COMPETITIVE EFFICIENCY IN AN OVERLAPPING-GENERATION MODEL WITH ENDOGENOUS POPULATION

by

*Elisha A.Pazner and Assaf Razin **

Working Paper No. 28 - 79
September, 1 9 7 9

* *The second author sadly notes the untimely passing of Elisha A.Pazner and dedicates the paper to his memory.*

FOERDER INSTITUTE OF ECONOMIC RESEARCH,
Tel-Aviv University, Ramat Aviv, I s r a e l.

Revised
September, 1979

COMPETITIVE EFFICIENCY IN AN OVERLAPPING-GENERATION MODEL WITH ENDOGENOUS POPULATION

by

Elisha A.Pazner and Assaf Razin *
Tel-Aviv University, Tel-Aviv,
Israel

INTRODUCTION

A puzzling result regarding market failure was obtained by Samuelson (1958) in his exact consumption-loan model. He showed that under the conventional assumptions on the economic environment, the fact that "each and every today is followed by a tomorrow" (e.g., Samuelson (1958, p.482)] may lead competitive markets to fail in achieving the standard Pareto-efficiency objective. As the standard sources for market failure (externalities and non-convexities) are absent from Samuelson's model, it was natural to "blame" the infinity of the time horizon as such for the resulting inefficiency.[1/]

Our purpose in this paper is to show that if certain exogenous features of the Samuelsonian model are treated as being endogenously determined by economic factors, the above-mentioned inefficiency, with its associated puzzles, does not arise.

More specifically, in the model presented below, population is an endogenous variable with parental preferences determining the number and "quality", (in a utility sense) of children. Importantly, endowments of children are viewed to be bequeathed to them by parents. Under the assumptions of perfect capital markets and perfect foresight, it is shown that every competitive equilibrium is Pareto-efficient (under any appropriate definition of Pareto-efficiency).[2/] While the pathological behavior of competitive markets in the Samuelsonian model must indeed be attributed to the infinity of the economy's time horizon (in the sense that in finite horizon economies the efficiency of competition is guaranteed even under Samuelson's formulation of exogenous population evolution), the fact that in our model each representative individual has an infinite time horizon (even though he himself is finitely lived) is shown to be sufficient to restore the efficiency properties of competitive markets. To the best of our knowledge, this paper presents one of the first attempts to analyze formally the properties of competitive markets insofar as the efficiency production of population is concerned.[3/] The final Section discusses possibilities of some genuine externalities and equity issues related to population changes, and indicates some public policy implications.

II. GALE'S FORMULATION OF SAMUELSON'S MODEL

In this section we briefly review the essence of Samuelson's inefficiency result as presented in Gale (1973).

Consider a Samuelsonian overlapping-generations model in which each individual lives for two periods and the population grows geometrically at a rate γ. Each individual is endowed with a vector $e = (e_o, e_1)$ where $e_i (i = 0,1)$ represents the (fixed) endowment of an individual in the i^{th} period of his life. All people of all generations are assumed to be identical in endowments and preferences. As in Samuelson, goods do not keep and production is ruled out.

Let $c(t) = (c_o(t), c_1(t+1))$ be the consumption vector of an individual born in period t, where $c_i(s)$ is the consumption of an individual of age i in time-period s.

On the assumption that nothing gets thrown away the economy's resource constraint in period t is given by

(2.1) $$\gamma(e_o - c_o(t)) + (e_1 - c_1(t)) = 0$$

since there are γ young individuals per an old one.

We assume that the representative person has a preference ordering on his lifetime consumption vector which can be represented by a continuous, monotone increasing and quasiconcave utility function. The utility function of an individual born in period t is denoted by $u(c_o(t), c_1(t+1))$. As usual, a perfect foresight competitive equilibrium is an infinite sequence of interest factors $(\rho_t)_{t=0}^{\infty}$ and a feasible consumption program such that each individual maximizes utility subject to the budget constraint defined parametrically by the interest-factors.

Restricting our attention in this section to steady-state equilibria it is easy to see that these must satisfy

(2.2) $$(\rho - \gamma)(e_o - c_o) = 0$$

where ρ is the (time-independent) interest factor associated with an equilibrium steady-state. Thus, steady-state equilibria are of two types:
(1) that for which $\rho = \gamma$, the golden-rule program, and (2) that for which $e_o = c_o$, i.e. autarkic (no-trade) equilibrium.

Denoting by $\bar{c}$ the golden-rule program, Figure 1 summarizes the possibilities for steady-state equilibrium.

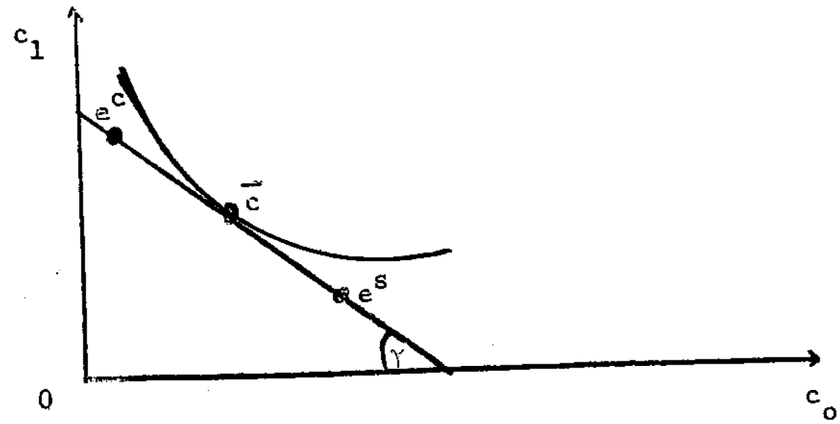

FIGURE 1

The figure demonstrates the possibility of steady-state competitive inefficiency. This obtains at the non-trade equilibrium represented by point e^s (and $\rho < \gamma$). The inefficiency is illustrated by the fact that in this situation the economy could instantaneously move to $\bar{c}$ making both the existing old and everybody else up to the indefinite future better off.

III. INFINITE INDIVIDUAL HORIZON AND THE IMPOSSIBILITY OF COMPETITIVE INEFFICIENCY

One possible objection to the Samuelsonian model is that while new generations are continually being produced by the older ones, there is nothing in the model that rationalizes this reproductive behavior. This difficulty can be resolved by appealing to the reproductive instincts of parents which implies that parents have preferences for children. Under such an interpretation, there will now be a utility link between any two successive generations. If the link is via parents having the utility function of their children as an argument in their own utility function then, recursively, the utility functions of <u>all</u> the (infinitely many) future generations become arguments in each representative individual's utility function.[4/] We will show now that under this specification of intergenerational preferences perfect foresight competitive equilibria are <u>always</u> efficient.

For each individual born in period t, $(t = 0,1,\ldots)$ the utility function is now assumed to be:[5/]

$$(3.1) \quad u(c_o(t), c_1(t+1), \gamma(t), u(c_o(t+1), c_1(t+2), \gamma(t+1), u(.,.,.,\ldots)))$$

where $\gamma(s)$ is the number of children of the representative individual of generation $s(s = t, t+1, \ldots)$. The utility function u is assumed to be monotone increasing and continuous. The dependence of u on $\gamma(t)$ stems from the fact that $\gamma(t)$ is an important endogenous variable of our model. But, even if $\gamma(t)$ were exogenous to the model, it should appear as an argument in u whenever it is desired to give a strictly positive utility weight to each (present and) future member of the family.

In order to define the representative individual's maximization problem in a competitive economy we must first define the budget set with which he is confronted. On the assumption that there exist perfect capital markets [6/] in which each individual of generation $t(t = 0, 1, 2,\ldots)$ can borrow and lend at the same (parametrically given) interest rates, we can lump up all the individual budget constraints with which each member of a given family $f(f = 1,\ldots,F,$ where F is the number of families, assumed to be fixed) will eventually be confronted with into a single family budget constraint given by:

$$(3.2) \quad \sum_{i=t}^{\infty} \prod_{s=t}^{i} \frac{\gamma^f(s)}{\rho(s)} (c_1^f(i) + \gamma^f(i) c_o^f(i) - e_1^f(i) - \gamma^f(i) e_o^f(i)) - B^f(t) \leq 0.$$

$$\text{where} \quad B^f(t) = \sum_{i=0}^{t} \prod_{s=0}^{t} \rho(s) [e_1^f(i) + \gamma^f(i) e_o^f(i) - c_1^f(i) - \gamma^f(i) c_o^f(i)],$$

$$t = 0,1,\ldots; \; f = 1,\ldots,F.$$

The meaning of $B^f(t)$ is the value, per individual of age 1 belonging to family f, at time t of the cumulative net intergenerational transfers of wealth between period 0 and period t.

In any period of time t, the economywide resource availability constraint is given by:

$$(3.3) \quad \sum_{f=1}^{F} \prod_{s=o}^{t} \gamma^f(s) [\gamma^f(t)(e_o^f(t) - c_o^f(t)) + e_1^f(t) - c_1^f(t)] \geq 0$$

for every $t = 0,1,\ldots$

Note that $\prod_{s=0}^{t} \gamma^f(s)$ is the number, at time t, of age 1 individuals belonging to family f. From (3.3) it is clear that for any t the aggregate cumulative net intergenerational transfers of wealth is non-negative, namely:

$$(3.4) \quad \sum_{f=1}^{F} \prod_{s=0}^{t} \gamma^f(s) B^f(t) \geq 0$$

A perfect foresight competitive equilibrium is defined by non-negative sequences $(c^f(s), \gamma^f(s), \rho(s))_{s=0}^{\infty}$ such that $(c^f(s), \gamma^f(s))_{s=t}^{\infty}$ maximize (3.1) s.t. (3.2) for each f and which satisfy (3.3). Such an equilibrium is referred to in the sequel as an <u>infinite-horizon equilibrium</u>. We are now ready to state the following:

THEOREM: An infinite-horizon equilibrium is Pareto-efficient.

Proof: The proof is standard. Suppose not, and let $(\hat{c}^{f_o}(s), \hat{\gamma}^{f_o}(s))_{s=t}^{\infty}$ for some f_o and some $t(t = 0,1,\ldots)$ be strictly preferred to the competitive sequence $(c^{f_o}(s), \gamma^{f_o}(s))_{s=t}^{\infty}$.

By the individual maximization property, it must then be true that:

$$(3.5) \quad \sum_{i=t}^{\infty} \prod_{s=t}^{i} \frac{\gamma^{f_o}(s)}{\rho(s)} (\hat{c}_1^{f_o}(i) + \hat{\gamma}^{f_o}(i)\hat{c}_o^{f_o}(i) - e_1^{f_o}(i) - \hat{\gamma}^{f_o}(i)e_o^{f_o}(i)) - \hat{B}^{f_o}(t) > 0$$

Aggregating (3.5) over population at time t, and using (3.4) it is then seen that a contradiction to (3.3) is obtained.

Q.E.D.

We will explain now why the no-trade (steady-state) allocation can never be an infinite-horizon equilibrium in the Samuelsonian case where the endowment vector e^s is to the right of the golden rule allocation in Figure 1. As noted by Samuelson (1958), and further elaborated upon by Gale (1973), the interest rate associated with the no-trade situation in such a case must be lower than the rate of population growth. In our model, however, such an equilibrium relationship between the rate of interest and the rate of population growth can never obtain since it would imply that the budget constraint (3.2) becomes unbounded which is inconsistent with the proper individual maximizations that underlie any competitive equilibrium. Sufficient conditions for the existence of individual maxima and of competitive equilibria are left for future research.

IV. THE NATURE OF INTERGENERATIONAL TRANSFERS IN THE MODEL

Suppose that $B^f(o) = 0$, i.e. that the "first" individual of age 1 in the economy has no net-claims on the present originating in the past, since there is no past. If it is also assumed that all individuals are identical in preferences and endowments, it follows that in equilibrium $B^f(t) = 0$, for all f and t. To see this, observe that $B^f(t) > 0$ for some t implies that every family in period t will consume less than the value of its initial endowments. Since goods are desirable and do not keep the resulting excess supply in period t is inconsistent with equilibrium. Likewise $B^f(t) < 0$ is impossible since it violates the feasibility condition (there can be no accumulation of goods from the past or decumulation from the future since goods are assumed to be nontransferable across time).

However, in the more general case where families differ, this will no longer be generally true. While for the economy as a whole the aggregate value in (3.4) is identically equal to zero at all t (the value of aggregate consumption during any time period equals the value of endowments in the same period) it is generally to be expected that some families overconsume in some periods (implying that some other families choose to underconsume in an offsetting way). From (3.2), it is then seen that the existence of overconsuming families until time $(t-1)$ implies that in time period t the budget constraint includes a negative $B^f(t)$ term.

We wish now to indicate how an institutional setting in which bequests might seemingly be "negative" is enforceable in a competitive market economy.[7/] Bequests from generation t to generation $t+1$ (intergenerational net-transfers of wealth) in our model are effectively always nonnegative in the following sense.

If we consider the endowments e_o and e_1 (say, productive abilities[8/] in each of the periods during which any individual lives) to be <u>inherited</u> (i.e. bequeathed by the previous generation), then the possibility that some generations might face a present value of consumption possibilities constraint smaller than the present value of endowments is not to be understood as involving a negative bequest. As long as each generation is able at all to consume (possibly by shifting debt to future generation), this is to be regarded as it having obtained a <u>positive</u> bequest from the previous generation.

Property rights in this model are in this sense assigned to parents. Since every individual in the model is a potential parent this assignment of property rights treats each generation symmetrically in a sense. Ths importance of this remark stems from the consideration that if each pair of e_o and e_1 were assumed to belong to the corresponding generation, then without outside legislative fiat requiring children to pay for their parents "debts" (i.e. without symmetric inheritance laws that treat debts and gifts in the same way) the above competitive program could not be sustained (unless parents' utility functions enter as arguments into those of their children).

Note that the enforcement of this system of property rights does not involve any more contrivance than the standard one implicitly assumed in finite horizon intertemporal economies where each individual is always required to pay his own debts even though this may be contrary to his self-interest (as is the case for instance when repayment of debt is due in the individual's last period of life). Finally, if the utility links are such that parents' welfare also enter childrens' utility function then there is an added-incentive for children to comply with repayments of their parents' debts.

V. CONCLUDING REMARKS

It has been shown that competition results in an efficient pattern of population production.

The two crucial features of the model are:

(1) The utility function depends strictly positively on every member of the family's utility in the indefinite future. The link into the indefinite future is obtained via the formulation that every member of a given generation attaches a strictly positive weight to the utility of his family in the next generation.

(2) Property rights are assigned to parents. Thus there are either positive bequests (forward handing over of endowments) or negative bequests (backward handing over of endowments).

In the Samuelson's formulation, with the assumption of a zero level of old-age endowment and its explicit notion of selfish utility, the "cure" for the inefficiency of competitive equilibrium can only come from negative bequests which are not compatible with individual incentives. Under our utility function and our notion of property rights the "old" can commit the "young" to negative bequests.

It is commonplace to regard the overall size and quality of population as one of the determinants of the welfare of any single member of the population. Often, considerations of congestion and crowding on the negative side or a sense of belonging or of security on the positive side are advanced. Such considerations of externalities imply that the overall (as opposed to family) size and quality of population enter as direct arguments in the individual utility functions, and that as a consequence, competitive markets would fail in achieving efficient outcomes.

In this connection, we wish to point out the following interesting problem.

Innate abilities are an important factor in determining the quality of population. Individuals do differ in their innate abilities. Now especially when considering individuals with unusual traits (e.g., geniuses who will contribute greatly to society's welfare or defective children who will become a net burden), we may expect that nobody is indifferent with respect to such possibilities. However, if the probability of such events is independent of the size of population and if the benefits (or costs) are fully appropriable by the families in which they happen, there will not arise a need for social intervention on efficiency grounds. That is to say, each family will have the correct incentives to make socially efficient choices regarding its decisions on number and quality of children. Full appropriability, however, may not be feasible, in which case some intervention, the nature of which requires some further investigation, is called for.

There are grounds for believing that the size of population is also a determinant of these probabilities. The probability in the population as a whole of any such individual being born is an increasing function of the size of the population (approaching one in the limit). In contrast, the probability with which each family is faced is negligibly small. Thus, there is a divergence between what may be termed social and private probabilities and the likelihood of market failure.

Let us consider now some equity aspects of population changes. The problem of equity deals with evaluating the relative desirability of alternative ways of distributing the economic resources of a society across its individual members. While there is no universally agreed-upon criterion of economic justice, there are now available several approaches which have received increased attention of economists, sociologists and moral philosophers in recent years.

One of the better-known criteria and one which has been applied to the problem of optimal population growth, is the Benthamite (utilitarian) criterion of maximizing the sum of utilities of society's members. While this criterion is not beyond dispute even when the number of individuals is fixed, the problem of population change introduces immediately a new dimension. For, while maximizing the sum of utilities is equivalent to maximizing the average utility level in the community when population is not subject to change, this, of course, is no longer true when the size of population is variable (this issue is discussed in detail in Meade (1976), Koopmans (1967), Arrow and Kurz (1970), and Rawls (1971) with most writers taking the viewpoint that the size of population is of ethical interest in itself, implying that the sum of utilities is a better normative guide than the average utility level). A related problem discussed in the above literature, is that of giving an appropriate representation in the social calculus to the welfare level of yet unborn generations. In order to overcome these problems we can specify utility functions of parents which are themselves a function of both the number and utility levels of future generations. The important remaining problem is to specify the precise conditions under which utilitarianism is a reasonable criterion for problems involving endogenous changes in population.

It is important to note also that under the standard specification of individual utility functions, the Rawlsian criterion of maximizing the welfare level of the least well off group of society is subject to the same kind of limitation as the criterion of maximizing average utility since no welfare weight is given to population size as such. Again, this particular criticism is overcome in our framework since individual utility functions are formulated in a way which gives positive weight to population size. The important remaining question for investigation in our present context is to assess the precise ethical meaning of maximum policies in the population area.

FOOTNOTES

* The second author sadly notes the untimely passing of Elisha A.Pazner and dedicates the paper to his memory.

The research was supported in part by a grant from the Rockefeller Foundation to Northwestern University for the study of the Economics of Population and Family Decision making. We wish to acknowledge helpful conversations with Richard Kihlstrom, Glenn Loury and Neil Wallace.

1. See Gale (1973), Shell (1971), Starrett (1973), and Thompson (1967). Particularly noteworthy for present purposes is Thompson's (1967) analysis of the source of inefficiency in Samuelson's model.

2. In a model with changing population, the definition of Pareto-efficiency warrants some discussion. Whose utility, current generation alone or current and all future (yet unborn) generations, does one wish to include in the efficiency calculus? In our model, in which there is a direct utility link between each generation and the one immediately following it (and thus an indirect utility link extending into the infinite future) the welfare of all generations as perceived by them is taken into account by the efficiency criterion in a natural way. Therefore, independently of whether or not the welfare of unborn generations is included in the efficiency criterion the analysis below implies that competitive markets always result in efficient allocations. In particular, the commodity "population" is also efficiently produced under the present competitive setting.

3. See Phelps (1968) for a related study of the welfare aspects of population changes.

4. See Nerlove (1974) and Razin and Ben-Zion (1975).

5. The function u should be thought of as a representation of the family's ordinal (Bergson-Samuelson) social welfare function as viewed by the current parent. We assume that each individual is consistent in his planning in the sense of Strotz (1955-56). Note that consistency with respect to plans of future generations is explicitly imbedded in the dynamic programming formulation of the utility function (3.1). We note here that the assumptions that utility functions are the same for all generations, and that individuals live only two-periods can be relaxed without affecting the results. Also a measure of the degree of altruism towards future generations (a rate of time preference) is already imbedded in the general utility specification.

6. On the nature of property rights in these markets see the discussion in Section IV overleaf.

7. Contrast this explanation to Barro (1974) who imposed a condition equivalent to $B^f(t) \geq 0$.

8. The model presented in Section III can be looked upon as being based on an implicit production technology by means of which initial endowments are transformed on a one-to-one basis into final consumption goods. We note here that intertemporal production possibilities can also be introduced without affecting the efficiency result.

REFERENCES

[1] Arrow, K. and Kurz, M., Public Investment, The Rate of Return and Optimal Fiscal Policy, Baltimore: The Johns Hopkins University Press, 1970.

[2] Barro, R.J., "Are Government Bonds Net-Wealth?", Journal of Political Economy, 82, (1974), 1095-1117.

[3] Gale, D., "Pure Exchange Equilibrium of Dynamic Economic Models", Journal of Economic Theory, 6, (1973), 12-36.

[4] Koopmans, T.C., "Intertemporal Distribution and "Optimal" Aggregate Economic Growth," in W.Fellner, et al., Ten Economic Studies in the Tradition of Irving Fisher. New York: John Wiley & Sons, 1967.

[5] Meade, J.E., The Just Economy, London: Allen and Unwin, 1976.

[6] Nerlove, M., "Household and Economy: Towards a New Theory of Population and Economic Growth", Journal of Political Economy, 82, (1974), 200-218.

[7] Phelps, E.S., "Population Increase", Canadian Journal of Economics, 1, (1968), 497-518.

[8] Rawls, J., A Theory of Justice, Cambridge, Mass.: Harvard University Press, 1971.

[9] Razin, A. and Ben-Zion, U., "An Intergenerational Model of Population Growth", American Economic Review, 65, (1975), 923-934.

[10] Samuelson, P.A., "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money", Journal of Political Economy, 66, (1958), 467-482.

[11] Shell, K., "Notes on the Economics of Infinity", Journal of Political Economy, 79, (1971), 1002-1011.

[12] Starrett, D., "On Golden Rules, the Biological Theory of Interest and Competitive Inefficiency", Journal of Political Economy.

[13] Strotz, R., "Myopia and Inconsistency in Dynamic Utility Maximization", Review of Economic Studies, 23, (1955-56), 165-180.

[14] Thompson, E.A., "Debt Instruments in Macroeconomic and Capital Theory," American Economic Review, 57 (1967), 1196-1210.

- - - - - - - - - - - - -