

Occupational Choice: Teacher Quality Versus Teacher Quantity

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Abstract

This article examines the relationship between skill-biased technological changes and the decline in both teacher quality and pupil–teacher ratio—called the “quality–quantity trade-off”—in the United States and other advanced economies during the past several decades. The study presents a theory of educational production that emphasizes teachers’ occupational choices. A key assumption is that talented agents have a comparative advantage in learning. The model endogenously generates a teachers sector with intermediate abilities between two types of skilled workers with tertiary education: highly skilled workers and vocational workers. This unique feature helps specify *which* technological changes may lead to quality–quantity trade-offs. In particular, a crucial element is that the ratio of incomes and thus income inequality rises *within* the skilled sector. In this case, the most talented teachers depart from the teachers sector to join the highly skilled sector, and as such, teacher quality declines. In other cases, both teacher quality and teacher quantity may increase. The results are consistent with the observed patterns of technology, educational attainment, educational expenditure, and wage inequality in advanced economies. Finally, other potential causes for the quality–quantity trade-off include a reduction in the teacher certification requirements and shifts in the distribution of initial endowments.

Keywords: Human capital accumulation; Skill-biased technological change; Teacher quality; Pupil–teacher ratio; Education policy;

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I. Introduction

What are the implications of skill-biased technological changes (SBTCs) on the quality and quantity of teachers? Which types of SBTCs might result in quality–quantity trade-offs? How is the rising income inequality among skilled workers related to the declining quality of teachers over time? To address these questions, this research develops a theory of educational production with compulsory education and tertiary education that emphasizes teachers’ occupational choices.

In the United States, education expenditure per student has increased since 1960. However, the pupil–teacher ratio has fallen from 25.8 in 1960 to 15.7 in 2005 (Digest of Education Statistics, 2007), and teacher quality has also declined relative to the educated labor force. The latter finding is robust to the sparse indicators used as a proxy for teacher quality.¹ This trade-off between the quality and quantity of teachers may have occurred in other OECD countries as well (Nickell and Quintini, 2002; OECD, 2005). The goal of this article is to investigate the quality–quantity trade-off within a general equilibrium overlapping-generations framework.

A common cause given in the literature for the quality–quantity trade-off is SBTC, which amplifies the demand for more college-educated workers, with a corresponding increase in their wages. As a result, decision makers tend to substitute quantity for quality in their resource allocation decisions.² This study takes this explanation one step further and argues that only *certain* types of SBTCs lead to quality–quantity trade-offs. The few theoretical models that address this question (Gilpin and Kaganovich, 2009; Lakdawalla, 2001; Stoddard, 2003), use a simplifying assumption that agents base their decisions only on income considerations. As a result, there is *excess* supply of low-ability teachers (whose earnings in the production sector are lower than teacher wages).

¹ Some of the proxies for teacher quality include teachers’ wages, the fraction of people entering teaching relative to other educated workers with standardized test scores above the 80th percentile, the fraction of prospective teachers being drawn from less selective institutions, the proportion of U.S. college graduates who earn lower salaries than the average teacher, and the relative fraction of married female teachers with top-earning or highly-educated husbands assuming positive assortative mating. (e.g., Bacolod, 2006; Dolton and Marcenaro-Gutierrez, 2011; Hanushek and Rivkin, 1997; Hanushek and Rivkin, 2006; Lakdawalla, 2001; Stoddard, 2003).

² Other explanations include the expansion in labor-market opportunities for women outside teaching, the compression of teachers’ incomes owing to unionization, the smaller number of children per household, rising incomes, the increased demand for educational services, and improved provision of public education (see Bacolod, 2006; Corcoran et al., 2004; Flyer and Rosen, 1997; Goldhaber and Liu, 2003; Hanushek and Rivkin, 1997, 2003; Hanushek and Rivkin, 2006; Hoxby and Leigh, 2004; Murnane et al., 1991; Stoddard, 2003).

Thus, the lower threshold of teachers is solely determined by the government. The current model is more comprehensive because it further emphasizes the occupational choice decisions of potential teachers taking into account the *leisure* implications of acquiring higher education (Betts, 1998; Costrell, 1994) as well. The model further assumes that the learning effort required to acquire a certain level of higher education is greater for low-ability workers. Under these assumptions, teachers endogenously have intermediate abilities between two types of skilled workers (with tertiary education): highly skilled and vocational. This unique division helps specify the crucial elements in SBTCs that lead to quality–quantity trade-offs—that is, the rising ratio of incomes between the highly skilled and vocational workers and the rising income inequality *within* the skilled sector.

The central finding is that quality–quantity trade-offs may occur because the SBTC amplifies the income inequality *within* the skilled sector.³ In the model, workers at the upper end of the ability distribution receive exponentially larger returns for their ability. In this case, the highly skilled sector attracts high-ability teachers, which in turn generates downward pressure on relative teacher quality. Moreover, as the pursuit of higher education becomes worthwhile for a broader population, workers with relatively low ability are added to the skilled sector. In other types of SBTCs, when the ratio of incomes (and, thus, the income inequality) does not change among skilled workers, both the supply of teachers and their quality increase. The results coincide with observed patterns in the United States and other advanced countries since 1960: increasing educational expenditures, rising wage inequality, increasing returns on ability, rising college attendance, and equalized teacher incomes (Autor et al., 1998; Berman et al., 1998; Goldin and Katz, 1999; Katz and Murphy, 1992).

Other potential causes for the quality–quantity trade-off include shifts in the distribution of initial endowments and a reduction in the teacher certification requirement. According to UNESCO (2006), several developing countries with limited budgets that face serious teacher shortages (e.g., Burkina Faso, Bangladesh, India) have decided that

³ Much of the literature attributes the evidence of rising residual wage inequality (within groups with similar education, labor market experience, age, race, and sex) to increased returns to unobserved learning abilities. Skill-biased revolutions trigger reallocations of capital from slow- to fast-learning workers, thus generating absolute gains for people with high cognitive ability (see Bartel and Sicherman, 1999; Caselli 1999; Galor and Moav, 2000; Juhn et al., 1993; Katz and Murphy, 1992; Murnane et al., 1995; Nelson and Phelps, 1966).

the most viable option is to lower entry standards for the teaching profession. This may also be the case in California, where its class-size reduction program came at a cost of hiring teachers with lower qualifications (Jepsen and Rivkin, 2002). This study further demonstrates the cost of requiring a relatively long time investment from teachers. In this case, in equilibrium even the top-quality teachers earn a higher income than their counterparts in the high-skilled sector, which compensates them for their greater effort in higher education.

A key insight in the analysis is that accounting for the heterogeneous learning effort in higher education is important for analyzing the quality–quantity trade-off. The evidence in Loewenstein and Thaler (1989) and Sizer (1984) suggests an extremely high discount rate of students on future incomes and a high emphasis of youth culture on current leisure and consumption. According to Costrell (1994), student time and effort are the most important inputs to education, given the level of ability. Hanushek et al. (2003) consider the disutility of labor (see also references there; Azariadis and Drazen, 1990; Glomm and Ravikumar, 1992, 2003; Tamura, 1991; Viaene and Zilcha, 2002). Huggett et al. (2006) show that differences in learning ability account for the bulk of the variation in earnings across agents. Accordingly, with heterogeneity in abilities, this study assumes that highly able agents have a comparative advantage in learning over low-ability workers. Therefore, low-ability workers are not interested in devoting the learning effort required for teacher certification. Instead, low-ability workers, who still desire tertiary education, enroll in shorter programs geared for entry into the labor market and designed to acquire practical/vocational/technical skills and know-how needed for employment in a particular occupation or trade (e.g., technicians, practical engineers, nurses). As a result, they earn lower incomes than teachers (but higher incomes than unskilled workers). The model further assumes that teachers are equally paid because of collective bargaining agreements. Hoxby and Leigh (2004) highlight the substantial contribution of teachers' unions to wage compression. It is well documented that unions tie teachers' incomes primarily to experience, oppose linking incomes to performance, and insist on raising incomes across the board. Wage compression in public schools imposes similar wage rigidity on the private school

teacher market (Lakdawalla, 2006).⁴ Under these assumptions, this study endogenously posits that teachers have intermediate abilities between vocational workers and highly skilled workers, who enroll in longer programs of higher education (typically academic and theoretically based/research preparatory). Thus, the main contribution of this article is the introduction of a more complete model of teacher self-selection that helps grasp the essential features of SBTCs that lead to a quality–quantity trade-off.

The paper is organized as follows: Section II develops a general equilibrium model. Section III defines the equilibrium and provides conditions for its existence and uniqueness. Section IV characterizes the time investment in higher education and incomes across sectors. Section V details the numerical example. Section VI derives the comparative static results on teacher quality and quantity, and section VII concludes. (Unless otherwise mentioned, the proofs are relegated to the Appendix).

II. The Model

A. Timeline

Consider an overlapping-generations model with a continuum of consumers in each period and no population growth. Assume that agents live for two periods. In the first period, childhood, they are not productive: their parents support them, and they acquire compulsory public education at a uniform level. In the second period, each agent allocates time to higher education. Then, agents work, pay taxes, give birth to one child and consume their after-tax income. Tax revenues are used by the government to support the children’s public education.

B. Human Capital Formation

Let $h_{i,t}$ be the human capital level in adulthood of an agent i born at date $t-1$. The term E_t denotes his or her level of compulsory public education (acquired in childhood). In adulthood, the agent chooses the fraction of time dedicated to higher education, $0 \leq e_{i,t} \leq 1$. This leads to the first assumption:

⁴ Gilpin and Kaganovich (2009) note that unionization is not the only factor responsible for the compression of teacher salaries. It is also attributed to the difficulty in measuring teacher productivity and determining criteria for performance-based pay.

(A1) A minimal level of time investment in higher education is necessary to attain some higher education degree, \hat{e} .

If this standard is not met, the human capital equals formal compulsory schooling. If the agent decides to acquire higher education above the minimal level, the human capital further depends on the time investment in higher education as well as the agent's innate ability, denoted by $\theta_{i,t}$. The term $\theta_{i,t}$ is i.i.d. and distributed as some random variable $\tilde{\theta}$ with values in the interval $[\underline{\theta}, \bar{\theta}]$, where $\underline{\theta} < \bar{\theta} < \infty$. To simplify the exposition (but at no cost to the essence of the matter), let $\underline{\theta} = 1$. Note that 'abilities' may reflect any unobserved initial endowments related to home background or school background.⁵ The production function of human capital is given by

$$(1) \quad h_{i,t} = \begin{cases} \rho E_t e_{i,t}^\beta \theta_{i,t}^\lambda, & \text{if } 1 \geq e_{i,t} \geq \hat{e} \\ E_t, & \text{if } 0 \leq e_{i,t} < \hat{e} \end{cases}$$

for some $\beta < 1$, $\lambda < 1$, $\rho \hat{e}^\beta > 1$. Thus, acquiring higher education (above the minimal level) increases the human capital. This functional form is consistent with the empirical evidence that under similar levels of compulsory schooling, higher education is more productive for agents with higher initial endowments. In the following discussion, for simplicity of presentation, I omit the time index.

C. Sectors of Workers: Teachers and Skilled and Unskilled workers

In the economy, teachers, skilled workers and unskilled workers differ in their income structure and time investment in education. Skilled workers are employed in professions that require various levels of abilities and higher education. Therefore, following Becker (1975), I assume that skilled workers are rewarded for their human capital. Their income equals $y_{s,i} = w_S h_{s,i}$, where w_S is the wage rate for an effective unit of human capital. At the same time, abilities and higher education are secondary determinants of income for unskilled workers. According to Bishop (1988), employers of high school graduates rely almost exclusively on the diploma, rather than the more complete information contained in school transcripts or employment tests. Thus, I assume that the income of unskilled workers is uniform, denoted by y_U . Furthermore, because teachers' collective bargaining agreements tend to equalize their incomes, I

⁵ Cunha and Heckman (2007) argue that abilities are created, not solely inherited. The family plays a powerful role in shaping abilities through genetics, parental investments and choice of child environments.

assume that teachers are equally paid, and their income is denoted by y_T . Note that income and time investment in all sectors are determined in equilibrium, except for the time investment of teachers. The government observes whether teachers meet the following requirement:

(A2) Teachers must invest at least $e_T \geq \hat{e}$ in higher education to attain a teacher certification. This level is exogenously given by governmental requirements. ◦

Note that the time investment of teachers acts in the model as both a sorting mechanism to the teachers sector and a source of human capital (Betts, 1998; Weiss, 1983).

D. Agents' Decisions: Allocation of Time and the Labor Supply

In adulthood, each agent is endowed with two units of time. One unit is inelastically devoted to labor, and the other unit is allocated between time investment in higher education, e_i , and leisure. Lifetime utility depends on consumption, denoted by c_i , and effective leisure, l_i , for some $\delta, \mu < 1$:

$$(2) u(c_i, l_i) = c_i^\delta l_i^\mu, \quad \text{for } c_i > 0, \quad l_i > 0.$$

(A3) For some parameter $Z > 0$, the effective leisure of agent i is given by

$$(3) l_i = 1 - \frac{Ze_i}{\theta_i},$$

where $0 \leq l_i \leq 1$. That is, $0 \leq e_i \leq \frac{\theta_i}{Z}$. ◦

The ratio $\frac{Ze_i}{\theta_i}$ represents the learning effort invested in higher education, where Z is the non-pecuniary cost of effort. I assume that highly talented agents have a comparative advantage in learning. Therefore, the learning effort required to achieve a given level of higher education diminishes with the level of ability. Accordingly, less talented agents have lower incentives to invest in higher education at the expense of leisure.

(A4) The following condition holds:

$$(4) e_T < \frac{\bar{\theta}}{Z}. \circ$$

This condition is necessary for the existence of the teachers sector. If this condition is not satisfied, no agent has a positive effective leisure as a teacher (recall (A3)). I assume that the government avoids this scenario by ensuring that teachers' time investment is sufficiently low. Note that if the cost of effort is sufficiently low, $Z < 1$, this condition holds for all e_T . I assume hereinafter that assumptions (A1)–(A4) hold. Given the income structure in the three sectors, $y_{s,i}$, y_T and y_U , each agent chooses whether to become a teacher, a skilled worker or an unskilled worker and how much time to invest in higher education by maximizing his or her utility, given in equation (2), such that his or her effective leisure, given in equation (3), and consumption are positive:

$$\text{Max}_{c_i, e_i} u_i = c_i^\delta \left(1 - \frac{Ze_i}{\theta_i} \right)^\mu$$

s.t.

$$c_i \geq 0 \quad \text{and} \quad 0 \leq e_i \leq \frac{\theta_i}{Z}$$

(5) *One of the following options can be chosen :*

(a) *Choose* $e_i \geq e_T$ *and* $c_i = (1 - \tau)y_T$ *(teachers)*

(b) *Choose* $e_i \geq \hat{e}$ *and* $c_i = (1 - \tau)w_s h_i$ *(skilled)*

(c) *Choose* e_i *and* $c_i = y_U$ *(unskilled)*

where h_i is defined in equation (1) and consumption equals the after-tax income. For simplicity, I assume the following progressive taxation: only the higher income sectors, teachers and skilled workers, pay taxes, and the tax rate, τ , is exogenously given. At the optimum, because teachers and unskilled workers are not rewarded for their human capital, teachers invest in their higher education exactly the time investment required to meet the standard, e_T , and unskilled workers exert zero effort, $e_U = 0$. I obtain the optimal time investment of skilled workers by rearranging the first-order condition that equates their marginal utility from time investment in higher education to the marginal cost:

$$(6) \quad e_i^* = \frac{1}{Z} \left(\frac{\delta\beta}{\mu + \delta\beta} \right) \theta_i.$$

The optimal time investment varies across skilled workers: highly able workers prefer to spend more time on higher education than less talented ones because of their comparative advantage in learning (recall (A3)). As a result, substituting equation (6) in equation (3), the optimal effective leisure is identical for all skilled workers:

$$(7) \quad I_S^* = \frac{\mu}{\mu + \delta\beta}.$$

Substituting equation (6) in equation (1), I derive the human capital of skilled workers as a function of the public education provision and ability:

$$(8) \quad h_i = \rho \left(\frac{\delta\beta}{Z(\mu + \delta\beta)} \right)^\beta (\theta_i)^{\beta+\lambda} E.$$

Accordingly, highly skilled workers accumulate larger levels of human capital directly (through λ) and indirectly (through β) by spending more time on higher education. Thus, they earn higher incomes than less talented skilled workers (recall equations (1), (3) and (6)). Note that teachers are also compensated for having higher ability through the lower learning effort required to attain teacher certification (recall (A2) and (A3)). Accordingly,

Corollary 1: The utility from skilled professions and from teaching increases with ability, while the utility in the unskilled sector is independent on ability. ◦

Thus, the least talented workers join the unskilled sector. Because of their insufficient talent for schooling, they prefer not to acquire higher education at all and enjoy the extra leisure. That is, acquiring higher education would reduce their utility because their learning effort as skilled workers or as teachers is too high relative to their incomes. Only sufficiently talented workers may acquire the minimal level of higher education required from skilled workers or teachers:

Corollary 2: Agents with sufficiently high ability, such that $\theta_i \geq Z\hat{e} \left(\frac{\mu}{\delta\beta} + 1 \right)$

($\theta_i > Ze_T$), are compatible with skilled professions (teaching), respectively. ◦

Corollary 2 is derived from (A1), (A2) and (A3). Namely, the optimal time investment of skilled workers given in equation (6) exceeds the minimal level necessary to attain some higher education degree, \hat{e} , and the effective leisure of teachers is positive.

Definition 1: Using equations (5)–(8), the utility from skilled professions relative to teaching is the utility of agent i from skilled professions divided by his or her utility from becoming a teacher:

$$(9) \quad \left(\frac{y_{i,S}}{y_T} \right)^\delta \left(\frac{I_S}{I_{i,T}} \right)^\mu,$$

where

$$\frac{y_{i,S}}{y_T} = \left(\frac{\delta\beta}{Z(\mu + \delta\beta)} \right)^\beta \frac{w_S \rho E}{y_T} (\theta_i)^{\beta+\lambda} \quad \text{and} \quad \frac{I_S}{I_{i,T}} = \left(\frac{\mu}{\mu + \delta\beta} \right) \left/ \left(1 - \frac{Ze_T}{\theta_i} \right) \right. . \circ$$

Recall that teacher income, y_T , and the optimal effective leisure of skilled workers, I_S , (see equation (8)) are uniform. However, when workers become more talented, their incomes as skilled workers, $y_{i,S}$, increase through $(\theta_i)^{\beta+\lambda}$ as a result of their greater time investment in higher education (recall equations (6) and (8)) and their greater ability. Moreover, their effective leisure as teachers, $I_{i,T} = 1 - \frac{Ze_T}{\theta_i}$, increases because they have a comparative advantage in making the exogenous time investment required to become teachers, e_T (recall corollary 1). Accordingly, the utility from skilled professions relative to teaching in equation (9) can be rewritten by gathering these ability-dependent factors in $\frac{u_S}{u_T}(\theta_i)$ and other elements independent of ability in π_1 :

$$(10) \quad \left(\frac{\pi_1 w_S}{y_T} \right)^\delta \left(\frac{u_S}{u_T}(\theta_i) \right)^\mu,$$

where

$$\frac{u_S}{u_T}(\theta_i) = \frac{(\theta_i)^{\delta(\beta+\lambda)}}{\left(1 - \frac{Ze_T}{\theta_i} \right)^\mu} \quad \text{and} \quad \pi_1 = \left(\frac{(\delta\beta)^\beta \mu^{\frac{\mu}{\delta}} \rho}{Z^\beta (\mu + \delta\beta)^{\frac{\mu+\beta\delta}{\delta}}} \right) E . \circ$$

π_t includes the skilled wage rate for an effective unit of human capital, teacher income, public education provision (given from the previous period) and parameters of the preference structure and the human capital formation.

Proposition 1: The utility from skilled professions relative to teaching is *convex* in ability, θ_i , and it attains a minimum at

$$(11) \quad \hat{\theta} = \chi Ze_T = \text{Argmin} \left(\frac{u_S}{u_T}(\theta_i) \right),$$

$$\text{where } \chi = \frac{\mu}{\delta(\beta + \lambda)} + 1.$$

Proposition 1 is easily proved by deriving the utility from skilled professions relative to teaching by ability. The proof is available on request.

Property 1⁶: The slope of $\frac{u_S}{u_T}(\theta_i)$ is steeper below $\hat{\theta}$ than above $\hat{\theta}$.

The convexity in ability implies that both highly skilled workers and low-ability workers prefer skilled professions rather than teaching and that the teachers sector consists of intermediate-ability workers. This feature is generated because ability contributes to utility through two channels: income and effective leisure. Skilled workers are compensated for higher ability through larger incomes, while teachers are compensated through lower effective effort (recall corollary 1). When the ability of highly skilled workers increases (above $\hat{\theta}$), the marginal utility derived from enlarging their incomes as skilled workers more than offsets the increase in their effective leisure as teachers. This occurs because their effective leisure as teachers (recall (A3)), which is already high, is bounded by 1. Nonetheless, their skilled incomes are unbounded and thus increase more substantially. As a result, highly skilled workers prefer skilled professions rather than teaching. However, when the ability of low-talented workers decreases (below $\hat{\theta}$), the decline in their skilled incomes is negligible relative to the increase in their learning effort as teachers. Specifically, workers with sufficiently low ability ($\theta_i \rightarrow Ze_T^+$) have almost no effective leisure as teachers (recall (A3) and corollary

⁶ Property 1 is proved under the sufficient (but not necessary) assumption: $\delta(\beta + \lambda) = \mu$, or $\delta(\beta + \lambda) < \mu$ (effective leisure is sufficiently important relative to income) and $P_T > \frac{2Ze_T}{\theta - 1} \sqrt{\chi^2 - 4}$ (the teachers sector is sufficiently large). In our numerical example, pupil-teacher ratio must be lower than 17 to satisfy this condition, which corresponds to the pupil-teacher ratios in primary and secondary education in most OECD countries. The proof is available on request.

2), and thus their marginal utility from effective leisure is infinite. As a result, not only low-ability workers prefer skilled professions to teaching, but also the slope of $\frac{u_S}{u_T}(\theta_i)$ is steeper below $\hat{\theta}$ than above $\hat{\theta}$. Instead of attaining the uniform time investment required for teacher certification (recall (A2)), low-ability workers optimally alleviate their learning effort by enrolling in shorter programs of higher education with fewer requirements than teaching (recall (6) and see proposition 6 hereinafter), such as community colleges, vocational training or any practical courses beyond high school with occupational orientation (e.g. nurses, nannies, dental assistants, technicians, computer/network/internet/technical operators, QA (quality assurance), paramedics, investigators, bookkeepers, policemen, firemen, medical secretaries). In contrast, highly skilled workers enroll in longer programs typically characterized by academic, theoretically based research preparation. Thus,

Definition 2: The skilled sector is also referred to as the 'total skilled' sector. It contains two sub-sectors (not interested in teaching):

The vocational sector consists of all skilled agents with lesser abilities than teachers.

The high-skilled sector consists of all skilled agents with greater abilities than teachers. ◦

The division of the labor force into unskilled workers, vocational workers, teachers and highly skilled workers is formalized in the following proposition 2 and illustration 1 using definitions 3–4. Note that the conditions that guarantee the existence of sectors are derived in section III.

Definition 3: The term θ_{jk} denotes the ability level of workers, who are indifferent between belonging to sector 'j' and sector 'k'. ◦

Definition 4: Assume that for some 'j' and 'k', θ_{jk} satisfies $\theta_{jk} < \bar{\theta}$. If all workers with ability below (above) θ_{jk} prefer sector 'j' ('k') to all other sectors, sectors 'j' and 'k' exist and θ_{jk} is the threshold level between them. ◦

Proposition 2: Sectors of workers are organized as follows:

- a. If both the high-skilled sector and vocational sector exist, then $\theta_{TH} > \hat{\theta} > \theta_{VT} > Ze_T$ and
- (1) The most talented agents, with abilities $[\theta_{TH}, \bar{\theta}]$, generate the high-skilled sector.
 - (2) Agents with abilities $[\theta_{VT}, \theta_{TH}]$ are teachers.
 - (3) The vocational sector comprises agents with abilities $[\theta_{UV}, \theta_{VT}]$ and $\theta_{UV} < \theta_{UT} < \theta_{VT}$.
 - (4) The lowest ability agents, with abilities lower than θ_{UV} , generate the unskilled sector.
- b. If the high-skilled (vocational) sector does not exist, then the upper (lower) threshold of the teachers sector is $\bar{\theta}$ (θ_{UT} and $\theta_{UV} > \theta_{UT} > \theta_{VT} > Ze_T$). ◦

Proof:

Because of the convexity of the utility from skilled professions relative to teaching (recall proposition 1), indifferent workers between these sectors are represented by a unique *pair* of abilities $\{\theta_{VT}, \theta_{TH}\}$, for which equation (10) equals ‘1’:

$$(12) \quad \left(\frac{y_T}{w_S \pi_1} \right)^\delta = \frac{u_S}{u_T}(\theta_{VT}) = \frac{u_S}{u_T}(\theta_{TH}), \quad \text{where } \theta_{TH} > \hat{\theta} > \theta_{VT} > Ze_T.$$

The ability of indifferent workers between the unskilled sector and the vocational sector (the teachers sector), denoted by θ_{UV} (θ_{UT}), is obtained by equating the utility from unskilled professions and skilled professions (teaching), respectively:

$$(13) \quad \frac{y_U}{w_S} = (1 - \tau) \pi_1 (\theta_{UV})^{\beta + \lambda}$$

$$(14) \quad \frac{y_U}{y_T} = (1 - \tau) \left(1 - \frac{Ze_T}{\theta_{UT}} \right)^\frac{\mu}{\delta}, \quad \text{where } \theta_{UT} > Ze_T.$$

θ_{VT}, θ_{UT} must be above Ze_T , because these workers are able to become teachers (recall corollary 2). The other inequalities are implied by consistency of preferences. This part of the proof is relegated to the Appendix. ◦

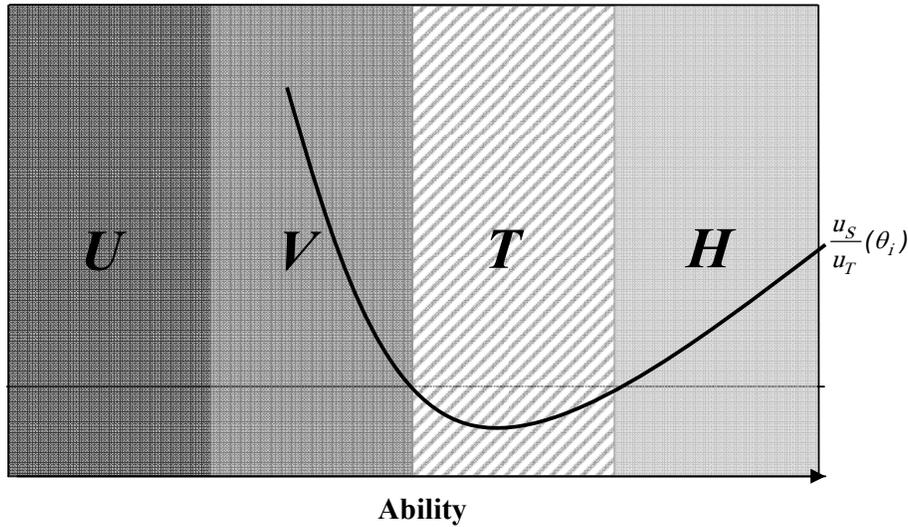


Illustration 1 – The composition of the labor force.

The numerical example is detailed in section V. Illustration 1 demonstrates the sectors defined in proposition 2: ‘H’, ‘T’, ‘V’ and ‘U’ denote the high-skilled sector, the teachers sector, the vocational sector and the unskilled sector, respectively. The X-axis denotes ability. The Y-axis denotes $\frac{u_S(\theta_i)}{u_T}$, which represents the utility from skilled professions relative to teaching (recall equation (10)). The intersection points define the threshold levels between skilled professions and teaching. The intermediate ability workers $(\theta_{VT}, \theta_{TH})$ prefer teaching.

In the other theoretical models, the utility from skilled professions relative to teaching monotonically increases with ability because agents gain utility purely from income. Consequently, there is an excess supply of low-ability teachers.⁷ The government’s choice of teacher income determines the top-quality teachers (with identical incomes as teachers and as skilled workers), and all college graduates with lower abilities (and thus lower incomes as skilled workers) are motivated to accept employment as teachers. Therefore, the results depend on the objective function of the government that determines the set of teachers. Accordingly, the government can decide to lower teacher income and still increase their numbers (i.e. substitute quantity for quality). This is a key element in generating the quality–quantity trade-off. In contrast, my framework

⁷ An exception is Bacolod (2006) that uses Roy’s model (1951) of self selection. However, her model does not take into account the fact that education is funded by the government.

emphasizes the self-selection of workers. Low-ability workers do not want to devote the exogenously given time investment required to gain teacher certification and thus do not want to become teachers. Instead, they join the vocational sector and alleviate their learning effort. Therefore, *ceteris paribus*, to attract more teachers, teacher income must grow.

E. Labor demand

a. Firms

Competitive identical firms produce one consumption good, q , using total skilled labor (recall definition 2) and unskilled labor. I denote the proportions of total skilled labor and unskilled labor in the working population used by firm j by P_S^j, P_U^j . I assume that the per capita production function of firm j is the following:

$$(15) \quad q^j(\bar{h}_S, P_S^j, P_U^j) = \left((\bar{h}_S)^\sigma (P_S^j)^r + (\bar{h}_S)^\phi (P_U^j)^r \right)^{\frac{1}{r}}, \text{ where } r < 1 \text{ and } 1 > \sigma > \phi > 0.$$

I also assume that the quality of skilled labor, \bar{h}_S , amplifies the productivity of skilled labor and unskilled labor with decreasing returns. This reflects the notion that skilled workers lead technological changes (e.g., Eicher 1996; Acemoglu, 1998; Galor and Moav, 2000; Nelson and Phelps, 1966). Though, the spillover is larger for skilled workers. Note that the results hold also under general functions $f_1(\bar{h}_S)$ and $f_2(\bar{h}_S)$ instead of $(\bar{h}_S)^\sigma$ and $(\bar{h}_S)^\phi$, respectively, assuming that they are strictly increasing, concave, and continuously differentiable.

Definition 5: The net productivity augmentation of skilled labor is given by $\sigma - \phi > 0$ ◦

Given the quality of skilled labor and incomes, each firm j chooses its demand for skilled and unskilled labor by maximizing its profits:

$$(16) \quad \underset{P_S^j, P_U^j}{Max} \pi^j = \left((\bar{h}_S)^\sigma (P_S^j)^r + (\bar{h}_S)^\phi (P_U^j)^r \right)^{\frac{1}{r}} - W_S \bar{h}_S^j P_S^j - Y_U P_U^j.$$

By rearranging the first-order conditions, I obtain the demand of firm j for skilled labor relative to unskilled labor:

$$(17) \quad \frac{\overline{W_S h_S^j}}{y_U} = (\overline{h_S})^{\sigma-\phi} \left(\frac{P_U^j}{P_S^j} \right)^{1-r}.$$

Because all firms are identical, by rearranging equation (17), I derive the total demand for skilled labor relative to unskilled labor as follows:

$$(18) \quad \frac{W_S}{y_U} = (\overline{h_S})^{\sigma-\phi-1} \left(\frac{P_U}{P_S} \right)^{1-r},$$

where P_S , P_U are the aggregate proportions of total skilled labor and unskilled labor, respectively, in the working population used by all firms.

b. The government

Recall that in this model, the taxation is progressive in the sense that unskilled workers are not taxed to finance education, and the tax rate on the other sectors, τ , is exogenously given. Tax revenues finance teachers' incomes at each date t , and the educational budget constraint is balanced:

$$(19) \quad y_T P_T = \tau (w_S \overline{h_S} P_S + y_T P_T).$$

By rearranging equation (19), teachers' incomes after tax are funded by the skilled sector:

$$(20) \quad (1-\tau) y_T P_T = \tau (w_S \overline{h_S} P_S).$$

That is, the teachers sector cannot exist without the funds from the skilled sector. On the other hand, if the skilled sector exists, then the tax revenues are positive. Because the educational budget is not disposed, the teachers sector must exist. Accordingly,

Corollary 3:

Given the budget constraint (20), the teachers sector exists if and only if the skilled sector exists (i.e. $P_T > 0 \Leftrightarrow P_S > 0$).

III. Equilibrium

A. Definition of Equilibrium

Let teachers' time investment in higher education, e_T , the tax rate, τ , the distribution of abilities, θ_i , and the provision of public education, E , be given in each period t . Then,

$\{e_i, P_U, P_T, P_H, P_V, y_U, y_T, W_S\}$, for $t=1,2,\dots$, constitutes an equilibrium, if it satisfies the following conditions:

- a. Given $\{y_U, y_T, W_S\}$, for all workers, $\{e_i\}$ is the optimal time dedicated to higher education and no worker can improve his or her position by moving to another sector.
- b. In production, $\{P_U, P_S\}$ are the optimal aggregate proportions of unskilled labor and total skilled labor, respectively, given $\{y_U, y_T, W_S\}$.
- c. The educational budget constraint (20) holds.
- d. The labor market clears. The demand for each sector equals supply. ◦

B. Existence of Equilibrium and Uniqueness

The following propositions 3–4 derive the conditions for the existence and uniqueness of equilibrium, respectively. In proposition 5, I add some technical assumptions to the model to ensure that the high-skilled sector and the vocational sector co-exist in the equilibrium.

Proposition 3

Under the aforementioned assumptions, equilibrium exists with at least three sectors: total skilled, teachers and unskilled (i.e. $P_S > 0$, $P_T > 0$, $P_U > 0$). ◦

Thus, at least one of the two sub-sectors exists in equilibrium: the vocational sector or the high-skilled sector.

Proposition 4: Under the aforementioned assumptions, and if the net productivity augmentation of skilled labor is sufficiently large, $\sigma - \phi > r$, equilibrium is unique with at least three sectors: total skilled, teachers and unskilled (i.e. $P_S > 0$, $P_T > 0$, $P_U > 0$). ◦

(A5) The distribution of abilities is uniform.⁸ ◦

Additional technical assumptions (A6)–(A9), specified in the Appendix, are sufficient to prove the following proposition 5.

⁸ The aim of the uniformity assumption (A5), in line with Galor and Moav (2000), is to obtain tractable analytical results. It does not seem to be essential for the overall intuitions and for proposition 5. Though, the effect of SBTC on relative teacher quality (see proposition 9 hereinafter) may alter, if the additional mass of vocational workers who choose to become teachers is relatively large. The sensitivity to alternative distributions is mitigated, however, if effective leisure is sufficiently important, see property 1.

Proposition 5:

Assume that assumptions (A5)–(A9) hold. Then, the number of highly skilled and vocational workers is positive in equilibrium. ◦

The intuition why assumptions (A6)–(A9) ensure that both types of skilled workers co-exist in the equilibrium is as follows: Assumption (A6) posits that $\bar{\theta}$ has intermediate levels. The intuition is that highly talented workers (with high θ s) prefer the high-skilled sector to teaching because of the returns to their ability (recall (8)) while low-ability workers (with low θ s) prefer vocational professions to teaching to alleviate their learning effort. Assumptions (A7)–(A8) guarantee that the total skilled sector is sufficiently attractive: If the provision of public education; the net productivity augmentation of skilled labor, $\sigma - \phi$; the intensity of ability, λ ; and the intensity of the time investment in higher education, β , are sufficiently large, incomes in the skilled sector are relatively amplified. Moreover, assumption (A9) guarantees that the vocational sector exists: Assumption (A9) posits that the effective effort is costly (i.e. Z is sufficiently large). In this case, the marginal utility from effective leisure increases. As a result, for low-ability workers, the teachers sector (with the exogenously given time investment in higher education) becomes less attractive relative to the vocational sector (in which they can optimally alleviate their learning effort; recall equation (6)).⁹

Note that in the following sections, I assume that the vocational sector and the high-skilled sector both exist. Nevertheless, I analyze the less probable case with no vocational sector in section VIA).

IV. Time Investment in Higher Education and Income

This section characterizes time investment and incomes in each sector. Typically, time investment in higher education and related incomes are weakly increasing in ability. The model generates this result in all sectors, though it may not hold at the threshold

⁹ Moreover, as Z rises, skilled workers become cheaper to firms relative to unskilled workers (because they reduce their time investment in higher education (recall equation (6)). Thus, the relative demand for vocational workers increases at the expense of unskilled workers. A weaker secondary effect is that the supply of vocational workers declines relative to the supply of unskilled workers (see equation (13)).

level between the teachers sector and the high-skilled sector, θ_{TH} , as proposition 6 depicts and Illustration 2 illustrates.

Proposition 6:

High-ability workers are more educated and earn higher incomes than low-ability workers, with the following exception: some highly skilled workers may be less educated and earn lower incomes than teachers. ◦

Illustration 2 depicts time investment in education and incomes as a function of ability:

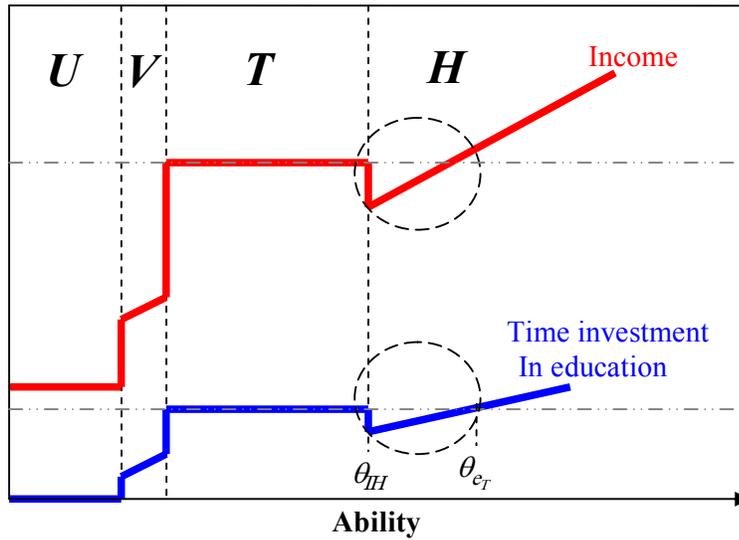


Illustration 2. – Time investment in education and income.

The numerical example is detailed in section V. ‘H’, ‘T’, ‘V’ and ‘U’ denote the high-skilled sector, the teachers sector, the vocational sector and the unskilled sector, respectively. Unskilled workers earn the lowest incomes (normalized to '1') and do not invest in higher education. Vocational workers (teachers) are more educated and earn higher incomes than unskilled (vocational) workers but enjoy less effective leisure. That is, they substitute income with effective leisure. In illustration 2, the time investment of highly skilled workers is identical to the *exogenously* given time investment of teachers, e_T , when ability equals θ_{e_T} . The most talented workers, with ability above θ_{e_T} , given their comparative advantage in learning, naturally choose to be more educated and thus earn higher incomes than teachers (recall (A2) and (A3) and equation (6)). However, because $\theta_{TH} < \theta_{e_T}$ in equilibrium, some highly skilled workers, with abilities between

$(\theta_{IH}, \theta_{e_T})$ (in the black circle), decide to acquire less higher education and thus earn lower incomes than teachers.

This phenomenon occurs when the intensity of ability, λ , is large. On the one hand, when the exponential intensity of ability is low (see the Appendix for $\lambda = 0$), the high-skilled sector is small (θ_{IH} is high) and includes only the most talented workers, who choose to be more educated and thus earn higher incomes than teachers. On the other hand, when the intensity of ability is sufficiently large, the high-skilled sector expands (because it becomes more attractive than the teachers sector). Accordingly, it attracts additional low-ability workers (θ_{IH} declines), who optimally choose to enroll in shorter higher education programs and enjoy more leisure but lower incomes than teachers. Devoting the *exogenously* given time investment of teachers in higher education, e_T , is sub-optimal for low-ability workers because their marginal cost in terms of learning effort is too high relative to the marginal utility from the income generated.¹⁰

V. Numerical Example

The baseline, used to draw illustrations 1 and 2, depicts the composition of the labor force and the time investment in education and income, respectively. Illustrations 3-5 demonstrate comparative static to examine potential explanations for the quality–quantity trade-off. The population size is 300. The income distribution is uniform (recall assumption (A5)) and calibrates a Gini coefficient close to developed OECD countries (0.24). In addition, the average tax rate is between 0.11–0.25, the range of national public expenditure per student for primary education as a percentage of GDP per capita in OECD countries. It lies in the range of the medium and high tax rates in Glomm and Ravikumar (2003), 0.05-0.6. I also set the standard parameters from the literature (see Table 1).

¹⁰ Reducing their time investment amplifies the returns to their learning effort as skilled workers, $\frac{y_i}{Ze_i/\theta_i} = \frac{W_s \rho E_i}{Z} (e_i^{\beta-1} \theta_i^{2+\lambda})$ (recall equation (1) and (A3)), as their income declines at a smaller rate $\beta < 1$ than their time investment. See section VI for more details on an increase in the intensity of ability λ .

Table 1: parameter values

	Parameters' description	Parameters' value	Parameters' source
Firms' production	Substitution between skilled and unskilled workers	$r = 0.925$	Hamermesh's (1979) and Johnson's (1970) estimates of the substitution between high school graduates and college graduates. These are close to the result of Bowles (1970), 0.995, for the substitution between secondary education and higher education, and in the range of Psacharopoulos's (1972) estimates.
Education technology	Intensity of higher education	$\beta = 0.16$	In the range of Card's (1995) IV estimates.
	Intensity of ability	$\lambda = 0.5$	Orazem and Tesfatsion (1997) and Loury (1981).
	Intensity of human capital	$\rho = 6$	Glomm and Ravikumar (1992) and Su (2004)
Utility	Weights of effective leisure and consumption	$\mu = 0.2$ and $\delta = 0.3$ $(\mu = 0.6, \delta = 0.4)$	Greenwood et al. (1997) and Tamura (2001)

VI. Comparative Static to Explain the Quality–Quantity Trade-Off

In this section, I examine the possible causes for the trends in teacher quality and teacher quantity in advanced countries. First, in the following sub-sections A and B, I discuss two types of SBTCs:

(a) The returns to ability rise *linearly*: In this case, skilled incomes are multiplied by the same constant factor (without changing the ratio of incomes within the skilled sector). Thus, the skilled sector grows (see equations (21)–(23)), though the income inequality (see definition 6 in the Appendix) within the skilled sector does not change. This factor-augmenting SBTC is common in the literature. It is carried out through the following comparative static in line with the evidence reviewed in the introduction:

- Augmented productivity of skilled labor, $\sigma - \phi$ (recall definition 5 and equation (18)). As a result, the demand for skilled workers increases in firms.
- Augmented human capital of skilled labor, through amplified intensity of human capital, ρ , or improved provision of public education, E (given at each period t).

As a result, skilled incomes increase because they are based on human capital (recall equation (1)), and thus the supply of skilled workers grows¹¹.

(b) The returns to ability rise *exponentially*: This SBTC is executed through an increase in the intensity of ability, λ . In this case, the marginal productivity of ability increases more than the marginal productivity of the other components of the human capital. As a result, workers at the upper end of the ability distribution receive exponentially larger returns for their ability. Consequently, the ratio of the incomes of highly skilled and vocational workers increases and the income inequality rises within the skilled sector.◦

In addition to the two types of SBTC, I consider in sections C and D a reduction of the teacher certification requirement and exogenous shifts in the distribution of initial endowments.

Definition 7: Relative teacher quality refers to teacher mean quality relative to the mean quality of the skilled sector (i.e. $\frac{\overline{h_T}}{h_S}$).◦

A. type (a) SBTC

This section demonstrates that when the returns to ability rise *linearly*, both teacher quality and teacher quantity increase. Thus, the quality–quantity trade-off does *not* occur (see illustration 3 and proposition 7). As the incomes of skilled workers increase, low-ability agents decide to acquire higher education and join the vocational sector (i.e., θ_{UV} declines). Therefore, the total skilled sector expands, the unskilled sector shrinks, and the mean ability declines in both sectors. Because the funds for public education increase, teacher income increases to balance the educational budget (20). As a result, the teachers sector becomes more attractive for both highly skilled workers and vocational workers (i.e., θ_{TH} increases and θ_{VT} declines). Thus, the teachers sector expands, and the high-skilled sector shrinks. Illustration 3 demonstrates type (a) SBTC.

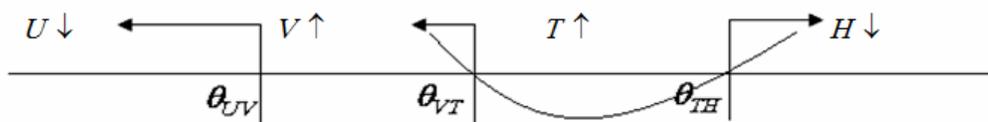


Illustration 3. – The effect of type (a) SBTC.

¹¹ A weaker secondary effect is a decline in the demand for skilled workers because they are more expensive to firms (see (18))

The corresponding simulation in figure 3 is relegated to the Appendix.

Proposition 7 (See Lemma 1 and its proof in the Appendix):

Under (A5), if type (a) SBTC occurs, then

- a. The total skilled sector and the teachers sector expand, and the unskilled sector shrinks.
- b. The vocational sector expands, and the high-skilled sector shrinks. ◦

It is not straightforward whether relative teacher quality increases or declines, because both high-ability workers (from the high-skilled sector) and low-ability workers (from the vocational sector) join the teachers sector. However, according to proposition 8, *more* highly skilled workers join the teachers sector than vocational workers (accordingly, the rise in θ_{TH} is larger than the decline in θ_{VT}). As such, relative teacher quality increases and the quality–quantity trade-off does *not* occur: SBTC increases the supply of teachers without sacrificing their quality.

Proposition 8:

Assume that (A5) holds and SBTC occurs. Then, relative teacher quality increases. ◦

Proposition 8 is derived by the following: In line with the empirical evidence in Bacolod (2006), the supply of vocational workers is less elastic than the supply of highly skilled workers with respect to shocks in their relative incomes. Because vocational workers are less talented, their learning effort as teachers and thus their marginal utility from effective leisure are higher than those of highly skilled workers¹². As a result, when teacher income increases, fewer vocational workers join the teachers sector than highly skilled workers. The other vocational workers prefer to remain in the vocational sector and reduce their learning effort. Thus, relative teacher quality rises. This result is in line with the empirical evidence about the positive relationship between teacher income and teacher quality (see for example Dolton and Marcenaro-Gutierrez, 2011).

¹² Proposition 8 reflects property 1, that the slope of $\frac{u_S}{u_T}(\theta_i)$ is steeper below $\hat{\theta}$ than above $\hat{\theta}$. Workers with sufficiently low ability ($\theta_i \rightarrow Ze_T^+$) have almost no effective leisure as teachers (recall (A3) and corollary 2), and thus their marginal utility from effective leisure is infinite.

In contrast, in the other models mentioned previously, there is *excess* supply of low-ability workers for the teachers sector. Thus, in such cases, a feasible policy would be to lower teacher income while increasing their numbers (i.e., substituting teacher quality with quantity). This policy may become optimal under SBTCs, because the cost of maintaining teacher quality rises. In the current framework, this policy is not feasible because of the existence of vocational workers who do not want to become teachers: if teacher income declines, the supply of low-ability teachers shrinks (they prefer to join the vocational sector).

Note that under this framework, if there is *no* vocational sector (the less probable case), the quality–quantity trade-off occurs: as the incomes of skilled workers increase, the high-skilled sector expands and pushes the teachers sector to the lower levels of the ability distribution (i.e., θ_{TH} declines). Then, because the funds for public education increase, teacher income also increases. As a result, the teachers sector expands towards the unskilled sector (i.e., θ_{UT} declines), and its relative quality declines, as proposition 9 proves:

Proposition 9: (Relative teacher quality)

Assume that (A5) holds and type (a) SBTC occurs. If the vocational sector does not exist, the teachers sector expands, and its relative quality declines. ◦

In the following sections, I analyze, through simulations, other potential causes for the quality–quantity trade-off, associated with observed trends in advanced countries: type (b) SBTCs, shifts in the distribution of initial endowments and reduction of the teacher certification requirement. Their common feature, in contrast with the type (a) SBTC, is that they generate quality–quantity trade-offs.

B. type (b) SBTC

When the intensity of ability, λ , increases, similarly to type (a) SBTC, more agents are attracted to the total skilled sector. Furthermore, because the funds for public education increase, teacher income also increases, and thus the teachers sector expands. However, while type (a) SBTC multiplies the incomes of skilled workers by the same factor, under type (b) SBTC the growth in incomes is highly disproportionate within the skilled sector. Because they are more talented, highly skilled workers enjoy exponentially

larger returns for their ability. That is, the ratio of the incomes of highly skilled and vocational workers increases, and thus income inequality rises within the skilled sector. As a result, in contrast with type (a) SBTC, the high-skilled sector *expands* and pushes the teachers sector towards the lower levels of the ability distribution. Top-quality candidates depart the teachers sector and join the high-skilled sector, leaving the government with less talented teachers (i.e., both the upper and lower thresholds of the teachers sector, θ_{TH} and θ_{VT} , decline). Thus, the quality–quantity trade-off emanates from the occupational choice of high-ability workers, whereas the government is forced (by the market) to recruit teachers with lower qualifications. Given its budget constraint, the government cannot attract better teachers. This result is consistent with the empirical finding that the decline in teacher quality was primarily driven by a decrease in the proportion of the most qualified teachers (see e.g., Corcoran et al., 2004; and Bacolod, 2006). Accordingly, the fundamental disadvantage of type (b) SBTC relative to type (a) SBTC in the current context is that it leads to a reduction in relative teacher quality. Illustration 4 demonstrates type (b) SBTC.

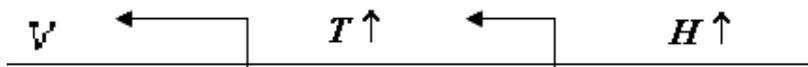


Illustration 4. – The effect of type (b) SBTC.

The corresponding simulation in figure 4 is relegated to the Appendix.

C. Reduction in the Teacher Certification Requirement

In this section, I discuss a reduction in the exogenous teacher certification requirement, e_T , or a reduction in the non-pecuniary cost of effort, Z .¹³ As a result, the learning effort required from teachers decreases (recall (3)), and the teachers sector becomes more attractive. Therefore, the supply of teachers grows. However, because they are less talented, with a higher marginal utility from effective leisure, more vocational workers are attracted to the teaching profession than highly skilled workers. Then, because the supply of skilled workers declines, funds for public education shrink. To balance the budget constraint (20), the government reduces teacher income. Because the supply of highly skilled workers is more elastic with respect to shocks in their relative

¹³ Note that Costrell (1994), considering a shock in student preference for leisure in high school, argues that shifts in preferences need not originate with the student. For example, changes in family structure might reduce the student's non-pecuniary cost of effort if no one is making him do the homework.

incomes (recall sub-section A), more top-quality candidates depart from the teachers sector to join the high-skilled sector than vocational workers. Combining these effects, both the high-skilled sector and the teachers sector grow, and the vocational sector shrinks, leaving the teachers sector with less talented candidates (i.e., both the upper and lower thresholds of the teachers sector, θ_{TH} and θ_{VT} , decline). Accordingly, relative teacher quality declines, and the quality–quantity trade-off occurs. The effects on the lower threshold of the vocational sector, θ_{VV} , are described in the Appendix. Illustration 4 in the previous section demonstrates the effect of reduction in the teacher certification requirement as well. The corresponding simulation in figure 5 is relegated to the Appendix.

D. Exogenous Shifts in the Distribution of Initial Endowments

Recall that ‘abilities’ may reflect any unobserved initial endowments related to home background or school background. As sub-section 0 demonstrates, a rising high-skilled sector may generate a quality–quantity trade-off. The growth of this sector may also originate from exogenous shifts in the distribution of initial endowments, e.g., the immigration of high-ability workers to a country, specifically to the United States; improved home background; evolutionary learning that increases the talent of children relative to their parents. When highly talented workers are added to the high-skilled sector, relative teacher quality declines. As the funds for public education increase, the teachers sector expands. Thus, a quality–quantity trade-off occurs.

VII. Conclusion

In the United States and other advanced economies, the pupil–teacher ratio and teacher quality have declined over time. Existing models have analyzed the causes for this trend, assuming an excess supply of low-ability teachers. Thus, in the face of rising costs of skilled workers, the government finds it optimal to opt for lowering teacher salaries while increasing their numbers. In contrast, the current study presents a theory of educational production that emphasizes self-selection in becoming teachers. I argue that agents consider their learning effort when allocating their time to higher education and that the learning effort diminishes with ability level. As a result, low-ability workers avoid becoming teachers because of the time required to acquire teacher certification. Instead, they enroll to shorter programs of tertiary education (with

vocational/practical orientation). Therefore, teachers endogenously have intermediate abilities between two types of skilled workers: vocational and highly skilled. This unique division helps grasp the essential features in SBTCs that lead to quality–quantity trade-offs.

This study suggests that the quality–quantity trade-off may have been generated by SBTCs that amplify the income inequality within the skilled sector, a reduction of teacher certification requirement or shifts in the distribution of initial endowments. When SBTCs increase incomes of highly talented workers exponentially relative to their less talented peers, the most talented teachers become attracted to the high-skilled sector. As a result, this sector expands and pushes the teachers sector to the lower levels of the ability distribution. That is, teacher quantity increases whereas teacher quality declines. However, when SBTCs does not affect the ratio of incomes (and the income inequality) among skilled workers, the quality–quantity trade-off does not occur. The results further coincide with other stylized facts in the United States, including rising educational expenditures, rising dispersion of earnings and rising college attendance over time. This study further demonstrates the cost of requiring a relatively long time investment from teachers. In this case, in equilibrium even the top-quality teachers earn a higher income than their counterparts in the high-skilled sector, which compensates them for their greater effort in higher education.

The main finding is that certain SBTCs increase in the income inequality within the skilled sector, increase teacher quantity and reduce relative teacher quality. As Bartel and Sicherman (1999) conclude, the observed effects of technological changes are due to the sorting of agents according to their unobserved characteristics, that may reflect innate ability, home environment, skills learned at home, school curriculum and quality of schooling. The policy implications of the current findings depend on the relative importance of these factors. For example, if the unobserved characteristics largely reflect innate abilities or home environment, public policy intervention will have a limited role in influencing the erosion of teacher quality and the rising wage differentials within the skilled sector induced by SBTC. If, however, unobservables largely reflect school curriculum or school quality (which can be viewed as somewhat endogenous), public policy could shape the allocation of these resources and thereby mitigate these effects. To increase teacher quality, the government should encourage

linear (as opposed to exponential) increases in the returns to ability. Policies should be directed at subsidizing the adaptation of low-ability workers to the SBTCs in order to increase their relevance in the labor market and driving technological changes in low-ability sectors.

The framework of analysis developed here is suitable to examine the spillover of income inequality and relative teacher quality across countries. For example, in the presence of imperfect technological diffusion, income inequality among technological leaders is likely to be higher while relative teacher quality is likely to be lower than among followers. Other intriguing questions for further research include the implications of the occupational choice of teachers on the supply of educated workers and more generally on growth in subsequent periods.

Note that in my framework, the salary for teachers is uniform because of collective bargaining agreements and the time investment of teachers is exogenously given. These assumptions are more suitable within a specific district or a small country and within the two periods of the model. Relaxing these assumptions requires a separate research article. In reality, teacher certification requirement may depend on endogenous governmental decisions. Specifically, its reduction may be due to the optimal policy of the government to substitute teacher quality with teacher quantity in response to the rising cost of skilled workers (potentially caused by SBTC). This may be related to California's class-size reduction program, in which the expansion of the teaching force required to staff the additional classrooms may have led to a decline in teacher quality (Jepsen and Rivkin, 2002) or to the case of several developing countries, (UNESCO (2006)), that due to serious teacher shortages have decided to lower the entry standards for teacher certification. Note also that the exclusion of physical capital from the model is natural (and common in educational production models) because the focus is on human capital. However, a more realistic assumption regarding physical capital should not qualitatively change the results.

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Appendix

Fig. 3-5 demonstrate type (a) SBTC, type (b) SBTC and a decline in the teacher certification requirement. The numerical example is detailed in section V.

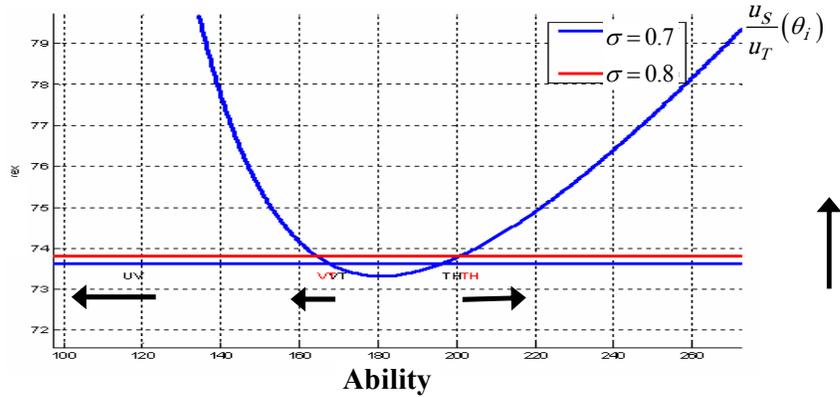


Fig. 3. – The effect of type (a) SBTC – an increase in the net productivity augmentation of skilled labor (recall equation (15)).

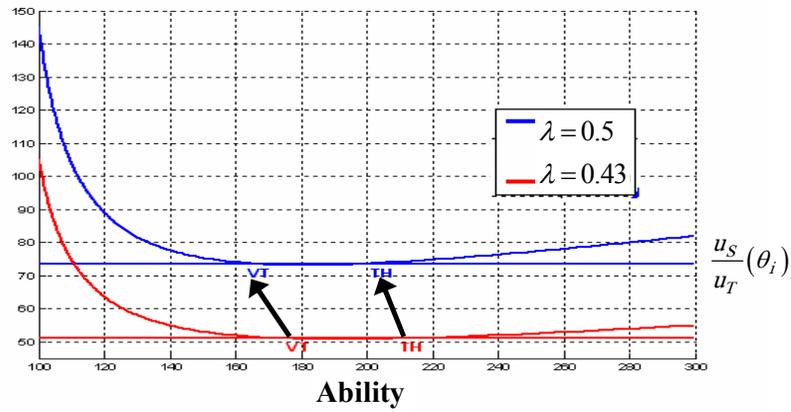


Fig. 4. – The effect of type (b) SBTC – an increase in the intensity of ability

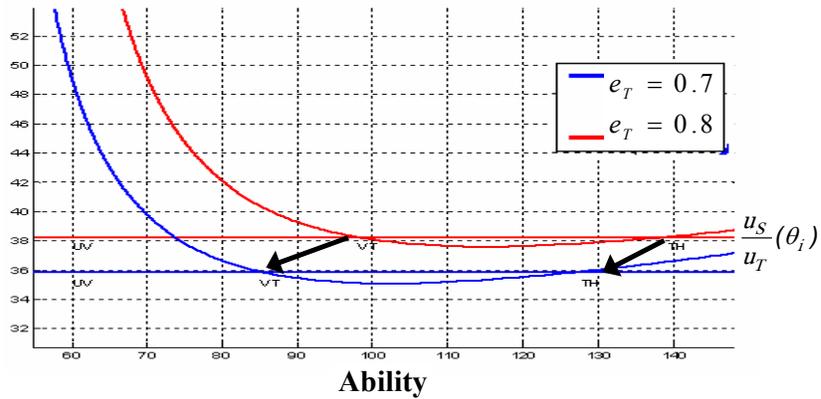


Fig. 5. – The effect of reduction in the time investment of teachers in higher education, e_T , (recall (A2))

Proof of proposition 2 – Consistency of preferences

If $\theta_{VT} \geq \theta_{UV}$, the vocational sector exists and agents θ_{VT}, θ_{UV} are the threshold levels between the vocational sector, the teachers sector and the unskilled sector. In this case, agents above θ_{VT} prefer teaching rather than vocational professions: $T \succ V$. They are also above θ_{UV} , and hence prefer vocational rather than unskilled professions $V \succ U$. Therefore, they prefer teaching rather than both vocational and unskilled professions: $T \succ V, V \succ U \Rightarrow T \succ U$. Similarly, agents below θ_{UV} prefer to become unskilled rather than vocational and unskilled workers, since $U \succ V, V \succ T \Rightarrow U \succ T$. Agents between $\theta_{VT} \geq \theta_{UV}$ prefer to become vocational workers rather than being teachers or unskilled. However, if $\theta_{VT} < \theta_{UV}$, there is no vocational sector, since all agents prefer other sectors rather than the vocational sector: Agents above θ_{UV} prefer teaching rather than both vocational and unskilled professions since $T \succ V, V \succ U \Rightarrow T \succ U$. Agents below θ_{VT} prefer to become unskilled rather than vocational and skilled workers, since $U \succ V, V \succ T \Rightarrow U \succ T$. Additionally, agents between θ_{VT}, θ_{UV} do not desire to become vocational workers, since $U \succ V, T \succ V$. In this case, agent θ_{UT} is the threshold level between the unskilled sector and the teachers sector, such that $\theta_{VT} < \theta_{UT} < \theta_{UV}$.

Proof of Proposition 3

As $r < 1$ (see equation (15)), the demand for skilled and unskilled workers must be positive (otherwise, their marginal productivity is infinite). Accordingly, since the skilled sector exists and the educational budget constraint holds, the teachers sector must exist either (recall corollary 3). Now, I prove by 3 steps that there is at least one feasible set of $\{P_S > 0, P_T > 0, P_U > 0\}$ that clears the labor market:

a. Assume that $P_S > 0, P_T > 0, P_U > 0$. To clear the labor market and find the threshold levels, the labor supply is intersected with the labor demand of firms and the educational budget constraint, as follows: The labor supply equation (12) is substituted in the educational budget constraint (20) to obtain:

$$(21) \quad \bar{h}_S P_S = \pi_1 \left(\frac{1-\tau}{\tau} \right) P_T \frac{u_S}{u_T} (\theta_{jk}), \quad jk = TH, VT$$

In case the vocational sector exists, substituting the labor supply (13) in the demand equation (18) obtains:

$$(22) \quad \left(\bar{h}_S \right)^{\sigma-\phi-1} \left(\frac{P_U}{P_S} \right)^{1-r} (\theta_{UV})^{\beta+\lambda} = \frac{1}{\pi_1(1-\tau)}$$

In case the vocational sector does not exist, multiplying the labor supply (12) in (14) and substituting in the demand equation (18) yields:

$$(23) \quad \left(\bar{h}_S \right)^{\sigma-\phi-1} \left(\frac{P_U}{P_S} \right)^{1-r} \left(\frac{\theta_{UT} - Ze_T}{\theta_{UT}} \right)^{\frac{\mu}{\delta}} \left(\frac{u_S}{u_T} (\theta_{TH}) \right) = \frac{1}{\pi_1(1-\tau)}$$

Then, substituting $P_S = 1 - P_U - P_T$ in equation (21) yields:

$$(24) \quad P_T = \pi_3 (1 - P_U)$$

$$\text{where } \pi_3 = \frac{\bar{h}_S}{\pi_1 \left(\frac{1-\tau}{\tau} \right) \left(\frac{u_S}{u_T} (\theta_{jk}) \right) + \bar{h}_S} \text{ and } jk = TH, VT$$

If the vocational sector exists, then $jk = VT$. If the high-skilled sector exists, then $jk = TH$, while if both sectors exist, then $jk = TH, VT$. Note that $\frac{u_S}{u_T} (\theta_{jk})$ is positive and finite and \bar{h}_S is positive

and finite (The value of \bar{h}_S has an upper bound since $\bar{\theta} < \infty$. Additionally, $\bar{h}_S > 0$, since even the human capital of the least talented agent, $\underline{\theta} = 1$ is positive as a skilled worker:

$h_i(1) = \left(\frac{\delta\beta}{\mu + \delta\beta} \right)^\beta \rho E > 0$. Moreover, recall that θ_{TH}, θ_{VT} , are bounded: If the high-skilled sector exists, then $\infty > \bar{\theta} > \theta_{TH} > \hat{\theta}$ (see (11) and (12)), while if the vocational sector exists, then

$\hat{\theta} > \theta_{VT} > Ze_T$ (see (12)). Therefore, the values of $\frac{u_S}{u_T}(\bar{\theta})$, $\frac{u_S}{u_T}(\hat{\theta})$ and $\frac{u_S}{u_T}(Ze_T + \varepsilon)$ for $\varepsilon > 0$, are positive and finite).

b. Assuming equation (24) holds and using $0 < \pi_3 < 1$ (recall that $\frac{u_S}{u_T}(\theta_{jk})$ is positive and finite), it is easy to verify that:

$$(25) \quad \left\{ \begin{array}{ll} 1 > P_T > 0, 1 > P_S > 0 & , \text{if } P_U \rightarrow 0 \\ 1 > P_T > 0, 1 > P_S > 0 & , \text{if } 1 > P_U > 0 \\ P_T \rightarrow 0, P_S \rightarrow 0 & , \text{if } P_U \rightarrow 1 \end{array} \right\}$$

Substituting $P_U \rightarrow 0$ in equation (24) yields $1 > P_T > 0$. Therefore, $1 > P_S = 1 - P_T - P_U > 0$. Substituting $1 > P_U > 0$ in equation (24) yields $1 > P_T > 0$. Therefore, $1 > P_S = 1 - P_T - P_U$. According to equation (21), $P_S > 0$. Substituting $P_U \rightarrow 1$ in equation (24) yields $P_T \rightarrow 0$. Therefore, $P_S = 1 - P_T - P_U \rightarrow 0$.

c. Recall that equations (24) and (25) combine the labor supply and the educational budget constraint, while labor demand is considered in equations (22) and (23). Now, assuming that the labor market clears, the results from (25) are substituted in equations (22) and (23). Then, it is easy to verify that

$$(26) \quad \left\{ \begin{array}{ll} LHS(23) \rightarrow 0, LHS(24) \rightarrow 0, & \text{if } P_U \rightarrow 0 \\ LHS(23) > 0, LHS(24) > 0, & \text{if } 1 > P_U > 0 \\ LHS(23) \rightarrow \infty, LHS(24) \rightarrow \infty, & \text{if } P_U \rightarrow 1 \end{array} \right\}$$

As the RHS of equations (22) and (23) is always positive, at least one intersection must occur with their LHS due to continuity. ◦

Lemma 1: Under assumption (A5), when the skilled sector expands, i.e., P_S increases, then the teachers sector expands, i.e., P_T increases and teacher income relative to the skilled sector, $\frac{y_T}{w_S h_S}$, grows. ◦

Proof of Lemma 1: Using equation (8),

$$P_S \bar{h}_S = P_S \left(\frac{P_H}{P_S} \bar{h}_H + \frac{P_V}{P_S} \bar{h}_V \right) = P_H \bar{h}_H + P_V \bar{h}_V = \pi \left(\left(\frac{\bar{\theta} - \theta_{TH}}{\bar{\theta} - 1} \right) \left(\frac{1}{\bar{\theta} - \theta_{TH}} \int_{\theta_{TH}}^{\bar{\theta}} \theta^{\beta+\lambda} \right) + \left(\frac{\theta_{VT} - \theta_{UV}}{\bar{\theta} - 1} \right) \left(\frac{1}{\theta_{VT} - \theta_{UV}} \int_{\theta_{UV}}^{\theta_{VT}} \theta^{\beta+\lambda} \right) \right)$$

$$\text{Thus, } P_S \bar{h}_S = \left(\frac{\pi}{\bar{\theta} - 1} \right) \left(\int_{\theta_{TH}}^{\bar{\theta}} \theta^{\beta+\lambda} + \int_{\theta_{UV}}^{\theta_{VT}} \theta^{\beta+\lambda} \right)$$

Thus, if the skilled sector expands, then $P_S \bar{h}_S$ increases (The high-skilled sector can expand at the expense of the teachers' sector (θ_{TH} decreases). If the vocational sector exists, the vocational sector can expand at the expense of the teachers sector (θ_{VT} increases) or at the expense of the unskilled sector (θ_{UV} decreases). In any case, $P_S \bar{h}_S$ increases). Accordingly, ceteris paribus, the average income of skilled workers $w_S P_S \bar{h}_S$ increases, and hence the funds for public education increase. ◦

Proof of proposition 4:

Along the labor supply, when the relative wage of skilled workers, $\frac{w_S}{y_U}$, increases, the skilled sector

becomes more attractive. Therefore, their relative supply, $\frac{P_S}{P_U}$, grows (see equations (12) and (13)).

According to Lemma 1, when the skilled sector expands, $P_S \bar{h}_S$ increases, while $\bar{\theta}_S$ declines. Because of the decreasing returns to ability and time investment (recall equation (1)), \bar{h}_S declines as well. Assuming that the labor market clears, substituting these results in the labor demand (18) and rewriting yields:

$$(27) \quad \frac{W_S}{y_U} = \left(\bar{h}_S\right)^{\sigma-\phi-r} \left(\frac{P_U}{P_S \bar{h}_S}\right)^{1-r}$$

Along the labor supply, the RHS of equation (27) is monotonically decreasing in $\frac{W_S}{y_U}$. Therefore, there

is one intersection between the labor supply and the demand, and $\frac{W_S}{y_U}$ that clears the market is unique.

Proposition 5 – assumptions (Note that these assumptions do not contradict the previous ones)

$$(A6) \quad \theta^{**} \geq \bar{\theta} \geq \theta^*, \text{ where } \theta^* = \bar{\theta} \left[1 + \left(\frac{\tau}{1-\tau} \right) \left(1 + \frac{\delta\beta}{\mu} \right)^{\frac{\mu}{\delta}} \right] \text{ and } \theta^{**} = \bar{\theta} Z e_T$$

$$(A7) \quad \text{Public education is sufficiently large, i.e., } E > \left(\frac{\nu}{\rho} \right) \left(\frac{I}{F} \right)^{\frac{1}{\sigma-\phi}} \left(\frac{1}{e_T} \right)^\chi \left(\frac{1}{Z} \right)^{\chi-\beta}$$

$$\text{where } \chi = \frac{(\beta+\lambda)(2(\sigma-\phi)-1)-(1-r)}{\sigma-\phi}, \quad I = \left(\frac{\mu}{\delta(\beta+\lambda)} + 1 \right)^{(\beta+\lambda)(1+\phi-\sigma)+1-r}$$

$$F = \left(\frac{\mu}{\mu+\delta\beta} \right)^{\frac{\mu}{\delta}} (1-\tau), \quad \nu = \left(\frac{\mu+\delta\beta}{\delta\beta} \right)^\beta \circ$$

(A8) The net productivity augmentation of skilled labor, $\sigma - \phi$, the returns to ability, λ , and the returns to time investment in higher education, β , are sufficiently large, such that $\sigma - \phi - 1 > \frac{1-r-\lambda}{\beta+2\lambda}$. In this case, $\chi - \beta > 0$, and hence (A7) does not contradict (A8).

$$(A9) \quad \text{Effective effort is costly, i.e., } Z > \frac{1}{e_T} \left[1 + \left(\frac{\tau}{1-\tau} \right) \left(1 + \frac{\delta\beta}{\mu} \right)^{\frac{\mu}{\delta}} \right].$$

Proof of Proposition 5

a. Let us assume by contradiction that the high-skilled sector does not exist in equilibrium, i.e., $P_H = 0$. Since the skilled sector exists in equilibrium (see proposition 4), it is composed of vocational workers only, i.e., $P_S = P_V$. Additionally, $P_H = 0$ implies that agent $\bar{\theta}$ prefers teaching rather than high-skilled professions. Therefore, substituting equation (7) in equation (5):

$$u^H(\bar{\theta}) = \left((1-\tau) w_S (\bar{\theta})^{\beta+\lambda} \pi \right)^\mu \left(\frac{\mu}{\mu+\delta\beta} \right)^\mu < u^T(\bar{\theta}) \rightarrow \left((1-\tau) y_T \right)^\delta \left(1 - \frac{Z e_T}{\bar{\theta}} \right)^\mu$$

$$\Leftrightarrow \frac{y_T}{w_S} > \left(\frac{\mu}{\mu+\delta\beta} \right)^{\frac{\mu}{\delta}} (\bar{\theta})^{\beta+\lambda} \left(\frac{\bar{\theta}}{\bar{\theta} - Z e_T} \right)^{\frac{\mu}{\delta}} \pi \quad \text{where } \pi = \left(\frac{\delta\beta}{Z(\mu+\delta\beta)} \right)^\beta \rho E$$

Substituting the educational budget constraint (20) in the LHS obtains

$$(28) \quad \frac{\tau \bar{h}_S P_S}{(1-\tau) P_T} > \left(\frac{\mu}{\mu+\delta\beta} \right)^{\frac{\mu}{\delta}} (\bar{\theta})^{\beta+\lambda} \left(\frac{\bar{\theta}}{\bar{\theta} - Z e_T} \right)^{\frac{\mu}{\delta}} \pi$$

Using equation (8), the skilled quality equals

$$(29) \quad \bar{h}_S = \pi (\bar{\theta}_S)^{\beta+\lambda}$$

where $(\bar{\theta}_S)^{\beta+\lambda}$ is the mean of $(\theta_i)^{\beta+\lambda}$ for all skilled workers.

Substituting in (28) and rearranging obtains

$$(30) \quad \left(\frac{\bar{\theta}}{\bar{\theta} - Z e_T} \right)^{\frac{\mu}{\delta}} \left(\frac{P_T}{P_S} \right) \frac{\bar{\theta}^{\beta+\lambda}}{(\bar{\theta}_S)^{\beta+\lambda}} < \left(\frac{\tau}{1-\tau} \right) \left(1 + \frac{\delta\beta}{\mu} \right)^{\frac{\mu}{\delta}}$$

Now, I prove that inequality (30) does not hold. Since $Ze_T > 0$, then $\frac{\bar{\theta}}{\bar{\theta} - Ze_T} > 1$. Moreover, using

assumption (A5), $P_S = P_V$ and $\hat{\theta} > \theta_{VT}$ (see equation (12)), then

$$(31) \quad \frac{P_T}{P_S} = \frac{P_T}{P_V} = \frac{\bar{\theta} - \theta_{VT}}{\theta_{VT} - \theta_{UV}} \geq \frac{\bar{\theta} - \hat{\theta}}{\hat{\theta}}$$

Furthermore, since $P_S = P_V$, all skilled workers are below θ_{VT} , and as $\hat{\theta} > \theta_{VT}$ (see equation (12)) they are below $\hat{\theta}$. As a result,

$$(32) \quad \frac{\bar{\theta}}{(\theta_S)^{\beta+\lambda}} < \frac{\hat{\theta}}{(\hat{\theta})^{\beta+\lambda}}$$

Using inequalities (31)-(32), the LHS of inequality (30) is bounded by

$$\left(\frac{\bar{\theta}}{\bar{\theta} - Ze_T} \right)^{\frac{\mu}{\delta}} \left(\frac{P_T}{P_S} \right) \frac{\bar{\theta}^{\beta+\lambda}}{(\theta_S)^{\beta+\lambda}} > \left(\frac{\bar{\theta}}{\hat{\theta}} - 1 \right) \frac{\bar{\theta}^{\beta+\lambda}}{(\hat{\theta})^{\beta+\lambda}}$$

Inserting the lower bound of $\bar{\theta}$, given in assumption (A6)

$$\left(\frac{\bar{\theta}}{\hat{\theta}} - 1 \right) \left(\frac{\bar{\theta}}{\hat{\theta}} \right)^{\beta+\lambda} \geq \left(\frac{\tau}{1-\tau} \right) \left(1 + \frac{\delta\beta}{\mu} \right)^{\frac{\mu}{\delta}} \left(1 + \left(\frac{\tau}{1-\tau} \right) \left(1 + \frac{\delta\beta}{\mu} \right)^{\frac{\mu}{\delta}} \right)^{\beta+\lambda} > \left(\frac{\tau}{1-\tau} \right) \left(1 + \frac{\delta\beta}{\mu} \right)^{\frac{\mu}{\delta}}$$

This contradicts inequality (30). Thus, the most talented worker, $\bar{\theta}$, prefers high-skilled professions rather than teaching. Hence, the high-skilled sector exists. ◦

b. According to corollary 2, all agents $[1, \bar{\theta}]$, where $\bar{\theta} = Ze_T$, are incompatible for teaching. Thus, they are vocational or unskilled workers. For $Ze_T > 1$ (under assumption (A9)), this set is not empty (recall that $\underline{\theta} = 1$). Assume by contradiction that the vocational sector does not exist in equilibrium, i.e., $P_V = 0$. Since the skilled sector exists in equilibrium (see proposition 4), it is composed of highly skilled workers only, i.e., $P_S = P_H$. Another implication of $P_V = 0$ is that the utility of agent $\bar{\theta}$ is larger as an unskilled worker than as a vocational worker. Substituting equation (8) in equation (5),

$$u^V(\bar{\theta}) = \left((1-\tau)w_S (\bar{\theta})^{\beta+\lambda} \pi \right)^{\delta} \left(\frac{\mu}{\mu + \delta\beta} \right)^{\mu} < u^U(\bar{\theta}) = (y_U)^{\delta}$$

$$\Leftrightarrow \frac{y_U}{w_S} > \left(\frac{\mu}{\mu + \delta\beta} \right)^{\frac{\mu}{\delta}} \pi (1-\tau) (\bar{\theta})^{\beta+\lambda}$$

Substituting the total demand for skilled relative to unskilled workers, (18), and $\bar{\theta} = Ze_T$ obtains

$$(33) \quad \frac{1}{\left(\frac{h_S}{h_U} \right)^{\sigma-\phi-1} \left(\frac{P_S}{P_U} \right)^{1-r}} > \left(\frac{\rho FE}{v} \right) (e_T)^{\beta+\lambda} Z^{\lambda}$$

Inserting equation (29) in inequality (33), derives:

$$(34) \quad \left(\frac{P_S}{P_U} \right)^{1-r} \left(\frac{\bar{\theta}}{(\theta_S)^{\beta+\lambda}} \right)^{1+\phi-\sigma} > Z^{\beta(\phi-\sigma)} (F) \left(\frac{\rho E}{v} \right)^{\sigma-\phi} (Ze_T)^{\beta+\lambda}$$

Now, I prove that inequality (34) does not hold. Since all workers are below $\bar{\theta}$ and the returns to skilled quality is decreasing in production, $1 + \phi - \sigma > 0$ (recall equation (15)),

$$(35) \quad \left(\frac{\bar{\theta}}{(\theta_S)^{\beta+\lambda}} \right)^{1+\phi-\sigma} < \bar{\theta}^{(\beta+\lambda)(1+\phi-\sigma)}$$

Using assumption (A5), $P_S = P_H$, $\theta_{TH} > \hat{\theta}$ (see equation (12)) and $\theta_{UT} > Ze_T$ (see corollary 2), derives

$$(36) \quad \frac{P_S}{P_U} = \frac{P_H}{P_U} = \frac{\bar{\theta} - \theta_{TH}}{\theta_{UT} - 1} < \frac{\bar{\theta} - \hat{\theta}}{Ze_T - 1}$$

Using inequalities (35) and (36), the LHS of inequality (34) is bounded by

$$\left(\frac{P_S}{P_U} \right)^{1-r} \left(\frac{\bar{\theta}}{(\theta_S)^{\beta+\lambda}} \right)^{1+\phi-\sigma} < \left(\frac{\bar{\theta} - \hat{\theta}}{Ze_T - 1} \right)^{1-r} \bar{\theta}^{(\beta+\lambda)(1+\phi-\sigma)}$$

Inserting the upper bound of $\bar{\theta}$, given in assumption (A6) and substituting $\hat{\theta}$, given in equation (11) yields

$$\left(\frac{\bar{\theta} - \hat{\theta}}{Ze_T - 1} \right)^{1-r} \bar{\theta}^{-(\beta+\lambda)(1+\phi-\sigma)} < I(Ze_T)^{2(\beta+\lambda)(1+\phi-\sigma)+1-r},$$

Under assumption (A7), we derive

$$I(Ze_T)^{2(\beta+\lambda)(1+\phi-\sigma)+1-r} < Z^{\beta(\phi-\sigma)} F(D)^{\sigma-\phi} (Ze_T)^{\beta+\lambda}$$

This contradicts inequality (34). Thus, agent $\bar{\theta} = Ze_T$ prefers vocational jobs rather than unskilled jobs and teaching. Hence, the vocational sector exists. Note that assumption (A8) guarantees that as Z or e_T increase, the RHS in assumption (A7) decreases. Thus, assumptions (A7) and (A9) co-exist. ◦

c. Under assumption (A6), the high-skilled sector and the vocational sector co-exist for $\frac{\bar{\theta}}{\theta}$ in the range:

$$(37) \quad \frac{\bar{\theta}}{\theta} \in \left[1 + \left(\frac{\tau}{1-\tau} \right) \left(1 + \frac{\delta\beta}{\mu} \right)^{\frac{\mu}{\delta}}, \quad Ze_T \right].$$

This set of $\frac{\bar{\theta}}{\theta}$ is not empty for sufficiently large Z , defined in assumption (A9). Note that assumption (A8) guarantees that as Z increases, the RHS of assumption (A7) decreases. Thus, assumption (A7) is compatible with assumption (A9). ◦

Proof of proposition 6:

Since unskilled workers choose not to invest at all in their higher education, and thus enjoy the maximum level of leisure, the other sectors must be compensated by larger incomes than in the unskilled sector. If the vocational sector exists, combining equations (12) and (13) yields:

$$(38) \quad \frac{y_U}{y_T} = (1-\tau) \left(\frac{\theta_{UV}}{\theta_{VT}} \right)^{\beta+\lambda} \left(\frac{\theta_{VT} - Ze_T}{\theta_{VT}} \right)^{\frac{\mu}{\delta}}, \text{ where } \theta_{VT} > Ze_T \text{ and } \theta_{VT} > \theta_{UV}$$

It is easy to verify that the RHS is lower than 1. Additionally, if the vocational sector does not exist, it is easy to see that the RHS of equation (14) is lower than 1. For $\lambda = 0$, the Argmin ability equals

$\hat{\theta}_0 = \left(\frac{\mu + \delta\beta}{\delta\beta} \right) Ze_T$ (recall equation (11)). Note that workers with the Argmin ability are teachers.

Workers with the Argmin ability have the highest utility from teaching relative to skilled professions. Thus, if workers with the Argmin ability are reluctant to become teachers, the teachers sector is empty. Though, if they were obligated to become skilled workers, their optimal time investment in higher education would be identical to the time investment of teachers, i.e., $e_i^*(\hat{\theta}_0) = e_T$ (recall equation (6)).

Accordingly, highly skilled workers, who are more talented than $\hat{\theta}_0$, spend more time on higher education than teachers, whereas vocational workers with lower ability than $\hat{\theta}_0$, spend less time on higher education than teachers. Now, the skilled incomes relative to teachers are derived easily. Since highly skilled workers (teachers) invest more time than teachers (vocational workers) in higher education, they must be compensated by relatively higher incomes. In the presence of $\lambda > 0$, it is easy to verify from equation (11) that $\hat{\theta}_1$ = the Argmin of $\left(\frac{u_s}{u_r}(\theta_i) \right)$ is lower than $\hat{\theta}_0$. Similarly, workers with the

Argmin ability $\hat{\theta}_1$ are teachers. Also note that $e_i^*(\hat{\theta}_0) = e_T$ for all λ , since the optimal time allocation in the skilled sector does not depend on λ (recall (6)). As a result, vocational workers who are less talented than $\hat{\theta}_1$, have lower ability than $\hat{\theta}_0$. Therefore, similar to the case of $\lambda = 0$, they spend less time on higher education and have lower incomes than teachers. However, workers with higher ability than $\hat{\theta}_1$ but lower ability than $\hat{\theta}_0$, i.e., with $e_i^* < e_T$, may become highly skilled workers.

Proof of proposition 8

Let us calculate the relative teacher ability:

$$\frac{\bar{\theta}_T}{\bar{\theta}_S} = \frac{\bar{\theta}_T}{\frac{P_H}{P_S}\bar{\theta}_H + \frac{P_V}{P_S}\bar{\theta}_V} = \frac{\theta_{VT} + \theta_{TH}}{\left(\frac{\bar{\theta} - \theta_{TH}}{\bar{\theta} - \theta_{TH} + \theta_{VT} - \theta_{UV}}\right)(\theta_{TH} + \bar{\theta}) + \left(\frac{\theta_{VT} - \theta_{UV}}{\bar{\theta} - \theta_{TH} + \theta_{VT} - \theta_{UV}}\right)(\theta_{UV} + \theta_{VT})}$$

Then, $\frac{\bar{\theta}_T}{\bar{\theta}_S} = \frac{\theta_{VT}^2 - \theta_{TH}^2 + (\bar{\theta} - \theta_{UV})(\theta_{VT} + \theta_{TH})}{\theta_{VT}^2 - \theta_{TH}^2 + (\bar{\theta} - \theta_{UV})(\bar{\theta} + \theta_{UV})}$. Recall that the lower threshold of the vocational sector,

θ_{UV} , declines. Moreover, the sum of teachers' thresholds, $\theta_{VT} + \theta_{TH}$, increases (recall that the rise in θ_{TH} is larger than the decline in θ_{VT}). As a result, the relative teacher ability increases.

Proof of proposition 9

When the vocational sector does not exist, relative teacher ability equals: $\frac{\bar{\theta}_T}{\bar{\theta}_S} = \frac{\theta_{TH} + \theta_{UT}}{\theta_{TH} + \bar{\theta}}$. It declines

since θ_{UT} declines.

Definition 6: Consider two income distributions represented by the random variables X and W. X is more equal than W if the Lorenz curve corresponding to X is everywhere above that of W. Thus, if X is more equal than W, it has a lower Gini coefficient. According to Atkinson (1970), a larger Lorenz curve is equivalent to second-degree stochastic dominance.

Section 0 – the effects on the lower threshold of the vocational sector, θ_{UV} :

Because the learning effort declines, the incentive to attain higher education rises. Thus, the supply of unskilled workers shrinks (see (13)). On the other hand, vocational workers increase their time investment in higher education, thus their quality rises and they become more expensive to firms. As a result, the relative demand for vocational workers declines. Under the assumptions in proposition 5, the latter effect dominates, i.e., θ_{UV} increases.