Demand-Deposit Contracts and the Probability of Bank Runs

Itay Goldstein

Ady Pauzner

Journal of Finance (2005)

The Model

- Three periods: $t = 0, 1, 2$. Single homogeneous good, no aggregate uncertainty

- Continuum $[0, 1]$ of *ex ante* identical, risk-averse consumers with an endowment of 1 unit of the good in period 0

- Beginning of period 1: idiosyncratic, **unobservable** preference shock:

  – with probability $\lambda > 0$: early type, $u = u(c_1)$

  – with probability $(1 - \lambda)$: late type, $u = u(c_1 + c_2)$

- $u(c)$ well-behaved, $u(0) = 0$, $CRRA > 1$ for $c \geq 1$

- One investment technology:

Date	$t = 0$	$t = 1$	$t = 2$
	1 $\longrightarrow$	1 $\longrightarrow$	$\begin{cases} R \text{ with prob. } p(\theta) \\ 0 \text{ with prob. } 1 - p(\theta) \end{cases}$

where $\theta \sim [0, 1]$ is the state of the economy (realized in period 1, revealed publicly in period 2).

- Assume: $p'(\theta) > 0$ and $E_\theta [p(\theta)] u(R) > u(1)$

- Autarky: early types consume 1 and late types consume R with prob. $p(\theta)$

- Social planner (can observe *ex post* types):

$$\max_{c_1} \lambda u(c_1) + (1-\lambda)\, u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) E_\theta [p(\theta)]$$

FOC: $u'\left(c_1^{FB}\right) = Ru'\left(\frac{1-\lambda c_1^{FB}}{1-\lambda} R\right) E_\theta [p(\theta)]$

In the optimum, $c_1^{FB} > 1$ since at $c_1 = 1$: $1 \cdot u'(1) > Ru'(R)\, E_\theta [p(\theta)]$

since $CRRA > 1$ and $E_\theta [p(\theta)] < 1$

- Types are unobservable. How to implement the first-best allocation? Set-up a bank offering a demand-deposit contract. Bank max expected utility of consumers

- Period-one return r_1 fixed and promised unless the bank runs out (sequential service), period-two return $\widetilde{r_2}$ stochastic

- Consumers give their entire endowment to a bank in exchange for an incentive compatible, $u(r_1) \leq u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) E_\theta [p(\theta)]$, demand deposit contract that sets

$$r_1 = c_1^{FB}.$$

- BUT: the solution above is implemented as **an** equilibrium. There is another, bank run equilibrium.

- Panic-based runs: if everyone withdraws in period 1: $r_1 \cdot \frac{1}{r_1} > 0$ (Nash equilibrium)

- Game played by late consumers has two Nash equilibria:

  - a "good", no run, equilibrium in which all late consumers withdraw in period 2

  - a "bad", run, equilibrium in which late consumers panic and try to withdraw in period 1

- When setting optimal r_1, need to know **how likely** each equilibrium is. Do banks increase welfare? Solve backwards: period one first, then period-zero decisions

Unique Equilibrium in Period 1

- θ is realized but not revealed at the beginning of period 1

- Private signals: each agent receives a private signal about $\theta_i = \theta + \varepsilon_i$, i.i.d. (uniform on $[-\varepsilon, \varepsilon]$). Note: no one has advantage in terms of the quality of the signal

- The signal θ_i has two effects:

  – provides info about R (the higher θ_i, the lower the incentive to run)

  – provides info about signals of others (the higher θ_i, the more probable others got a high signal, too, the lower incentive to run)

- Early consumers always withdraw in period 1

- Late consumers compare the expected payoffs from withdrawing in period 1 and 2. This payoff depends on θ and proportion n of consumers demand early withdrawal.

- Signal θ_i provides info on both $\rightarrow$ actions depend on signals

- Assume there are two extreme ranges of fundamentals at which agents' behavior is known: the lower range $[0, \theta_{LB}(r_1)]$ and the upper range $\left[\theta^{UB}(r_1), 1\right]$

- For $\theta < \theta_{LB} - 2\varepsilon$, all late consumers receive signals below $\theta_{LB} - \varepsilon$ and everybody runs: $n = 1$

- For $\theta > \theta^{UB} + 2\varepsilon$, all late consumers receive signals above $\theta^{UB} + \varepsilon$ and only early consumers withdraw: $n = \lambda$

- When choosing the equilibrium action, a consumer must take into account the equilibrium actions at nearby signals etc.

- Theorem 1: There is a unique equilibrium in which late consumers withdraw of they observe a signal below threshold $\theta^* (r_1)$ and do not run above

- Strategic complementarity property: an agent's incentive to take an action is influenced by how many other agents take that action

- A late consumer's utility differential is:

$$\nu(\theta, n) = \begin{cases} p(\theta)\, u\left(\frac{1-nr_1}{1-n}R\right) - u(r_1) \text{ if } \frac{1}{r_1} \geq n \geq \lambda \\ \\ 0 - \frac{1}{n}\frac{u(r_1)}{r_1} \text{ if} 1 \geq n \geq \frac{1}{r_1} \end{cases}$$

- Proof: Show that there exists a unique threshold equilibrium, i.e. equilibrium in which all late consumers run if their signal is below some common threshold and do not run above. Need to show that the utility differential is equal to zero when θ equals the threshold

- What proportion of consumers runs at every realization of θ?

- Function $n\left(\theta, \theta^{'}\right)$: specifies the proportion of agents who run when fundamentals are θ and all consumers run at signals below $\theta^{'}$ and do not run at signals above $\theta^{'}$

- $n\left(\theta, \theta^{*}\left(r_1\right)\right) = \begin{cases} 1 \text{ if } \theta \leq \theta^{*}\left(r_1\right) - \varepsilon \\ \lambda + (1-\lambda)\left(\frac{1}{2} + \frac{\theta^{*}(r_1) - \theta}{2\varepsilon}\right) \text{ if } \theta \in \theta^{*}\left(r_1\right) \pm \varepsilon \\ \lambda \text{ if } \theta \geq \theta^{*}\left(r_1\right) + \varepsilon \end{cases}$

Do Banks Increase Welfare?

- Threshold signal $\theta^* (r_1)$: a late type must be indifferent between withdrawing in period 1 and 2

- His posterior distribution of θ: uniform over $[\theta^* (r_1) - \varepsilon, \theta^* (r_1) + \varepsilon]$

- Beliefs: the proportion of people who run is $n (\theta, \theta^* (r_1))$;

- Posterior distribution of n is uniform over $[\lambda, 1]$

- Indifference condition:

$$\int_{n=\lambda}^{1/r_1} u(r_1) + \int_{n=1/r_1}^{1} \frac{1}{nr_1} u(r_1) = \int_{n=\lambda}^{1/r_1} p(\theta^*)\, u\left(\frac{1-nr_1}{1-n}R\right)$$

Solving for θ^*: $\lim_{\varepsilon \to 0} \theta^*(r_1) = p^{-1}\left(\frac{u(r_1)}{r_1}\frac{(1-\lambda r_1+\ln(r_1))}{\int_{n=\lambda}^{1/r_1} u\left(\frac{1-r_1 n}{1-n}R\right)}\right)$

- Theorem 2: $\theta^*(r_1)$ is increasing in r_1

Decision in Period 0

- Choose r_1 to max expected utility:

$$\lim_{\varepsilon \to 0} EU(r_1) = \int_0^{\theta^*(r_1)} \frac{1}{r_1} u(r_1)\, d\theta + \int_{\theta^*(r_1)}^{1} \lambda u(r_1) + (1-\lambda) \cdot p(\theta)\, u\left(\frac{1-\lambda r_1}{1-\lambda} R\right) d\theta$$

- Theorem 3: If θ_{LB} is not too large, the optimal r_1 must be larger than 1

- Liquidity provision is optimal and banks increase welfare even though panic-based bank runs occur in the optimum: $\theta^*(r_1) > \theta_{LB}(r_1)$

- FOC for r_1:

$$\lambda \int_{\theta^*(r_1)}^{1} \left[u'(r_1) - p(\theta) R u'\left(\frac{1-\lambda r_1}{1-\lambda} R\right)\right] d\theta = \frac{\partial \theta^*(r_1)}{\partial r_1} \left[\lambda u(r_1) + (1-\lambda) \cdot \right.$$

$$\left. p(\theta^*(r_1)) u\left(\frac{1-\lambda r_1}{1-\lambda} R\right) - \frac{u(r_1)}{r_1}\right] + \int_{0}^{\theta^*(r_1)} \left[\frac{u(r_1) - r_1 u'(r_1)}{r_1^2}\right] d\theta$$

- Theorem 4: The optimal r_1 is lower than c_1^{FB}

- Need to cut back on liquidity provision because of the possibility of runs