

EQUITY, NONFEASIBLE ALTERNATIVES AND SOCIAL CHOICE:
A RECONSIDERATION OF THE CONCEPT OF SOCIAL WELFARE

by

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ABSTRACT

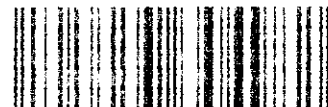
This paper introduces a new condition of independence of irrelevant alternatives. This new condition is seen to be conducive only to limited possibilities in the general social choice theoretic framework. However, standard Bergson-Samuelson social welfare functions are shown to be consistent with this new condition. As the analysis is carried out within the extended sympathy framework of social choice, some recent concepts of economic justice can be related in a natural manner to the present results.

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I. Introduction

The subject matter of this paper is best introduced by considering the following excerpt from Arrow [1], pp. 109-110:

"The essential point of the modern insistence on ordinal utility is the application of Leibniz's principle of the identity of indiscernibles. Only observable differences can be used as a basis for explanation. In the field of consumers' demand theory, the ordinalist position turned out to create no problems; cardinal utility had no explanatory power above and beyond ordinal.

It is the great merit of Bergson's 1938 paper to have carried the same principle into the analysis of social welfare. The social welfare function was to depend only on indifference maps; in other words, welfare judgments were to be based only on interpersonally observable behavior.

The Condition of Independence of Irrelevant Alternatives extends the requirement of observability one step farther. Given the set of alternatives available for society to choose among, it could be expected that, ideally, one could observe all preferences among the available alternatives, but there would be no way to observe preferences among alternatives not feasible for society.

The austerity imposed by this condition is perhaps stricter than necessary; in many situations, we do have information on preferences for nonfeasible alternatives. It can be argued that, when available, this information should be used in social choice... "

This quote reminds us that the concept of what came to be known as an individualistic Bergson-Samuelson social welfare function was intended to equip normative economics with well-defined orderings of social states based on indifference maps of individuals (see [4], pp. 318-320 and [11], p. 228). It was understood that such orderings would involve value judgments in the form of

ordinal interpersonal welfare comparisons. Arrow's classic impossibility theorem, on the other hand, made it clear what kind of ordinal interpersonal comparisons ought to be ruled out on grounds of logical (and ethical) inconsistency. (On Bergson-Samuelson social welfare functions, see [4], [11], and [12]; on their relation to Arrow's impossibility theorem, see [1], pp. 61-73, pp. 103-106, and p. 110 fn. 50 and [5], [6], and [13]).

If one accepts the conditions imposed by Arrow on social welfare functions, there is no escape from the conclusion that such functions do not exist. As is well known, of all of Arrow's conditions, the one that has been most controversial is that of independence of irrelevant alternatives. The main objective of the present paper is to advance an ordinal consistency condition which does not rule out the potential "relevance" of nonfeasible alternatives. This condition, called independence of non-indifferent alternatives (INIA), is based on additional information contained in individual indifference maps (i.e. inclusive of some pertaining to nonfeasible alternatives). It thus is a condition of independence of irrelevant alternatives with a different emphasis on what is or ought to be considered as irrelevant. Since INIA is weaker than IIA (IIA implies INIA), an interesting question is how weak it really is, i.e. how does it affect the pessimistic conclusions of Arrow's analysis.

The general possibilities for social choice introduced by the INIA condition are shown to be rather limited. In the general social choice theoretic framework of Arrow, if the nondictatorship requirement is replaced by the requirement of anonymity (symmetry across individuals), then replacing IIA by INIA still leads to impossibility. In this sense, INIA is quite a strong condition, and the disturbing consequences of Arrow's impossibility theorem remain essentially unchallenged. However, insofar as the specific economic context is concerned, INIA is considerably more productive. First, standard Bergson-Samuelson social welfare functions satisfy INIA almost by definition. In addition, any symmetric Bergson-Samuelson social welfare function satisfies the Pareto

condition, INIA and anonymity. Since the standard economic framework is not immune to Arrow's impossibility theorem, the possibilities introduced in this framework by weakening IIA to INIA are encouraging. This is the more so since INIA seems particularly meaningful in the economic context. As it is a purely ordinal condition, I believe that the welfare economic content of INIA is to be found in its shedding some light on the possibility of meaningful ordinal interpersonal comparisons in the economic context.

Another feature of the present paper is the formulation, within the social choice theoretic framework, of equity criteria which have been the subject of some recent economic literature. As the standard social choice framework does not seem amenable to the incorporation of equity criteria, the analysis is presented throughout within the interesting extended sympathy framework of social choice. This also gives me the opportunity to briefly mention at the outset what I consider to be rather serious limitations of recent results within that framework. For the sake of completeness, I have included a self-contained Appendix in which all the results pertaining to the possibility of social welfare analysis under INIA in the standard social choice theoretic framework are formally stated. In order to avoid needless duplication of formal results, the analysis in the text is informal and seeks to bring out intuitively the normative aspects of the present approach.

The plan of the paper is as follows. In Section II, the assumption of an objective extended sympathy ordering which underlies some recent literature is critically discussed. Section III introduces the INIA condition and discusses its implications. In Section IV, some equity criteria which have been the subject of recent economic literature are defined within the extended sympathy framework and are related to the discussion in Section III. In Section V, some brief remarks on the normative significance of IIA and INIA are offered. The Appendix concludes the paper.

II. On the Existence of an Objective Extended Sympathy Ordering

Let Z be a set of conceivable social states and I a set of individuals. Consider the Cartesian product $Z \times I$. Let (x, i) be an element of $Z \times I$. Following Suppes [15] and Arrow [1], pp. 114-115, we interpret (x, i) to mean "being individual i in social state x ". Let Σ be the set of weak orderings (total, transitive and reflexive binary relations) on $Z \times I$, the generic element of which is represented by R . Every i in I is assumed to have an ordering $R_i \in \Sigma$. For all j, k, ℓ in I and all x, y in Z , we shall understand $(x, j) R_k (y, \ell)$ to mean "in individual k 's opinion, it is at least as desirable to be individual j in state x as to be individual ℓ in state y ".

The extended ordering R_i reflects thus some explicit (ordinal) interpersonal welfare comparisons by individual i . In order to form his R_i , every i in I is required to imagine himself assuming the full personality of every other individual (including preferences, innate and acquired abilities, etc.). This, at any rate, seems to be the most general interpretation of R_i . In the economic context of the next section, a more limited but more intuitive interpretation of R_i will be considered.

In line with the individualistic tradition of modern welfare economics, it is assumed that for all i in I and all x, y in Z , $(x, i) R_i (y, i)$ implies, for all j in I , $(x, i) R_j (y, i)$. That is, each individual is required to have a non-paternalistic view of other individuals' welfare (in the sense that the judgment of any individual regarding his own relative welfare positions in any two social states is being respected). In what follows, I shall refer to the orderings R_i as extended sympathy orderings, interpersonal welfare comparisons orderings or simply interpersonal orderings.

Denote by B the set of weak orderings on Z . As proved recently by Hammond [8], there exists a function $H: \Sigma \rightarrow B$ satisfying Arrow's conditions of independence of irrelevant alternatives and Pareto, and the condition of anonymity.

On the other hand, if we denote by Σ^I the set of all I -lists of orderings

in Σ , as a direct consequence of Arrow's impossibility theorem we know that there exists no function $A: \Sigma^I \rightarrow \Sigma$ satisfying the above conditions (with all the conditions suitably adapted to orderings in Σ).¹ The impossibility result is actually stronger, since it continues to hold when nondictatorship is substituted for anonymity.² Even if the domain is restricted to I-lists of orderings in Σ^I satisfying the previously-mentioned non-paternalism property, the impossibility theorem still applies, since (as long as $\#I \geq 3$) the free-triple version of the condition of unrestricted domain is nevertheless satisfied.

Thus when the individuals are allowed to have different interpersonal orderings (different R_i 's), there is no consistent way (in Arrow's sense) to aggregate them. It follows that Hammond's starting point of postulating the existence of an "objective" interpersonal ordering (R_i independent of i) is crucial for his results. Since a common R cannot be derived, it must be assumed and a nontrivial restriction of domain is effectively involved (even though every i in Hammond's problem is free to order Z in any logically conceivable way under the interpretation that the restriction of R_i to $Z \times \{i\}$ conforms to i 's ordering of Z in the original Arrowian formulation).

The reason that the postulate of an objective interpersonal comparison ordering common to all individuals seems restrictive to me is essentially the same as that underlying the difficulties with this assumption which Arrow raises in the last two paragraphs in [3]. For, the case for this postulate (which is presented in Section 7 of [3]) essentially asks one to engage in a mental experiment at the end of which it is not clear "who" or "what" precisely it is that is assumed to have identical preferences on the enlarged set $Z \times I$. Since the individuals have to be stripped in this thought experiment from every possible personal characteristic (preferences, abilities, looks, etc.) that may serve to distinguish among them, there effectively seems to be "nobody" or "nothing" left to whom we can impute the ability of having an ordering of any kind.

That is to say, it seems natural to presume that there is a limit to one's ability to put oneself into somebody else's shoes. The implication of such a limit is that interpersonal orderings become a subjective matter and will generally differ for different individuals. In the next section, we shall mainly be concerned with the possibilities for social welfare functions that arise under subjective interpersonal orderings of a more limited scope.

III. Subjective Extended Sympathy, Nonfeasible Alternatives, and Social Welfare

The discussion of the previous section indicates that the extended sympathy framework is, in a formal sense, subject to all the difficulties raised by Arrow's impossibility theorem. Yet, especially from the viewpoint of equity analysis, this framework seems richer in possibilities than that used by Arrow in his original formulation of the problem of social choice. The possibility results that emerge under the assumption of an objective extended sympathy ordering are, of course, an important indication that such indeed is the case when Arrow's strict ordinality requirement is weakened to co-ordinality (see [3] and [7]). What I want to argue here is that if the "irrelevance" part of the condition of independence of irrelevant alternatives is weakened in a plausible way without giving up strict ordinalism, then interesting possibilities arise even when interpersonal orderings are not required to be identical.

Consider first the following statement by Arrow on the plausibility of the condition of independence of irrelevant alternatives which is even stronger than the one quoted in the introduction:

"I now feel, however, that the austerity imposed by this condition is stricter than desirable. In many situations, we do have information on preferences for nonfeasible alternatives. It can certainly be argued that when available this information should be used in social choice..." ([2], p. 134).

The extended sympathy framework is then presented ([2], p. 135) as one type of use of irrelevant alternatives ("irrelevant" since it is nonfeasible to be somebody else). I will briefly outline now a further possible use of irrelevant alter-

natives which seems especially appropriate in the economic context.

In the standard economic framework, each individual is postulated to have preferences over the set of all conceivable commodity bundles in his consumption set. His choice in any situation is dictated by his preferences and by the set of commodity bundles which is feasible in the particular situation under consideration. It is obvious that Arrow's condition of independence of irrelevant alternatives (abbreviated to IIA in the sequel) is, in the case of a single decision-maker, equivalent to the requirement of ordinality. In the multiperson framework, however, it is important to observe that ordinality can be retained even when the irrelevance aspect is weakened.

In other words, ordinal consistency conditions which are allowed to make use of the entire information contained in individual indifference maps (i.e. of preferences for nonfeasible alternatives) can, in principle, be advanced. The condition of independence of non-indifferent alternatives presented below should primarily be regarded as an attempt in that direction.

As before, let Z be the set of conceivable social states and I that of individuals. For every i in I , let $R_i \in \Sigma$ be his weak preference ordering on $Z \times I$ and let E_i be the indifference relation derived from R_i . As I will primarily have the economic context in mind in what follows, Z can be thought to be the set of conceivable I -lists of consumption bundles, i.e. the set of conceivable allocations. For all i in I and all z in Z , when all goods are purely private, (z,i) can simply be taken to mean "the consumption bundle assigned to individual i in social state z ". For all i and j in I and for all z in Z , we may then interpret the statement $(z,i) R_i(z,j)$ in the economic context to mean "individual i considers the consumption bundle that is assigned to him in social state z to be at least as desirable as that assigned in this state to individual j ". This interpretation is specifically advanced for the case of pure private good economies (in which each consumer is assumed to have selfish preferences) and is admittedly restrictive in that it explicitly excludes any interpersonal evaluations of individual charac-

teristics, such as preferences and abilities. As my main purpose here is to illustrate in a concrete manner the possibilities for social welfare functions introduced by weakening the IIA requirement and since the set of pure private good economies is a well-structured framework which is subject to all the difficulties raised by Arrow's impossibility theorem, I believe that this particular interpretation will prove to be of special interest. It will be made clear, though, that some of the formal possibility implications of the INIA condition which will now be formulated are completely independent of this particular interpretation.

Independence of Non-Indifferent Alternatives (INIA):

For every subset S of Z and every I -list of preferences $\underline{R} = (R_1, \dots, R_I)$ in Σ^I , let $C(S, \underline{R})$, a subset of S , represent the corresponding (social) choice set. The social choice correspondence $C(\cdot)$ is said to satisfy Independence of Non-Indifferent Alternatives (INIA) if for every pair $\underline{R} = (R_1, \dots, R_I)$, $\underline{R}' = (R'_1, \dots, R'_I)$ in Σ^I and for every subset S of Z :

if for all i in I and for all pairs x, y in S :

$(x, i) R_i(y, i)$ if and only if $(x, i) R'_i(y, i)$,

and if, in addition, for all (z, j) in $Z \times I$: $(z, j) E_j(x, i)$ if and only if

$(z, j) E'_j(x, i)$ and $(z, j) E_j(y, i)$ if and only if $(z, j) E'_j(y, i)$,

then $C(S, \underline{R}) = C(S, \underline{R}')$.

It might be worthwhile to mention first that the INIA condition is presented here (as in Arrow's original formulation) in terms of properties of the social choice function (called here correspondence since it maps social environments to subsets of the set of social states). It might perhaps have been more elegant to formulate INIA directly in terms of a (generalized) binary relevance condition on the social welfare function itself. Such a formulation can be found in the Appendix (for the case of the standard Arrowian framework without extended sympathy considerations). The reason I chose this (equivalent) way here is that the equity criteria discussed below can then be related in a very natural way to the present framework.

The difference between INIA and Arrow's IIA is that in any pairwise comparison of feasible states in S , the former takes into account information for each individual concerning preferences for those nonfeasible alternatives in Z which are indifferent according to his extended ordering to the implications for him of the social states which are members of the pair under consideration. It is clear that INIA is a purely ordinal condition. In the case of strong orderings, INIA is equivalent to IIA. However, when indifference between distinct alternatives is possible, INIA is consistent with the remaining Arrowian conditions on social welfare functions. However, the existence of a dictator over the subdomain of strong orderings implies that substituting anonymity for nondictatorship (in addition to substituting INIA for IIA) leads again to formal impossibility over the unrestricted domain of weak orderings in the general social choice theoretic framework. Thus, as a general truth about the conundrum of social choice, the distributing power of Arrow's impossibility theorem remains almost intact in the presence of INIA.

In the context of pure private good economies (with the above interpretation of the extended orderings), INIA implies that in comparing two allocations such that each individual's indifference curves (surfaces) through the bundles assigned to him at each are unchanged, then, independently of changes in the individual indifference maps over all other consumption bundles, the social ranking of the two allocations in question is required to be unchanged. Thus, the notion of INIA is consistent with the Bergsonian and Samuelsonian viewpoints that a social welfare function essentially ranks I-lists of indifference curves (surfaces). In pure private good economies, the monotonicity of individual preferences over commodity space, together with the assumption that allocation space is ranked by each individual in accordance with the bundle accruing to him, will now be shown to imply that standard Bergson-Samuelson social welfare functions satisfy INIA and anonymity.

Since Z in the economic context is the set of all conceivable I-lists

of consumption bundles, we can think of $Zx(i)$ as being the consumption set of individual i , and of $Sx(i)$ as being a compact subset of $Zx(i)$. In the pure private good economy case, we can further think of $Zx(i)$ as being the non-negative orthant of Euclidean space of some finite dimension. For all i in I , the restriction of R_i to $Zx(i)$ is assumed to be monotone increasing and continuous. From the selfishness assumption, it then follows that R_i is monotone increasing over ZxI . This monotonicity implies that whenever an indifference surface remains unchanged (when preferences are changing) so do the sets of elements of ZxI that are strictly preferred to all of its elements and those to which all of its elements are strictly preferred. Also, by continuity, strong preference orderings are ruled out.

It follows then that any standard Bergson-Samuelson social welfare function of the individualistic variety (i.e. satisfying the Pareto condition) satisfies INIA (since, for each individual, all elements that belong to the same indifference set are assigned identical utility numbers in any particular utility representation). Furthermore, whenever the same method of assigning utility numbers to indifference curves is used for each individual, anonymity can be guaranteed. If INIA is agreed to be a plausible condition, then the existence of (plausible) Bergson-Samuelson social welfare functions (which has been in doubt ever since the appearance of Arrow's impossibility theorem) can no longer be denied.

Note also that the presence of pure public goods does not create any special problems for the existence of individualistic Bergson-Samuelson social welfare functions that satisfy INIA and anonymity. The general consumption-externalities case, however, is subject to all the limitations of the general social choice framework mentioned above.

IV. Equity and Social Choice

Consider now the following equity criteria which are formulated here in a way which will help relate the present extended sympathy framework to some

recent literature on economic justice.

Equity Criteria:

State x in Z is said to be fair if, for all i, j in I , $(x,i) R_i(x,j)$; if for all i,j in I , $(x,i) E_i(x,j)$, then state x is said to be egalitarian. State x in Z is said to be fair-equivalent if there exists a fair state y in Z such that for all i in I , $(x,i) E_i(y,i)$; if the state y is egalitarian, then state z is said to be egalitarian-equivalent.

Observe first that an egalitarian state is in particular both fair and egalitarian equivalent; each of the latter two criteria satisfies in turn the fairness-equivalence criterion. Also note that only the equivalence criteria make potential use of the kind of nonfeasible alternatives used in INIA.

While under unrestricted domain in the general social choice framework, none of the equity criteria can be satisfied at all logically conceivable I -list of preferences, in the standard economic framework or pure private goods this problem does not arise. The idea of using equivalence criteria of equity in economics was motivated by the disturbing fact that allocations that are both Pareto-efficient and either egalitarian and/or fair (envy-free) need not exist in standard economic environments. As shown in [9] and [10], the equivalence criteria on the other hand are always consistent with Pareto efficiency. Since INIA and anonymity are, by definition, satisfied by either equivalence criterion, this implies (together with our earlier discussion) that there exist Bergson-Samuelson social welfare functions satisfying the Pareto condition, INIA and anonymity and having the additional property that every allocation that belongs to any of the choice sets generated by them satisfies either equivalence criterion.

Specifically, for a fixed ray through the origin of commodity space assign to each individual indifference surface a utility value which is equal to the Euclidean norm of the unique point on the ray which also is on the surface. As shown in [10], the set of best elements of the (Bergson-Samuelson) maximin

social welfare function under this normalization of utilities consists only of allocations that are egalitarian-equivalent. Note, by the way, that the apparent co-ordinality of this result is misleading, since, rather than using any particular utility representation, we can look at the underlying ordering of allocation space induced by this maximin function as being the Bergson-Samuelson ordering in question.³ That Pareto, INIA and anonymity are all satisfied is clear from our earlier discussion.

It may be useful to summarize the argument by means of a formal proposition. Thus if we denote by Σ_E^I the set of I-lists of extended preference orderings consistent with the standard assumptions on the economic environment, by Z the conceivable set of allocations, and by B the set of weak orderings on Z, we have:

Proposition: There exist Bergson-Samuelson social welfare functions $f: \Sigma_E^I \rightarrow B$ satisfying the Pareto condition, INIA and Anonymity, with the additional property that for any feasible compact subset S of Z, the choice set in S induced by f consists only of states in S that satisfy either equivalence criterion of equity.

At the risk of belaboring the obvious, let me mention here that throughout the discussion of the economic context, whenever referring to allocations as I-lists of consumption bundles, it should be understood that economies in which production is possible are included. Formally speaking, this means that the consumption bundles in question include the consumption of leisure. With this interpretation, a state of the society in the economic context (i.e. an allocation) includes a full description of the consumption and work activities by each individual.

V. On the Normative Significance of Nonfeasible Alternatives

The condition of IIA requires social choices to be independent of preferences for nonfeasible alternatives. In light of the ethical undertones of the term "irrelevant", it might perhaps have been preferable had IIA been christened "independence of nonfeasible alternatives". Since the difference between IIA and INIA is that some (nonfeasible) alternatives that are deemed "irrelevant"

under the first condition are not deemed so under the second, a brief discussion of the potential ethical relevance of nonfeasible alternatives might be indicated.

The set of social states which happens to be feasible at any given time is (or at any rate might be regarded as being) a morally irrelevant fact in itself. If the starting point (as in modern welfare economics) is that individual preferences over (all) conceivable social states ought to "count", then this (and not feasibility considerations) is in effect the sole benchmark of general normative significance. The fact that any actual social choice has to be feasible is under this viewpoint nothing more than an objective constraint. To attach separate normative significance to this particular fact can then legitimately be claimed to be alien to the spirit of welfare economics. This does not mean, however, that consistency conditions on social choices across different feasible sets are normatively meaningless. All it means is that, in principle, information about the entire preference orderings should be used when advancing such conditions.

Viewed from this perspective, the ethical significance of both IIA and INIA may be questioned. The additional normative content of INIA, though, is to be found I think in its "truncating" individual preferences over the prevailing feasible set in a less arbitrary way than IIA. Whereas IIA sharply truncates all the individual orderings directly over the feasible set of social states, the truncation under INIA is, so to speak, over the feasible "utility" set. (This follows from the observation that under INIA all the social states that belong to the indifference class of any feasible alternative are taken into account, irrespective of whether they themselves are feasible or not). INIA, that is to say, is an ordinal consistency condition on the choice of individual "utilities"; it is a translation of IIA to utility space.

Weaker ordinal consistency conditions than INIA, i.e. weaker "relevant truncations" of preference orderings (if at all), are, of course, conceivable and some might open up interesting new possibilities for social choice in general.

Insofar as the specific economic context is concerned, I am not sure, though, whether weaker conditions than INIA will be needed. The reason is that, in this particular context, INIA already effectively takes into account the entire individual orderings. In effect, the fixed structural properties of allocation space, together with the monotonicity of preferences over this space, imply that whenever any particular indifference set is left unchanged, then the set of allocations that were previously preferred to any of its members and the set of those to which any of its members were strictly preferred remain unchanged as well. This is the crucial feature of the standard economic framework which enabled me to show that when IIA is replaced by INIA, the existence of Bergson-Samuelson social welfare functions is no longer inconsistent with the general validity of Arrow's impossibility theorem.

APPENDIX

The INIA Condition in the Standard Social Choice Theoretic Framework

The discussion in the text has been confined to the extended sympathy framework of social choice. The reason for that is that this framework is amenable to the explicit introduction of equity considerations. In particular, it is not clear whether and how the equity criteria advanced in Section IV could have been presented and analyzed within the standard framework of social choice theory. However, in order to evaluate the broader social choice implications of INIA in proper perspective, it is important to present the analysis within the standard framework. This is the purpose of the Appendix. As the Appendix is essentially self-contained, I expect that no confusion will arise from the fact that the notation related to preference orderings (i.e. R , P , E and Σ) is the same as in the text, even though the sets on which the orderings are defined are different.

Let X be a set of (conceivable) social states and I a finite set of individuals. A weak preference ordering on X is a binary relation R which is reflexive, complete and transitive. Denote by P and E the strict preference relation and the indifference relation derived from R , respectively. Let Σ denote the set of weak preference orderings on X . For each i in I assume that there is a weak preference ordering R_i on X . Let Σ^I denote the set of I -lists of weak preference orderings on X .

A social welfare function is a mapping $f: \Sigma^I \rightarrow \Sigma$. For any given I -list of individual orderings, (R_1, \dots, R_I) the corresponding social ordering is denoted by $R = f(R_1, \dots, R_I)$.

Consider the following conditions on f :

Pareto: If (R_1, \dots, R_I) is an I -list of weak orderings on X such that for any pair x, y in X and all i in I : $xR_i y$, then xRy . If, in addition, for some j in I : $xP_j y$, then xPy .

Independence of Irrelevant Alternatives (IIA): For any pair x, y in X ,

if (R_1, \dots, R_I) and (R'_1, \dots, R'_I) are two I-lists of individual orderings such that $xR_i y$ if and only if $xR'_i y$ and $yR_i x$ if and only if $yR'_i x$, then xRy if and only if $xR'y$ and yRx if and only if $yR'x$.

Non-Dictatorship (ND): There is no i in I such that for every I-list in Σ^I and every ordered pair x, y in X , $xP_i y$ implies xPy .

Theorem (Arrow [1]): If there are at least three alternatives in X , then there exists no function $f: \Sigma^I \rightarrow \Sigma$ satisfying Pareto, IIA and ND.

Consider the following conditions:

Independence of Non-Indifferent Alternatives (INIA): For any pair x, y in X , if (R_1, \dots, R_I) and (R'_1, \dots, R'_I) are two I-lists of individual orderings such that $xR_i y$ if and only if $xR'_i y$ and if, in addition, for all z in X : $zE_i x$ if and only if $zE'_i x$ and $zE_i y$ if and only if $zE'_i y$, then xRy if and only if $xR'y$ and yRx if and only if $yR'x$.

Anonymity: If (R_1, \dots, R_I) and (R'_1, \dots, R'_I) are two I-lists of individual orderings with the property that there exists a permutation σ on I such that for all i in I : $R_i = R'_{\sigma(i)}$, then $R = R'$.

Denote by Σ^I_P the set of I-lists of strong orderings (reflexive, complete, transitive and anti-symmetric binary relations) on X and by Σ^I_W that of I-lists of orderings such that for at least one i in I and at least one pair of distinct alternatives x, y in X , $xE_i y$. (Note that $\Sigma^I_P \cup \Sigma^I_W = \Sigma^I$ and $\Sigma^I_P \cap \Sigma^I_W = \phi$.)

Proposition 1: If there are at least three alternatives in X , then there exists no function $F: \Sigma^I_P \rightarrow \Sigma$ satisfying Pareto, INIA and ND.

Proof: Over the domain Σ^I_P , INIA is equivalent to IIA. Since Arrow's Theorem holds over Σ^I_P , the proposition directly follows. Q.E.D.

Corollary: If there are at least three alternatives, then there exists no function $f: \Sigma^I \rightarrow \Sigma$ satisfying Pareto, INIA and Anonymity.

Proof: By Proposition 1, if Pareto and INIA are satisfied, then there exists a dictator over Σ^I_P , precluding the possibility of anonymity over all of Σ^I . Q.E.D.

Remark 1: It is clear that there exists a function $f: \Sigma^I \rightarrow \Sigma$ satisfying Pareto,

INIA and ND. For example, let i be the dictator over Σ_P^I and let $j \neq i$ be a dictator over Σ_W^I .

In the context of the standard economic model, INIA as formulated here requires that the social ordering of any two allocations will remain invariant whenever for each individual the indifference curves passing through each of the two allocations are unchanged (when moving from one set of individual preferences to another). As explained in the text, the fact that INIA takes into account the entire indifference sets of the two allocations involved in any binary comparison ensures its consistency with standard Bergson-Samuelson social welfare functions. Since utilities can be measured symmetrically (e.g. along a common ray in commodity space) and since strong orderings are ruled out under the standard assumptions on the economic environment, it follows that if we denote by Σ_E^I the set of admissible I-lists of preferences in the economic model, we have:

Proposition 2: In the economic context, there exist Bergson-Samuelson social welfare functions BS: $\Sigma_E^I \rightarrow \Sigma$ satisfying Pareto, INIA and Anonymity.

Remark 2: The presence of pure public goods does not affect the validity of Proposition 2 (provided that for any given public good or bad, all individuals have monotone preferences in the same direction). The general case of consumption-externalities, however, is governed by Proposition 1.

In sum, insofar as welfare economics proper is concerned, Proposition 2 justifies Bergson's and Samuelson's insistence on the possibility of defining normatively appealing and consistent (economic) social welfare functions. Proposition 1, however, confirms once more that there is no easy way out of the disturbing consequences of Arrow's general impossibility theorem.

FOOTNOTES

* Helpful conversations with my good friend David Schmeidler are gratefully acknowledged.

¹ For every list of orderings $(R_i)_{i \in I}$ in Σ^I let $R \in \Sigma$ represent the corresponding social ordering and let P denote the strict preference relation derived from R . The conditions adapted to social orderings in Σ (i.e. on $Z \times I$) are then:
Pareto: If $(R_i)_{i \in I}$ is a list of orderings on $Z \times I$ such that for all k in I $(x, i) R_k(y, j)$, then $(x, i) R(y, j)$. If, in addition, there is an h in I such that $(x, i) P_h(y, j)$ then $(x, i) P(y, j)$.

IIA: For every ScZ, if $(R_i)_{i \in I}$ and $(R'_i)_{i \in I}$ are two lists of orderings on $Z \times I$, then for all pairs $(x, i), (y, j)$ in $S \times I$: if for all h in I : $(x, i) R_h(y, j)$ if and only if $(x, i) R'_h(y, j)$ and $(y, j) R_h(x, i)$ if and only if $(y, j) R'_h(x, i)$, then: $(x, i) R(y, j)$ if and only if $(x, i) R'(y, j)$ and $(y, j) R(x, i)$ if and only if $(y, j) R'(x, i)$.

Anonymity: If $(R_i)_{i \in I}$ and $(R'_i)_{i \in I}$ are two lists of orderings on $Z \times I$ with the property that there exists a permutation σ on I such that for all i in I : $R_i = R'_{\sigma(i)}$, then $R = R'$.

² It may be interesting to observe, though, that Arrow's dictator in the present framework "only" dictates the value judgments which underly his extended sympathy ordering. That is to say, if we select an extended sympathy dictator, and then impose Hammond's conditions (inclusive of Hammond's maximin oriented Equity axiom), then society will be ruled by the (lexical) maximin criterion as perceived by the dictator. (From references to [14] which I have not yet had a chance to see, I understand that Strasnick has reached results similar to Hammond's.) If every individual is assumed to be nonpaternalistic in his extended sympathy ordering, then the unavoidable dictatorship result seems less disturbing than that obtained in the standard social choice framework (where the dictator's ordering is totally independent of that of other individuals). Nevertheless, even dictatorship in the present sense is troublesome, especially if one wishes to interpret extended sympathy orderings in terms of value judgments of a higher order.

³ Recall that the subjective extended sympathy orderings in the present case of pure private goods economies involve only evaluations of bundles according to individual preferences for bundles. Therefore, for all i , any monotone utility stretching over $Z \times \{i\}$ involves a simultaneous (consistent) stretching over all of $Z \times I$. Thus, the analysis is purely ordinal since independent positive monotone transformations over all of $Z \times I$ will not alter the results.



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