of x^* (rather than $n_B + 1$), maximizes $V_B(n; \hat{S})$. By choosing δ sufficiently close to one $(\hat{f}, \hat{S})$ can be made arbitrarily close to (f_B, S_L^h) in an E3 equilibrium, where $f_B = (S_L^h)^{n_B}$, (f_B, S_L^h) satisfies (34) and n_B maximizes $V_B(n; S_L^h)$. This can be seen as follows. Let $f_o = \hat{S}^{n_B}$. Substitute f_o for f in (34) and denote the value which obtains by S_o. If $\lim_{\delta \to 1} S_L^h > 1$ then $\lim_{\delta \to 1} x^* = \infty$ (the proof of this follows closely the proof of proposition 1 and

its corollary). THen f_o can be made arbitrarily close to $\hat{f} = \hat{S}^{x^*}$, and hence S_o can be made arbitrarily close to $\hat{S}$, so that (a) n_B maximizes $V_B(n; S_o)$, and (b) (f_o, S_o) satisfies (34) and $f_o \approx (S_o)^{n_B}$ with an arbitrary degree of precision. As we have replaced B_L^l by $B(\delta)$, the root of $\Delta V_{SL} = 0$, we are free to choose any $n_{SL} \in \{0, \ldots, \infty\}$ in order to satisfy (25). The other variables of the E3 equilibrium B_H^l, K_H, and K_L are determined by the equilibrium equations. A similar argument holds for $\lim_{\delta \to 1} S_L^h = 0$ (in which case $\lim_{\delta \to 1} x^* = \infty$.)
Fix $x_B \in (0,1)$, and choose δ close to 1. For $\alpha_H > \overline{\alpha}(x_B)$ there must exist E3 equilibria with f_B very close to zero. Denote this set by U. Similarly for $\alpha < \underline{\alpha}(x_B)$. Denote this set by T. For $\alpha_H \in [\underline{\alpha}(x_B), \overline{\alpha}(x_B)]$ there may be E3 equilibria with f_B very close to the value given by (32). By (32) the relation between f_B and α_H is strictly monotonic. Hence, the set of these wrong price E3 equilibria can connect either with U or with T but not with both. It follows that the graph of right price E3 equilibria must contain a connected set which projects onto the domain of α_H.$\|$

Discussion. In equilibrium E1 sellers play "soft" ($\Delta V_{SL} < 0$), i.e. force MI (Misrepresentation of Information) is not active. Force CL (Cost of Learning) induces uninformed buyers to play "tough," and trade occurs at the right price. In equilibrium E2 sellers also play "soft," but α_H is sufficiently high to induce uninformed buyers to play "soft," so they trade at the wrong price. Force CL is not strong enough to overcome the prior belief regarding the state of the world. When $\delta \to 1$ force CL becomes stronger, breaking this equilibrium. In equilibrium E4, when $x_B = 0$, sellers play "tough" ($\Delta V_{SL} > 0$) - force MI. As in equilibrium E2, α_H is sufficiently high to induce buyers to play "soft," and trade occurs at the wrong price.

Along a sequence of E3 equilibria as $\delta \to 1$, sellers have, other things equal, a greater incentive to misrepresent their information and play "tough" - force MI. But as $\delta \to 1$ buyers are also becoming very "tough" ($\lim_{\delta\to 1} n_B = \infty$ and $\lim_{\delta\to 1} B_L^l = 1$) - force CL. By construction of an E3 equilibrium this exactly offsets the incentive of sellers to be "tougher," rendering them indifferent between playing "tough" or "soft" ($\Delta V_{SL} = 0$ and $n_{SL} \in \{0, \ldots, \infty\}$).

The question is what happens to the "toughness" of sellers in the limit, i.e. to $\lim_{\delta\to 1} S_L^h$. If n_B approaches infinity very fast, the only way to maintain a steady state is for some sellers to switch from "tough" to "soft" so that $\lim_{\delta\to 1} S_L^h < 1$ (implying that $\lim_{\delta\to 1} n_{SL} < \infty$). This can be interpreted as force CL overcoming, in the limit, force MI, with trade occurring at the right price. When the speed at which n_B approaches infinity is not sufficiently big, S_L^h may approach 1 (with $\lim_{\delta\to 1} n_{SL} = \infty$) along a sequence of (steady state) equilibria, and trade might occur at the wrong price.

We provide some intuition for the shaded regions in figure 3, where trade always occurs at the right price. When x_B is big most buyers are informed, so in state L sellers face a population which is predominantly "tough." This induces sellers to switch from "tough" to "soft," as the probability of extracting surplus from an uninformed buyer is low. Hence $\lim_{\delta\to 1} S_L^h < 1$. This is *a fortiori* true when α_H is low, which contributes to greater "toughness" on the part of the uninformed buyers. This explains the shaded region in the lower right hand side of figures 3a and 3b.

The other region may be explained as follows. When x_B is small, most of the buyers are uninformed. If S_L^h were to approach 1 uninformed buyers would find it optimal to switch from "tough" to "soft" after a very short stay in the market (as in equilibrium E4 where they enter the market playing "soft"). But then it would not be possible to sustain a steady state with B_L^l approaching 1, which must hold in any E3 equilibrium by construction. This is *a fortiori* true when α_H is high, which contributes to greater "softness" on the part of the uninformed buyers. This explains the shaded region in the upper left hand side of figures 3a and 3b.

Equilibria E1, E4, and E3 are reminiscent of the types of equilibria that one finds in signaling models. The E1 equilibrium can be thought of as a separating equilibrium, where the behavior of informed agents fully reveals their private information. The E4 equilibrium can be viewed as a pooling equilibrium, where informed agents behave in the same way regardless of their private information. E3 equilibria are similar to semi-separating equilibria, where informed agents are indifferent between revealing and not revealing their private information.

Finally, note that in the non-shaded area in figure 3 trade may still occur at the right price. Curiously, if it occurs at the wrong price, the limiting fraction of wrong price trades is a precise number determined by equation (32).

The relation to Gale's work. Gale studies a one sided information model where all sellers are informed and all buyers are uninformed. His model differs from our model in that (a) $p^{hl} = p^{ll}$, and (b) A fixed fraction $\pi < 1$ of sellers has high costs in state L.[10] The first difference contributes to trade at the right price, as in our model a meeting between two "soft" agents results in trade at price p^{hl} which is always wrong, whereas in Gale's model it results in trade at price p^{ll} which in state L (the only state in which wrong price trade is possible) is the right price. The second difference, though, contributes to trade at the wrong price, as the fixed fraction of sellers who always have high costs play "tough" also in state L, increasing the

[10] Another difference is that Gale fixes market size exogenously. The agents who trade are replaced by the same number of agents with the same characteristics as those who traded. This turns out to be equivalent to our formulation, although it raises a difficulty at the level of interpretation: As $\delta \to 1$ the number of transactions becomes negligible in absolute terms (not only relatively to market size).

likelihood of disagreement or of wrong price trade.

Gale studies a particular equilibrium where sellers with low costs always call the low price (for sellers with high costs it is a dominant strategy to call the high price). Then the proportion of sellers who call the high price in state L is $\pi < 1$. Gale shows that the equilibrium becomes fully revealing as $\delta \to 1$.

This result is perfectly compatible with our analysis. In the particular equilibrium which Gale studies sellers are truth-telling, i.e. force MI is not active. The proportion of "tough" sellers is $S_L^h = \pi < 1$, for all $\delta < 1$, and therefore, in the spirit of proposition 1, $\lim_{\delta \to 1} f_B = 0$.

4 Wolinsky's Two Sided Information Model

Let $x_S \in (0,1)$ be the fraction of informed sellers who enter the market each period. The rest of the sellers are uninformed and have the same prior α_H as uninformed buyers. There is an additional variable in the model, n_S, the strategy of an uninformed seller. The equilibrium system when best responses are singletons is as follows.[11]

$$M = K_H(1 - S_H^h B_H^l). \tag{35}$$

$$M = K_L(1 - S_L^h B_L^l). \tag{36}$$

$$K_H(1 - S_H^h) = M[x_S(B_H^l)^{n_{SH}} + (1 - x_S)(B_H^l)^{n_S}]. \tag{37}$$

$$K_H(1 - B_H^l) = M[x_B(S_H^h)^{n_{BH}} + (1 - x_B)(S_H^h)^{n_B}]. \tag{38}$$

$$K_L(1 - S_L^h) = M[x_S(B_L^l)^{n_{SL}} + (1 - x_S)(B_L^l)^{n_S}]. \tag{39}$$

$$K_L(1 - B_L^l) = M[x_B(S_L^h)^{n_{BL}} + (1 - x_B)(S_L^h)^{n_B}]. \tag{40}$$

[11] We present the simplest version of the two sided information model.

$$n_{SH} \in \arg\max_n V_{SH}(n; B_H^l). \quad (41)$$

$$n_{SL} \in \arg\max_n V_{SL}(n; B_L^l). \quad (42)$$

$$n_S \in \arg\max_n V_S(n; \alpha_H, B_H^l, B_L^l). \quad (43)$$

$$n_{BH} \in \arg\max_n V_{BH}(n; S_H^h). \quad (44)$$

$$n_{BL} \in \arg\max_n V_{BL}(n; S_L^h). \quad (45)$$

$$n_B \in \arg\max_n V_B(n; \alpha_H, S_H^h, S_L^h). \quad (46)$$

Wolinsky shows that the following configurations are possible in equilibrium:

$$\begin{array}{ccc} n_{SH} = \infty & n_{SL} = 0 & n_S < \infty \\ n_{BH} = 0 & n_{BL} = \infty & n_B < \infty. \end{array} \quad (47)$$

$$\begin{array}{ccc} n_{SH} = \infty & n_{SL} = 0 & n_S < \infty \\ n_{BH} \in \{0, \ldots, \infty\} & n_{BL} = \infty & n_B = \infty. \end{array} \quad (48)$$

$$\begin{array}{ccc} n_{SH} = \infty & n_{SL} \in \{0, \ldots, \infty\} & n_S = \infty \\ n_{BH} = 0 & n_{BL} = \infty & n_B < \infty. \end{array} \quad (49)$$

Substituting according to these configurations in (35) through (40) we obtain the various versions of the steady state conditions in Wolinsky's model.

Let the fraction of transacting uninformed sellers who in state H end up trading at the wrong price be

$$f_S = \frac{K_H(1 - S_H^h)}{M(1 - x_S)}. \quad (50)$$

Similarly for buyers in state L, as in equation (31).

Wolinsky's main result is that along a sequence of equilibria where $\delta \to 1$, and such that $\lim_{\delta \to 1} f_S$ and $\lim_{\delta \to 1} f_B$ exist, at least one of these limits is positive.

The intuition for this result is as follows. Suppose the state of the world is H and

$f_S \approx 0$. Therefore, sellers aren't switching from h to l, as any seller who switches trades at price p^{hl} or p^{ll}, which are both wrong. As the market is in steady state we know that trade is occurring, and since all sellers are playing "tough" transactions are being executed at price p^{hh}. This means that some buyers must be switching from l to h. Moreover, almost all of these buyers are uninformed, as the fraction of informed buyers in the market tends to zero as $\delta \to 1$. For example, for the configuration in (47) the fraction of informed buyers is $\gamma_H = \frac{Mx_B}{K_H}$ (as $n_{BH} = 0$). Using (35), $\gamma_H = (1 - S_H^h B_H^l)x_B$. Wolinsky shows that[12] $B_H^l = O\left(1 - \frac{1}{n_B}\right)$ and $S_H^h > S_L^h = O\left(1 - \frac{1}{n_S}\right)$. As $\delta \to 1$, n_S and n_B are increasing and approaching infinity (due to the lower cost of learning). Thus $S_H^h B_H^l \to 1$ and $\gamma_H \to 0$. It follows that although both n_S and n_B are increasing and approaching infinity, n_S must be increasing faster, so sellers exit the market before they get a chance to switch from "tough" to "soft." Thus $\lim_{\delta\to 1} \frac{n_S}{n_B} = \infty$.

Repeating this reasoning for state L with $f_B \approx 0$, we get $\lim_{\delta\to 1} \frac{n_S}{n_B} = 0$. But n_S and n_B are the same numbers in both states of the world (as they are chosen by uninformed agents), so their ratio cannot approach infinity and zero at the same time. Thus, either $\lim_{\delta\to 1} f_S > 0$ or $\lim_{\delta\to 1} f_B > 0$, i.e. in at least one state of the world some agents don't learn even when the market becomes approximately frictionless.

Remark. A particular case of Wolinsky's model occurs when $x_S \in (0, 1)$ and $x_B = 0$. This is a two sided information model as there are uninformed agents on both sides of the market. The intuitive argument for Wolinsky's result goes through. In fact,

[12] This can be explained as follows. Take δ sufficiently close to 1. Then n_S is sufficiently close to ∞. As $n_{SH} = \infty$, almost all sellers say h and therefore almost no buyer leaves the market before having completed his tough phase. Thus $B_H^l \approx \frac{n_B M(1-x_B)}{(n_B+1)M(1-x_B)+Mx_B} = 1 - \frac{1}{(n_B+1)(1-x_B)+x_B} = O\left(1 - \frac{1}{n_B}\right)$. Similarly for S_L^h. The inequality simply states that a message h has informative content.

it is even simpler, as in state H the population behind force MI (informed buyers) is not present. The case $x_S = 0$ and $x_B \in (0, 1)$ is symmetric.

In the two sided information model the agents behind force MI become a negligible fraction of the market as $\delta \to 1$. What drives the economy towards the wrong price is the Noise force, namely the fact that as $\delta \to 1$ most agents on both sides of the market are uninformed. In the one sided information model the agents behind force MI cannot become a negligible fraction of the market. Whether the economy is driven to the wrong price depends on the relative strength of the Misrepresentation of Information force and the Cost of Learning force.

Thus, the *reason* for wrong price trades is not the same in the two models. In the two sided information model wrong price trades occur due to the low informational content of the pairwise meetings. Uninformed agents do not learn from informed agents because the informed agents disappear from the market before the uninformed learn. In the one sided information model, whenever there is trade at the wrong price, this happens because informed agents prefer to misrepresent their information rather than play the role of teachers. Propositions 1 and 2 show that "very often" this is *not* the case, with the Cost of Learning force overcoming the Misrepresentation of Information force, driving the economy to an ex-post individually rational price.

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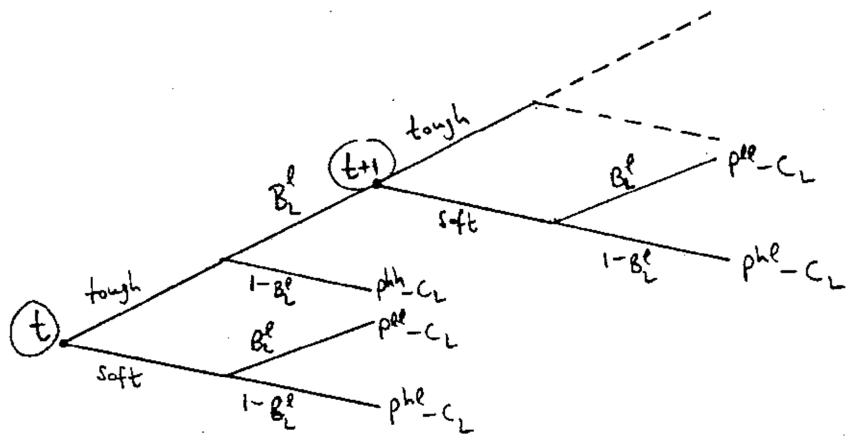
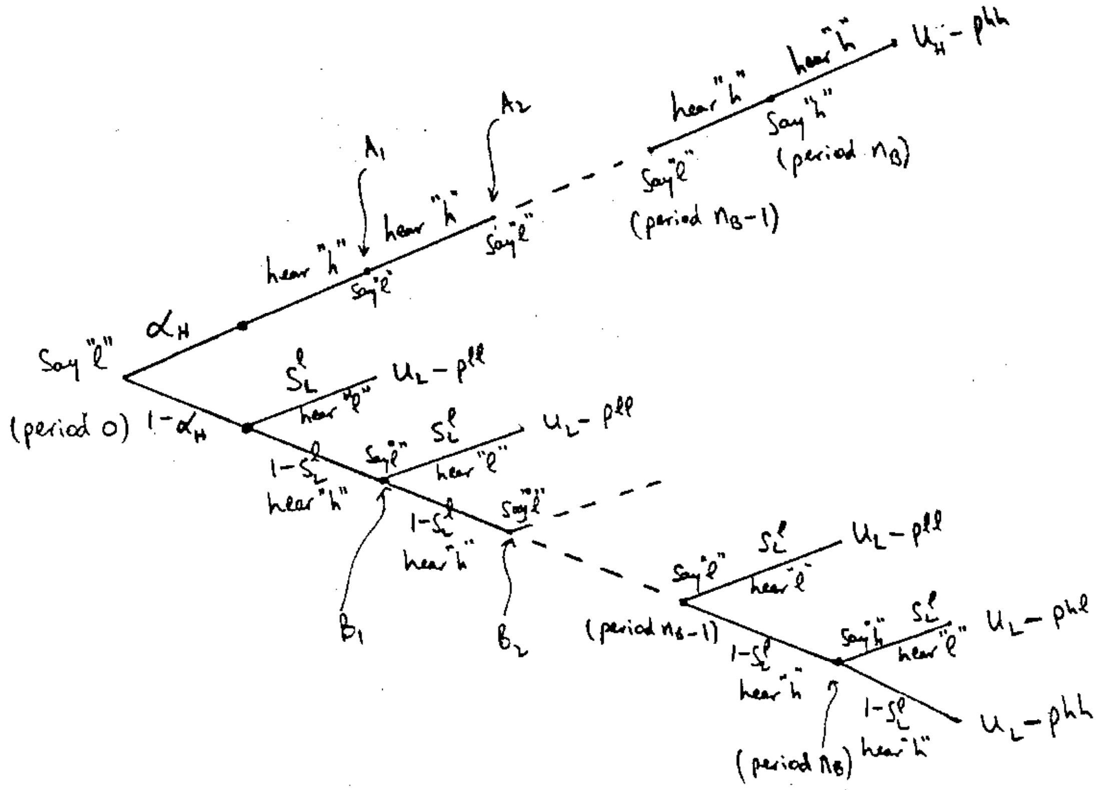


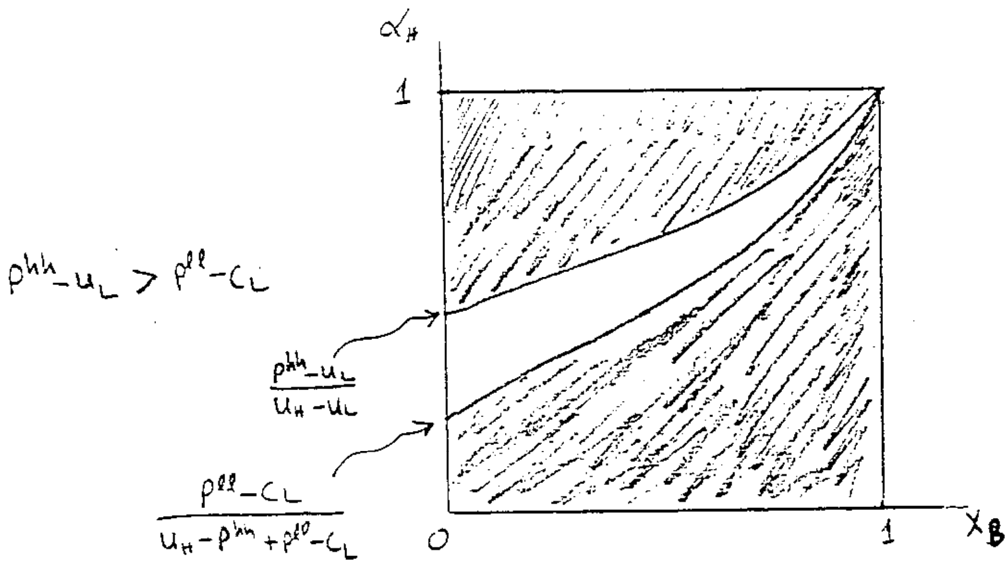
t
t+1
tough
B_L^ℓ
tough
soft
$1-B_L^\ell$
$p^{hh}-C_L$
B_L^ℓ
$p^{\ell\ell}-C_L$
soft
$1-B_L^\ell$
$p^{h\ell}-C_L$
B_L^ℓ
$p^{\ell\ell}-C_L$
$1-B_L^\ell$
$p^{h\ell}-C_L$

Figure 1



Say "l"
(period 0)
α_H
$1-\alpha_H$
hear "h"
Say "l"
hear "h"
Say "l"
A_1
A_2
Say "l"
(period n_B-1)
hear "h"
Say "h"
(period n_B)
hear "h"
$U_H - p^{hh}$
S_L^l
hear "l"
$U_L - p^{ll}$
$1-S_L^l$
hear "h"
Say "l"
S_L^l
hear "l"
$U_L - p^{ll}$
$1-S_L^l$
hear "h"
Say "l"
B_1
B_2
Say "l"
S_L^l
hear "l"
$U_L - p^{ll}$
(period n_B-1)
$1-S_L^l$
hear "h"
Say "h"
S_L^l
hear "l"
$U_L - p^{hl}$
$1-S_L^l$
hear "h"
$U_L - p^{hh}$
(period n_B)

Figure 2


α_H
1
$p^{hh}-u_L > p^{ll}-c_L$
$\frac{p^{hh}-u_L}{u_H-u_L}$
$\frac{p^{ll}-c_L}{u_H-p^{hh}+p^{ll}-c_L}$
0
1
x_B

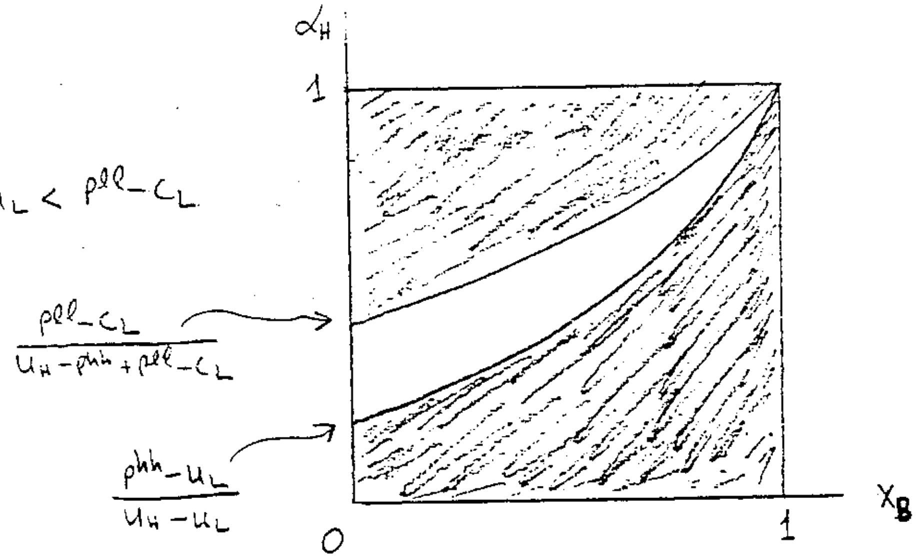

α_H
1
$p^{hh}-u_L < p^{ll}-c_L$
$\frac{p^{ll}-c_L}{u_H-p^{hh}+p^{ll}-c_L}$
$\frac{p^{hh}-u_L}{u_H-u_L}$
0
1
x_B

Figure 3