HUMAN CAPITAL AND INEQUALITY DYNAMICS:THE ROLE OF EDUCATION TECHNOLOGY

JEAN-MARIE VIAENE AND ITZHAK ZILCHA

11-08

OCTOBER, 2008

THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH
AND
THE SACKLER INSTITUTE OF ECONOMIC STUDIES

Human Capital and Inequality Dynamics: The Role of Education Technology

By Jean-Marie Viaene[†] and Itzhak Zilcha[⊤⊤]

[†]Erasmus University, Tinbergen Institute and CESifo

[‡] The Foerder Institute for Economic Research, Tel Aviv University

July, 2008

Abstract

The paper offers a unified way to examine several puzzles on inequality dynamics. It focuses on differences in the education technology and their effects on income distributions. Our overlapping generations economy has the following features: (1) consumers are heterogenous with respect to ability and parental human capital; (2) intergenerational transfers take place via parental direct investment in education and, public education financed by taxes (possibly, with a level determined by majority voting). We explore several variations in the production of human capital, some attributed to 'home-education' and others related to 'public-education', and indicate how various changes in education technologies affect the intragenerational income inequality along the equilibrium path.

JEL *classification:* D91; E25; H52

Keywords: Human Capital; Innate Ability; Inequality Dynamics.

INTRODUCTION

Statistical offices of international organizations have compiled lists of indicators that compare scholastic achievements across countries. A primary common element of these indicators is that the processes of training and knowledge acquisition differ in various parts of the world. Significant differences between countries arise mainly in the following areas: the level and efficiency of public education, involvement of parents in the education process of their children, the human capital of teachers and the use of existing technologies such as computers and internet. Since human capital formation affects output and the intragenerational distribution of human capital, it is essential to explore how these differences in the provision of education matter. In particular, we explore in this paper how variations in the education technology affect the distribution of earnings.

Though human capital formation is a complex process, theoretical economic models in the literature have assumed various restricted mechanisms governing this process. Due to tractability reasons, these processes have concentrated only on very few parameters (see, e.g., Glomm and Ravikumar, 1992; Laitner, 1997; Orazem and Tesfatsion, 1997; Hanushek, 2002). The implications of these simplified processes of the human capital production function are far reaching, since the dynamics of the human capital distribution is significantly affected.

We shall consider a human capital production process that exhibits two important properties. First, the parental human capital plays an important role in the process of generating the human capital of the offspring. Evi-

dence for that is well established in the literature (see, e.g., Hanushek, 1986). Glaeser (1994) finds that children from families with educated parents obtain better education. Burnhill *et al.* (1990) find that parental education influences entry into higher education in Scotland over and above parental social status. Lee and Barro (2001) and Brunello and Checchi (2003) find that family characteristics, such as income and education of parents, enhance student's performance. A reason that is put forward is that parental education elicits more parental involvement (including related private investment) at home. Second, the contribution of public education to human capital formation depends on both the level of provision and the quality of teachers. Individuals from below-average human capital families will have a greater return to investment in public schooling than those from above-average families. In addition, the cost of acquiring human capital will be smaller for societies endowed with relatively higher levels of average human capital.

Income distribution is a key economic issue and a large literature has improved our understanding of its underlying determinants. Besides trade and technical progress, some believe that social norms are crucial determinants of earnings inequality (e.g., Atkinson, 1999; Corneo and Jeanne, 2001). Others have thoroughly studied the role of human capital accumulation on income distribution in various contexts (see, e.g., Loury, 1981; Becker and Tomes, 1986; Galor and Zeira, 1993; Chiu, 1998; Fernandez and Rogerson, 1998; Rubinstein and Tsiddon, 2004). However, as the information and communication technology advances and computors are being integrated into the learning process, new issues like the increasing technological contribution to learning arise. The literature also contains work on how education systems

come about. For example, Glomm and Ravikumar (1992) establish that majority voting results in a public educational system as long as the income distribution is negatively skewed. Cardak (1999) strengthens this result by considering a voting mechanism where the median preference for education expenditure, rather than median income household, is the decisive voter. This paper examines the effects of technological changes in public and private education processes on income inequality in equilibrium. We consider first the case where the level of public education is predetermined and later (in Section II) we apply the median-voter theorem to generalize these results.

Our analysis is conducted in an OLG economy in which physical capital and human capital are factors of production. Young individuals in each generation are heterogeneous due to the human capital distribution of parents, as well as (random) innate ability. Education/ learning take place via two channels: the time invested by parents at home educating their child (motivated by altruism) and the provision of *public education* by the government financed by taxing wage incomes. Home education is carried out mainly through parental tutoring, social interaction and the learning devices available at home (such as computor and internet). In this case the human capital of parents and the time dedicated to tutoring are important factors. Public education includes public expenditures related to schooling, in particular, the time children are studying at school, as well as the quality of teachers, size of classes, social interactions, etc. Our framework will generate endogenous growth in human capital, due to investments in education/ training, and will allow for a political equilibrium regarding the provision of public education. In our model intergenerational transfers take place via

investments in education only; there are no physical capital transfers even though altruism between each child and his/her parents exists. In the US the main channels for intergenerational transfers are education-related expenditures, the 'bequest' part being rather weak [see, e.g., Gale and Scholz, 1994; Laitner and Juster, 1996).

Using our general process of human capital formation we derive the following results. Comparing dynamic equilibrium paths period by period we obtain: (i) When the government does not supply public education, income inequality declines (increases) over time under decreasing (increasing) returns to parental human capital; (ii) Higher provision of public schooling reduces inequality in the equilibrium distribution of income; (iii) Initial human capital distribution matters. A country starting from a lower level of human capital has a lower return to public education and, hence, experiences more inequality; (iv) When the provision of public education becomes "more efficient" the intragenerational income inequality declines in all subsequent periods. If, instead, only the process of private provision of education becomes "more efficient" it results in higher inequality in all subsequent periods; (v) If the level of provision of public education is determined by *majority voting* the above results are strengthened; (vi) Unlike cases studied in the literature, majority voting may uphold the public education system even if the income distribution is positively skewed; (vii) Different measures of household incomes provide different predictions regarding how openness affects income inequality; (viii) The relationship between growth and income inequality can be positive or negative depending on the source of change in the human capital formation process. Hence, this paper offers another angle in the search

for better understanding several puzzles related to inequality dynamics.

The remainder of the paper is organized as follows. Section I presents an OLG model with heterogeneous agents and analyzes the properties of this framework. Section II studies the effects of variations in the education technology on intragenerational income inequality. Section III concludes the paper. We shall relegate the proofs to the appendix to facilitate the reading.

I. THE DYNAMIC FRAMEWORK

Consider an overlapping generations economy with a continuum of consumers in each generation, each living for three periods. During the first period each child is engaged in education/training, but takes no economic decisions. Individuals are economically active during the working period which is followed by the retirement period. We assume no population growth, hence population is normalized to unity. At the beginning of the 'working period', each parent gives birth to one offspring. Each household is characterized by a family name $\omega \in [0,1]$. Denote by $\Omega = [0,1]$ the set of families in each generation and by μ the Lebesgue measure on Ω.

Agents are endowed with *two units* of time in their working period. One unit is inelastically supplied to labor, while the other is allocated between leisure and self-educating the offspring.[1] Consider generation t, denoted G_t, namely all individuals ω born at the outset of date $t-1$, and let $h_t(\omega)$ be the level of human capital of $\omega \in G_t$.We assume that the production function for human capital consists of two components: informal education initiated and provided by parents at home and public education provided by the government at schools by hiring 'teachers'. The 'home-education'

depends on the time allocated by the parents to this purpose, denoted by $e_t(\omega)$, and the 'quality of tutoring' represented by the parent's human capital level $h_t(\omega)$. The time allocated to public schooling (i.e., the level of public education) is denoted by e_{gt}. The human capital of the teachers determines the 'quality' of public education in the formation of the younger generation's human capital. We assume that the (random) innate ability of individual $\omega \in G_{t+1}$, denoted by $\theta_t(\omega)$, enters multiplicatively in the production function of the child's human capital.

We take the human capital formation process to depend on both components of education: the 'home education' as well as the public education. Thus our process generalizes the processes of human capital production in most models used in the theoretical literature in this field. We assume that for some parameters $\beta_1 > 1$, $\beta_2 > 1$, $\upsilon > 0$ and $\eta > 0$, the evolution process of a family's human capital is given as follows. For all $\omega \in G_{t+1}$:

$$h_{t+1}(\omega) = \theta_t(\omega)[\beta_1 e_t(\omega) h_t^{\upsilon}(\omega) + \beta_2 e_{gt} \overline{h}_t^{\eta}]$$

where the average human capital involved in the public schooling system, denoted $\overline{h}_t$, is the *average* human capital of generation t. This is justified if, for example, the instructors in each generation are chosen at random from the population of that generation. The parameters υ and η measure the externalities derived from parents' and society's human capital respectively. The constants β_1 and β_2 represent how efficiently parental and public education contribute to human capital: β_1 is affected by the home environment while β_2 is affected by facilities, the schooling system, size of classes, neighborhood,

social interactions, and so forth[2].

Regarding innate ability, Viaene and Zilcha (2007) model $\theta_t(\omega)$ as a random and independently distributed variable across individuals in each generation and over time. They show that when ability is known to parents before they make their decision about investment in education, the introduction of child ability has no effect in all the subsequent analysis. Therefore, we assume $\theta_t(\omega) = \theta$ for all t and ω and, hence, the evolution process of a family's human capital becomes:

$$h_{t+1}(\omega) = \theta[\beta_1 e_t(\omega) h_t^v(\omega) + \beta_2 e_{gt} \overline{h}_t^{\eta}] \tag{1}$$

The production function of human capital given by (1) exhibits the property that public education dampens the family attributes. As it is common to all, individuals from below-average families have, therefore, a greater return to human capital derived from public schooling than those born to above-average human capital families. In addition, the effort of acquiring human capital is smaller in countries endowed with relatively higher levels of human capital. An important difference between our process of generating human capital and most cases discussed in the literature is the representation of the private and the public inputs in the production of human capital via allocation of time.[3] Our approach assumes that the *time spent learning*, coupled with the human capital of the instructors, rather than the expenditures on education, are more relevant variables in such a process although there may exist a relationship between the quality of public education and public

expenditure on education.[4].

Consider the lifetime income of individual ω, denoted by $y_t(\omega)$. Since the human capital of a worker is observable, it depends on the effective labor supply. Let w_t be the wage rate in period t and τ_t is the tax rate on labor income, then:

$$y_t(\omega) = w_t(1 - \tau_t)h_t(\omega) \tag{2}$$

Under the public education regime taxes on incomes are used to finance education costs of the young generation. Making use of (1) and (2), balanced government budget means:

$$\int_\Omega w_t e_{gt} \overline{h}_t d\mu(\omega) = \int_\Omega \tau_t w_t h_t(\omega) d\mu(\omega)$$

or equivalently,

$$e_{gt} = \tau_t \tag{3}$$

that is, the tax rate on labor is equal to the proportion of the economy's effective labor used for public education.[5]

Dynamic equilibrium

Production in this economy is carried out by competitive firms that produce a single commodity, using effective labor and physical capital. This commodity is both consumed and used as production input. Physical capital fully depreciates and the per-capita effective human capital in date t, $\overline{h_t}$, is an in-

put in aggregate production. In particular we take this production function to be:

$$q_t = F(k_t, (1 - e_{gt})\overline{h_t}) \tag{4}$$

where k_t is the capital stock and $(1 - e_{gt})\overline{h_t} = (1 - \tau_t)\overline{h_t}$ is the effective human capital used in the production process. F(·,·) is assumed to exhibit constant returns to scale; it is strictly increasing, concave, continuously differentiable and satisfies $F_k(0, (1 - \tau_t)\overline{h_t}) = \infty$, $F_h(k_t, 0) = \infty$, $F(0, (1 - \tau_t)\overline{h_t}) = F(k_t, 0) = 0$.

Given the public education provision and factors' prices, an agent ω at time t maximizes lifetime utility, which depends on consumption, leisure and income of the offspring. Thus:

$$\max_{e_t, s_t} u_t(\omega) = c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} y_{t+1}(\omega)^{a_3} \left[1 - e_t(\omega)\right]^{\alpha_4} \tag{5}$$

subject to

$$c_{1t}(\omega) = y_t(\omega) - s_t(\omega) \geq 0 \tag{6}$$

$$c_{2t}(\omega) = (1 + r_{t+1}) s_t(\omega) \tag{7}$$

where $h_{t+1}(\omega)$ and $y_{t+1}(\omega)$ are given by (1) and (2). The α_i 's are known parameters and $\alpha_i > 0$ for $i = 1, 2, 3, 4$; $c_{1t}(\omega)$ and $c_{2t}(\omega)$ denote, respectively, consumption in first and second period of the individual's economically active

life; $s_t(\omega)$ represents savings; leisure is given by $(1 - e_t(\omega))$; $(1 + r_{t+1})$ is the interest factor at date t. The offspring's income $y_{t+1}(\omega)$ enters the parents' preferences directly and represents the motivation for parents' investment in tutoring and formal education expenditure. Given some tax rates (τ_t), k_0 and the initial distribution of human capital $h_0(\omega)$, a *competitive equilibrium* is $\{e_t(\omega), s_t(\omega), k_t; w_t, r_t\}$ which satisfies: For all t and all individuals $\omega \in G_t$, $\{e_t(\omega), s_t(\omega)\}$ are the optimum to the above problem given $\{w_t, r_t\}$. And, the following market clearing conditions hold:

$$w_t = F_h(k_t, (1 - e_{gt})\overline{h_t}) \tag{8}$$

$$(1 + r_t) = F_k(k_t, (1 - e_{gt})\overline{h_t}) \tag{9}$$

$$k_{t+1} = \int_\Omega s_t(\omega) d\mu(\omega) \tag{10}$$

Equations (8) and (9) are the clearing conditions in the factors market. After substituting the constraints, the first-order conditions that lead to the necessary and sufficient conditions for an optimum are:

$$\frac{c_{1t}}{c_{2t}} = \frac{\alpha_1}{\alpha_2(1 + r_{t+1})} \tag{11}$$

$$\frac{\alpha_4}{(1 - e_t(\omega))} \geq \frac{\beta_1 \alpha_3 (1 - \tau_{t+1}) w_{t+1} h_t^v(\omega) \theta_t(\omega)}{y_{t+1}(\omega)}, \text{with } = \text{if } e_t(\omega) > 0 \tag{12}$$

From (6), (7) and (11) we obtain:

$$c_{1t}(\omega) = \left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right) y_t(\omega) \tag{13}$$

$$s_t(\omega) = \left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right) y_t(\omega) \tag{14}$$

Equation (12) allocates the unit of nonworking time between leisure and the time spent on education by the parents. In fact, we find that whenever $e_t(\omega) > 0$:

$$e_t(\omega) = \left(\frac{\alpha_3}{\alpha_3 + \alpha_4}\right) [1 - \frac{\alpha_4}{\alpha_3} \frac{\beta_2 \tau_t \overline{h}_t^{\eta}}{\beta_1 h_t^{v}(\omega)}]$$

Hence, $e_t(\omega)$ increases with the parents' human capital $h_t(\omega)$ but decreases with the tax rate τ_t. By applying (12) and making use of (1) , (2) and (3) we obtain the reduced-form solution of the model:

$$y_{t+1}(\omega) = (1 - \tau_{t+1}) w_{t+1} h_{t+1}(\omega) \tag{15}$$

$$h_{t+1}(\omega) = \left(\frac{\alpha_3}{\alpha_3 + \alpha_4}\right) \theta_t [\beta_1 h_t^{v}(\omega) + \beta_2 \tau_t \overline{h}_t^{\eta}], \text{ whenever } e_t(\omega) > 0 \tag{16}$$

$$h_{t+1}(\omega) = \beta_2 \theta_t \tau_t \overline{h}_t^{\eta}, \text{ whenever } \ e_t(\omega) = 0 \tag{17}$$

Equations (15)-(17) determine the income at the future date in terms of the net wage at date $t+1$, the parents' human capital, society's level of human capital at date t, the current education input ($\tau_t = e_{gt}$) and the externalities in education. More importantly, (15) shows that, in our framework, the intragenerational distribution of income is similar to that of human capital.

Non-participation of parents

The non-participation of parents in the education process is an important characteristic of the education systems in some OECD countries like Germany[6]. This situation, where utility maximization is attained at $e_t(\omega) = 0$, occurs under certain conditions. To derive these circumstances recall that (12) establishes a negative relationship between the two types of education: public education *substitutes* for parental tutoring. For each individual there exists a particular tax rate such that $e_t(\omega) = 0$, namely, when the marginal utility of leisure is larger than the marginal utility gained by increasing the offspring's human capital due to parental tutoring. Consider the families which *optimally* choose $e_t(\omega) = 0$, and denote this set of families in generation t by $A_t \subset G_t = [0,1]$. In fact, condition (12) holds if:

$$1 - e_t(\omega) < \frac{\alpha_4}{\beta_1 \alpha_3}[\beta_1 e_t(\omega) + \beta_2 e_{gt} \frac{\overline{h}_t^{\eta}}{h_t^{v}(\omega)}]$$

Hence, for each individual in G_t we obtain $e_t(\omega) = 0$ and $\omega \in A_t$ if:

$$h_t^{v}(\omega) < \frac{\alpha_4 \beta_2 e_{gt}}{\alpha_3 \beta_1} \overline{h}_t^{\eta} \tag{18}$$

Parental and public education being substitutes, inequality (18) shows that the set A_t increases in societies with strong preference for leisure and/or with a high provision of public education e_{gt}. In both cases, families in A_t delegate the task of education to the public sector. It is clear that these families include individuals with low levels of human capital.

II. EDUCATION AND INCOME INEQUALITY

We rank income inequality of any two income distributions according to their Lorentz ordering, namely, using the **second degree** stochastic dominance criterion (Atkinson, 1970).

The role of initial endowments

The literature that studies the connection between trade and income inequality provides mixed empirical evidence regarding the sign of this relationship. It depends on the sample of countries, but more importantly, on the definition of income that is used in the computation of inequality (see e.g., Francois and Rojas-Romagosa, 2005). Also, the relationship is often conditional on the given factor endowments. For example, Spilimbergo *et al.* (1999) finds that openness increases inequality but its effect depends on the initial factor endowments. Fischer (2001) finds that labor-abundant countries are more equal. To demonstrate that initial conditions matter in our framework let us consider two economies that differ only in their initial endowments of *human capital*: one economy has higher *levels* of human capital but the measure of inequality in the *initial* human capital distributions is *the same*. The

reason we start with endowments is to uncover conditions under which international trade based on endowment differences, or differences in educational technology, does not affect income inequality in equilibrium. These conditions provide a justification for our approach which is based on comparing countries' educational systems in isolation. The next proposition compares the equilibrium paths of these two countries.

Proposition 1 *Consider two economies which differ only in their initial human capital distributions, $h_0(\omega)$ and $h_0^*(\omega)$. Assume that $h_0^*(\omega) > h_0(\omega)$ for all ω, but the initial distributions have the* ***same*** *level of inequality. Then, the equilibrium from $h_0^*(\omega)$ will have lower income inequality than that from $h_0(\omega)$ at all dates.*

The result has the following policy implications: a country that starts with higher levels of human capital, not necessarily more equal, has a higher return to public education and, hence, has a better chance to maintain less inequality in its future income distributions. We relegate all the proofs to the appendix.

Given different endowments of human capital let us consider the introduction of international trade and mobility of physical capital between these two economies, keeping labor immobile internationally. These assumptions about trade and factor mobility guarantee factor price equalization. In this setting, we can show the following:

Proposition 2 *Consider two economies which differ only in their initial conditions. Trade in goods and physical capital mobility will not alter the intragenerational income inequality obtained under autarky.*

Hence, though two economies differ in their initial conditions, introducing trade in goods and capital mobility in our framework will not alter the income inequality measure under autarkic regime. Variations in the equilibrium factor prices do not affect the income distribution since labor incomes vary in the same proportion. In contrast, trade and capital mobility have significant impact on wages, interest rates and outputs of the two countries and, in this regard, affect the *intergenerational* distribution of income as follows. At date t, total income of family ω is given by:

$$q_t(\omega) = c_{2(t-1)}(\omega) + y_t(\omega) \tag{19}$$

where the first term is consumption at date t by the family member who was economically active at date t-1 and the second term is the labor income generated by the active member of the family. Using equations (7), (14) and (15) we obtain:

$$q_t(\omega) = (1 + r_t)[\frac{\alpha_2}{\alpha_1 + \alpha_2} y_{t-1}(\omega) + \frac{w_t(1 - \tau_t)}{1 + r_t} h_t(\omega)] \tag{20}$$

with $h_0^*(\omega) > h_0(\omega)$ and assuming similar stocks of physical capital (i.e., $k_0 = k_0^*$) and $\tau_t = \tau_t^*$ for all t. It is clear that, in isolation, $\frac{w_t(1-\tau_t)}{1+r_t} > \frac{w_t^*(1-\tau_t^*)}{1+r_t^*}$ for all t. As a result, when capital markets are integrated physical capital will flow from the low return domestic economy to high return foreign country until equality in wage-rental ratios is obtained. Using the results of

Propositions 1 and 2 we can show the implication of capital mobility to the *intergenerational income distribution*:

Proposition 3 *Consider two economies which differ only in their initial human capital distributions. Assume that* $h_0^*(\omega) > h_0(\omega)$ *holds for all* ω, *but the initial income inequality is the same. Trade in goods and physical capital mobility result in a lower intergenerational income inequality for the home country and a higher intergenerational income inequality for the foreign country.*

As in the empirical literature, the above proposition stresses the importance of factor endowments in explaining equilibrium income inequality. In addition, the last two propositions show that different measures of household income generates different predictions regarding the effect of openness on income inequality. Also, as trade plays no role in explaining *intragenerational* income inequality in our framework, we can compare countries' education systems separately and ignore how these systems affect the comparative advantage of nations.

Public education

Let us consider first a situation in which the government does not contribute to human capital formation. Thus, we take $\tau_t = 0$ for all t. In this case:

$$y_{t+1}(\omega) = w_{t+1} h_{t+1}(\omega)$$

From (18) we know that the set A_t is empty, and from (12) we obtain

that:

$$e_t(\omega) = e^*(\omega) = \frac{\alpha_3}{\alpha_3 + \alpha_4} \text{ for all } \omega$$

Hence, in the absence of public education the only source of income inequality is the initial distribution of human capital. This is clear from:

$$y_{t+1}(\omega) = [\beta_1 w_{t+1} e^*(\omega) h_t^v(\omega)]\theta$$

We conclude from these observations that:

Proposition 4 *In the absence of public education : (i) income inequality declines over time under decreasing returns to parental human capital (i.e., if $v < 1$), (ii) income inequality increases over time under increasing returns (i.e., if $v > 1$), and (iii) income inequality remains constant over time under constant returns (i.e., if $v = 1$).*

Our economy generates, in equilibrium, an intragenerational income distribution whose inequality is endogenously determined by the externality in the home-component of the education process. Inequality may decrease even in the absence of public schooling. When $v > 1$ a family 'poverty trap' arises in that $h_t(\omega)$ goes to zero for some families whose initial endowment of human capital is below some benchmark level. More precisely, this occurs for family ω such that:

$$h_0(\omega) < \left[\frac{\alpha_3 + \alpha_4}{\beta_1 \alpha_3 \theta}\right]^{\frac{1}{v-1}} \equiv \underline{h}$$

It segments the population's human capital into two groups: families below this benchmark level $\underline{h}$ which face a permanent decline in human capital while those to the right of it experience a permanent increase. This result is

applicable to China where increasing returns in parents' human capital have been observed (see Knight and Shi, 1996).

Let us now look at the effect of public education on income inequality assuming that its level is exogenously given. Let us reconsider expression (18): it is clear that as e_{gt} increases more parents may stop educating their children. It is therefore important to further characterize the role of public education, its effect on accumulation of human capital and the distribution of income. We do not choose explicitly the social decision mechanism underlying its determination by the government. The level at date t is e_{gt} and it is financed by taxing labor income at a fixed rate $\tau_t (= e_{gt})$. In the sequel we assume that $\upsilon \leq 1$ and that $\eta \leq 1$ and, to simplify our analysis, we also assume that $\upsilon \leq \eta$. Does public education reduce income inequality in equilibrium?

Proposition 5 *Let $h_0(\omega)$ be any initial human capital distribution and assume that the tax rate that finances public education is constant over time. Increasing this tax rate results in a lower intragenerational income inequality in all subsequent periods.*

This proposition extends similar results in the literature (see, e.g., Glomm and Ravikumar, 1992) to our setup under active *public* and *private* education. It may not seem surprising since public education in our framework dampens family attributes as it is provided equally to all young individuals (of the same generation), while it is financed by a flat tax rate on wage income. However, its importance lies in the fact that: (i) it is proved in *equilibrium,* (ii) it holds for *all periods*, and (iii) it allows for the non-participation of

some parents after public education is introduced. Hence, if one compares two countries which are *similar* in all respects except for the level of public education, the country which invests less in public schooling will face a higher inequality along the equilibrium path.

Efficiency in human capital formation

Let us consider the information and communication technology (ICT) revolution, seen as a technological improvement that enhances knowledge. According to the World Bank (2001, Table 19), the diffusion of information technology across countries is highly uneven. The 1998 figures on the number of computers per 1000 people range between 458.6 in the US and 0.2 in Niger. A more comprehensive ranking by the International Telecommunication Union measures, besides availability, also the innate and financial abilities of individuals to use ICT (ITU, 2003). A similar gap has been observed in this case as well where Niger is ranked at the bottom but the US is positioned now as 11th. These observations raise the following question: does the home component of human capital formation benefit more than the public education component from the ICT revolution? We believe that this is the case for two reasons. First, in many countries computers and internet access have enchanced home education considerably, while the benefits to public schools are significantly less. Second, within countries there are wide gaps between the wealthier and poorer families. Thus, the use of the ICT raises the issues of affordablity of education and emphasizes the importance of families' human capital. In terms of our model, the first argument means a rise in β_1 that is proportionately larger than the rise in β_2, while the second

means an increase in v.

Let us concentate upon cross-country differences in processes describing human capital formation and focus on technological variations assuming that the human capital is generated by the process in (1). Such improvements can be represented by an increase in the 'efficiency' of the education environment; namely, via the introduction of more sophisticated teaching facilities (computors, for example), reducing class size, better organization of schools and so forth. This amounts to increasing the parameters β_1 and/or β_2. Another form of technological improvement in this process is to enhance the effectiveness of the 'teachers' or 'tutors' through, for example, better training for teachers and advising parents about tutoring their own child. Such an improvement amounts to increasing the parameters v and η, which brings into expression the effectiveness of the human capital of the parents and/or that of the 'teachers'. Let us assume in the sequel that $v \leq 1$ and $\eta \leq 1$, even though these assumptions can be relaxed in most cases[7] .

An improvement in one country (vs. the other) in the production of human capital may result in a more efficient home education or a more efficient public education, or both. We say that the provision of *public* education is *more efficient* if either β_2/β_1 is *larger* (without lowering neither β_1 nor β_2) or η is larger, or both. We say that the *private* provision of education becomes *more efficient* if β_1/β_2 becomes *larger* (while neither β_1 nor β_2 declines) or v becomes larger, or both. It is called *neutral* in the case where both parameters β_1 and β_2 increase while the ratio β_2/β_1 remains unchanged. The next proposition considers the effect of each type of technological change in the education process on intragenerational income inequality.

Proposition 6 *Consider improvements in the production process of human capital, then: (a) If the public provision of education becomes more efficient the inequality in intragenerational distribution of income declines in all periods; (b) If the private provision of education becomes more efficient then inequality increases in all periods; (c) If the technological improvement is neutral inequality remains unchanged at period 1 but declines for all periods afterwards.*

This result demonstrates the asymmetry between a technological change that affects primarily the efficiency of the public schooling system and the one that affects primarily the home environment of learning. The inequality in human capital distribution increases when the private-component of education/learning becomes more efficient because the family attributes are magnified. In contrast, a more efficient public education reduces inequality because all children are exposed to instructors with the same level of 'average' human capital: below-average families have a greater return to public schooling than above-average families. When the technological advancement in the education technology is neutral, then along the 'better' equilibrium inequality declines, except for the first date, since, after the first period, the effectiveness of public schooling outweighs that of home education.

An extension of Proposition 6 is to examine how inequality relates to economic growth as various parameters in the education process vary. In our framework the sole source of income is generated by the aggregate production which applies both physical capital and human capital. Thus, variations of the parameters tied to educational technology affect growth significantly.

Let us consider the implication of a technological change in the production of human capital has to output in equilibrium. From process (1) we call $\beta_1 e_t(\omega) h_t^v(\omega)$ the *home component* and the second term $\beta_2 e_{gt} \overline{h}_t^{\eta}$ the *public component*. An improvement in the production of human capital which makes either the public provision more efficient or the private provision more efficient, implies a higher human capital stock as of date 1 onwards. Since the initial human capital stock is given it implies a higher output and a higher capital stock as of date 2. Does such a technological progress, which results in higher growth, mean more income inequality? Let us combine our results to obtain:

Corollary 1: *Consider improvements in the production process of human capital, then:*

(a) If the technological progress occurs only in the home component it results in higher growth coupled with higher income inequality in all subsequent periods;

(b) If the technological progress occurs in the public component of education it results in higher growth accompanied by lower income inequality in all subsequent periods.

The issue of co-movements of economic growth and income inequality has been widely debated in the literature, mainly by using empirical evidence, and this debate is inconclusive (see, e.g., Persson and Tabellini, 1994; Barro, 2000; Forbes, 2000). Corollary 1 provides some interpretation to these empirical findings. It establishes conditions on *endogenous processes* under which growth can be accompanied by either more or less income inequality.

Political equilibrium

Thus far, the analysis in our framework was carried out under the assumption that the education tax rate, hence, the level of public education, is exogenously given. However, the assumption that the tax rate is independent of the technology parameters is very questionable. The exogeneity of τ_t can be relaxed by introducing a voting scheme into our model. As families are heterogenous, each has a different preference regarding the amount of resources that should be invested in public education. The choice of the 'optimal' level of public schooling should therefore be the outcome of a certain political equilibrium.

The political equilibrium we consider here is an application of the median-voter theorem, widely used in economic theory (see, e.g., Persson and Tabellini, 2000, Section 3.3). Let us substitute conditions (11)-(12) in (5) to obtain an expression for the lifetime utility of agent $\omega \in G_t$ in terms of the tax rate τ_t:

$$U_t(\omega) = B_t \theta^{\alpha_3} [1 - \tau_t]^{\alpha_1 + \alpha_2} [\beta_1 h_t^{\upsilon}(\omega) + \beta_2 \tau_t \overline{h}_t^{\eta}]^{\alpha_3 + \alpha_4} \tag{21}$$

where B_t groups parameters and variables given to this individual at the outset of date t (including τ_{t+1}).[8] Since $U_t(\omega)$ is concave in τ_t there is a unique maximum for each individual's lifetime utility denoted by $\tau_t(\omega)$. It is obtained directly from the first order (necessary and sufficient) condition:

$$(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)\beta_2 \tau_t(\omega) \overline{h}_t^{\eta} = (\alpha_3 + \alpha_4)\beta_2 \overline{h}_t^{\eta} - (\alpha_1 + \alpha_2)\beta_1 h_t^{\upsilon}(\omega)$$

It is clear that the heterogeneity in voter's optimal policy $\tau_t(\omega)$ results

from the heterogeneity in their human capital $h_t(\omega)$. In particular, the median voter's choice is:

$$(22) \qquad \tau_t(m) = [\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4]^{-1}[(\alpha_3 + \alpha_4) - (\alpha_1 + \alpha_2)\frac{\beta_1 h_t^{\upsilon}(m)}{\beta_2 \overline{h}_t^{\eta}}]$$

Some monotonicity results can be verified from the expression in (22):

$$(23) \quad \frac{\partial \tau_t(m)}{\partial \alpha_1} = \frac{\partial \tau_t(m)}{\partial \alpha_2} < 0 \quad , \quad \frac{\partial \tau_t(m)}{\partial \alpha_3} = \frac{\partial \tau_t(m)}{\partial \alpha_4} > 0 \text{ and } \frac{\partial \tau_t(m)}{\partial (\frac{\beta_1}{\beta_2})} < 0$$

Observed cross-country differences in education expenditures can be explained by (22) and (23). For example, as $h_t(m)$ drops relative to $\overline{h}_t$, $\tau_t(m)$ rises: A below-average median voter favors a higher tax rate than an above-average median voter. Also, an increase in υ and β_1/β_2 [or a decrease in η] imply a lower tax rate for financing education [other educational efficiency implications of political equilibria can be found in De Fraja (2001, 2002)].

Given these observations, let us illustrate how using the median-voter theorem strengthens our previous results regarding income inequality. Table 1 examines how various parameters in our model affect income inequality. The first column contains the results of (23); the second column applies part (ii) of Proposition 5 to infer the effects on income inequality of column one. The third column applies Proposition 6, while the total effect is given in the last column. Consider, for example, a marginal increase in β_1: by Proposition 6 it leads to a higher inequality while majority voting implies a lower tax rate $\tau_t(m)$. In turn, applying part (ii) of Proposition 5 leads to even more inequality.

Corollary 2: *When the resources invested in public education are determined by a political equilibrium, applying the median-voter theorem strengthens the results regarding income inequality attained under exogenously given tax rates.*

The proof of Corollary 2 follows directly from the preceding propositions, hence it is omitted. It is important to note that under majority voting consumer preference parameters become determinants of income inequality[9].

Table 1: Median Voter and Income Inequality[a]

	(1) $\tau_t(m)$	(2) Prop. 5	(3) Prop. 6	Total: (2)+(3)
α_1	$-$	$+$		$+$
α_2	$-$	$+$		$+$
α_3	$+$	$-$		$-$
α_4	$+$	$-$		$-$
β_1	$-$	$+$	$+$	$+$
β_2	$+$	$-$	$-$	$-$
v	$-$	$+$	$+$	$+$
η	$+$	$-$	$-$	$-$

[a] Notes: Column (1) reports the monotonicity results of (23); column (2) uses Proposition 5 to infer variations in inequality resulting from column (1); column (3) applies Proposition 6 to obtain how marginal changes in technology parameters affect inequality.

Another characteristic of (22) is that $\tau_t(m)$ is zero for combinations of parameters representing consumer preferences and the education technology. Our political equilibrium has therefore the following implications for the types of income distribution that can support a public education system.

Proposition 7 *Majority voting results in the provision of public education as long as:*

$$h_t(m) < (\frac{\alpha_3 + \alpha_4}{\alpha_1 + \alpha_2})^{\frac{1}{\upsilon}} (\frac{\beta_2}{\beta_1})^{\frac{1}{\upsilon}} (\overline{h_t})^{\frac{\eta}{\upsilon}} \tag{24}$$

Thus, a negative skewness of the income distribution is neither a necessary nor a sufficient condition for the emergence of public education.

This result follows directly from (22) by solving for $h_t(m)$ such that $\tau_t(m) = 0$. Assume for a moment that all parameters are equal to one. Then $h_t(m) < \overline{h_t}$ in (24) which reproduces the "negative skewness" result. Hence, what condition (24) does is to correct mean human capital for parameters of the utility function and education technology. A strong preference for leisure and a strong inclination for altruism [*i.e.*,when $(\alpha_3 + \alpha_4) > (\alpha_1 + \alpha_2)$], a relatively effective public education ($\beta_2 > \beta_1$) and a low values of υ will increase the right hand side of (24). Hence, $h_t(m)$ can be larger than $\overline{h_t}$, a case of positively skewed human capital and income distribution. The implication of Proposition 7 is that, in contrast to established knowledge, for a broad range of income distributions we obtain support for a public education system.

III. CONCLUSION

This paper attempts to study, within a general equilibrium framework with human capital accumulation, the cross-country differences in income distribution. Our analysis is carried out in the following framework: an overlapping-generations economy with heterogenous households, where heterogeneity results from (random) innate abilities and the nondegenerate initial distribution of human capital. We derive a number of results which provide explanations for observed cross-country differences in income inequality based on variations in the human capital formation process. In particular, our analysis suggests certain hypotheses regarding the education technology that generates a cross-country variation in the equilibrium income distributions: (a) externalities of family's (and society's) human capital; (b) the effective level of public education; (c) the efficiency of public schooling and parental home-education; (d) initial conditions, represented here by the initial stock of physical capital and initial distribution of human capital; (e) the skewness of income distributions; and (f) market openness.

This work illustrates explicitly the role of family attributes (assuming altruism between parents and their children) in the production of human capital. Any education system that elevates the role of a family, such as private education or home education, would lead to higher income inequality. Alternative models, that would include the financing of private education by parents, would magnify our results on the sources of income inequality.

Our framework includes some specific assumptions and, therefore, the results are subject to the issue of robustness. First, the selection of our func-

tional forms, which facilitate our analysis, was strongly motivated by stylized facts. For example, incorporating parental role in the production process of human capital is justified due to the repeatedly reported evidence that it has an empirical relevance in a large number of countries (see, e.g., Checchi, 2006). Second, the assumption that each agent supplies inelastically one unit of his time to the labor market is not essential. Munandar (2006) shows that our results hold qualitatively for the case of elastic supply of labor. Thus, the effect of relaxing the assumption of inelastic labor supply is not trivial as each family's supply of human capital becomes endogenous. Since population rate of growth is zero, our assumption is less stringent due to the time required to raise children which is equal at all generations. Third, the model assumes away taxation of non-wage income, i.e., the interest income from savings. However, expanding the tax base to include this type of income will not alter the qualitative results concerning income inequality. Moreover, our framework allows for additional generalizations, including other redistributive measure by the government, such as social security. Some of the results may vary, however, in this case since intergenerational transfers will take place via both govermental programs: public education and social security.

ACKNOWLEDGEMENTS

We gratefully acknowledge the helpful comments of seminar participants at AUEB (Athens), IIES (Stockholm) and the Tinbergen Institute (Rotterdam). We thank especially two anonymous referees and our Editor-in-charge, Frank Cowell, for detailed comments.

Notes

[1]Though the supply of labor is inelastic, each family's supply of human capital is the result of utility maximization. Also, Munandar (2006) shows that our results hold qualitatively for the case of an elastic supply of labor. Thus the assumption of inelastic labor supply is less severe since, due to our assumption of no population growth, the time required to raise children is equal at each date and is insensitive to the number of young-age children.

[2]Empirical support for (1) is abundant, but let us point out to Brunello and Checchi (2003) who demonstrate, using Italian data, the importance of both 'home' and 'public' education in human capital formation. The family background in human capital formation has been shown to be empirically significant in the case of East Asia by Woessmann (2003). Card and Krueger (1992) established, using US data, that differences in school quality matters when we consider the rate of return to education. A lower pupil/teacher ratio results in a higher return.

[3]Home and public education play different roles in the literature. For example, in Eckstein and Zilcha (1994) there is investment in home education on the part of parents in terms of time. In Eicher (1996), young agents must decide whether to enter the private education sector as students or to work in production as unskilled workers. In Orazem and Tesfatsion (1997), there is private investment in terms of effort and in Viaene and Zilcha (2002, 2003) there is a time input for public education only. In Restuccia and Urrutia (2004), children in their first period of life acquire human capital through public education financed by income

taxes and through private education via additional personal expenditures.

[4]This is in line with Hanushek (2002) who argues in favor of considering the 'efficiency' in the public education provision rather than 'expenditure' on public education. This distinction is important since in a dynamic framework the cost of financing a particular level of human capital fluctuates with relative factor rewards.

[5] Under a decentralized system, namely under a fully private education regime, both $\tau_t(\omega)$ and $e_{gt}(\omega)$ are decision variables of each agent, hence the individual's budget constraint on private education is: $\tau_t(\omega)w_t h_t(\omega) = w_t e_{gt}(\omega)\overline{h}_t$, where the level of teachers' instruction $e_{gt}(\omega)$ is chosen freely while their average human capital is the same as their corresponding generation.

[6]See, e.g., Der Spiegel (2001) and DICE Reports (2002) for attempts at explaining the poor performance of German adolescents in the 2000 study of the Programme for International Student Assessment (PISA) of the OECD.

[7]Throughout this paper we ignore the effect of technological change in the aggregate production function upon inequality. The reason is that even though such changes affect labor income it does not affect inequality in income distribution, since all incomes are varied in the same proportion.

[8]Self-interested agents vote myopically in this model in that they ignore the effect of current political decision on future political outcomes. Voters may induce the end of public education this period but a constituency for an education policy can regenerate next period. See Hassler *et al.* (2003) for a model of rational dynamic voting.

[9]Likewise, it can be shown that the application of the median-voter the-

orem increases the likelihood of a negative co-movement between economic growth and income inequality. Consider a marginal increase in β_2 : the higher tax rate $\tau_t(m)$ implied by this increase leads to higher endogenous growth. Also, the public-component of education becomes more efficient and it enhances growth as well. Thus, all effects on growth are positive and all effects on inequality (see Table 1) are negative.

REFERENCES

Atkinson, A.B. (1970). On the measurement of inequality. J*ournal of Economic Theory*, **2**, 244-263.

Atkinson, A.B. (1999). Is rising income inequality inevitable? A critique of the Transatlantic concensus. UNU/WIDER Annual Lecture.

Barro, R.J. (2000). Inequality and growth in a panel of countries. *Journal of Economic Growth,* **5,** 5-32.

Becker, G.S. and Tomes, N. (1986). Human capital and the rise and fall of families. *Journal of Labor Economics,* **4**(3), S1-39.

Brunello, G. and Checchi, D. (2003). School quality and family background in Italy. *Working Paper #705*, IZA, Bonn, Germany.

Burnhill, P., Garner, C. and McPherson, A. (1990). Parental education, social class and entry to higher education 1976-86. *Journal of the Royal Statistical Association*, Series A, **153**(2), 233-248.

Card, D. and Krueger, A.B. (1992). Does school quality matter? Returns to education and the characteristics of public schools in the United States. *Journal of Political Economy,* **100**(1), 1-40.

Cardak, B.A. (1999). Heteregeneous preferences, education expenditures and income distribution. *Economic Record,* **75** (228), 63-76.

Checchi, D. (2006). **The Economics of Education**. Cambridge: Cambridge University Press.

Chiu, H. (1998). Income inequality, human capital accumulation and economic performance. *The Economic Journal,* **108**, 44-59.

Corneo, G. and Jeanne, O. (2001). Status, the distribution of wealth, and growth. *Scandinavian Journal of Economics,* **103**(2), 283-293.

De Fraja, G. (2001). Education policies: equity, efficiency and voting equilibrium. *Economic Journal,* **111** (May), C104-C119.

De Fraja, G. (2002). The design of optimal education policies. *Review of Economic Studies,* **69**, 437-466.

Der Spiegel (2001). PISA-Studie: Die neue Bildungskatastrophe, Nr. **50** (Dec. 12), 60-75.

DICE Reports (2002). Results of PISA 2000: the case of Germany. *CESifo Forum*, **3**, 49-52.

Eckstein, Z. and Zilcha, I. (1994). The effects of compulsory schooling on growth, income distribution and welfare. *Journal of Public Economics,* **54**, 339-359.

Eicher, T. S. (1996). Interaction between endogenous human capital and technological change. *Review of Economic Studies,* **63**, 127-144.

Fernandez, R. and Rogerson, R. (1998). Public education and income distribution: a quantitative dynamic evaluation of education- finance reform. *American Economic Review,* **88**(4), 813-833.

Fischer, R.D. (2001). The evolution of inequality after trade liberalization. *Journal of Development Economics,* **66**, 555-579.

Forbes, K.J. (2000). A reassessment of the relationship between inequality and growth. *American Economic Review,* **90** (4), 865-887.

Francois, J.F. and Rojas-Romagosa, H. (2005). The construction and interpretation of combined cross-section and time-series income inequality datasets. Discussion Paper 05-079/2, Tinbergen Institute.

Gale, W.G. and Scholz , J.K. (1994). Intergenerational transfers and accumulation of wealth. *Journal of Economic Perspectives,* **8**(4), 145-160.

Galor, O. and Zeira, J. (1993). Income distribution and macroeconomics. *Review of Economic Studies,* **60**, 35-52.

Glaeser, E.L. (1994). Why does schooling generate economic growth?. *Economics Letters,* **44**(3), 333-337.

Glomm, G. and Ravikumar, B. (1992). Public versus private investment in human capital: endogenous growth and income inequality. *Journal of Political Economy,* **100**, 818-834.

Hassler, J., Rodríguez Mora, J.V., Storesletten, K. and Zilibotti, F. (2003). The survival of the welfare state. *American Economic Review*, **93** (1), 87-112.

Hanushek, E.A. (1986). The economics of schooling: production and efficiency in public schools. *Journal of Economic Literature*, **XXIV**, 1141-1177.

Hanushek, E.A. (2002). Publicly provided education. In A. Auerbach and M. Feldstein (eds.). *Handbook of Public Economics.* Amsterdam: North Holland, 2047-2143.

International Telecommunication Union (2003). ITU digital access index: world's first global ICT ranking. In www.itu.int/newsroom/press_releases.

Karni, E. and Zilcha, I. (1995). Technological progress and income inequality. *Economic Theory,* **5**, 277-294.

Knight, J. and Shi, L. (1996). Educational attainment and the rural-urban divide in China. *Oxford Bulletin of Economics and Statistics,* **58**(1), 83-117.

Laitner, J. and Juster, F.T. (1996). New evidence on altruism: a study of TIAA-CREF retirees. *American Economic Review,* **86**, 893-906.

Laitner, J. (1997). Intergenerational and interhousehold economic links. In M.R. Rosenzweig and O. Stark, (eds.). *Handbook of Population and Family Economics.* Amsterdam: North Holland, 189-238.

Lee, J.-W. and Barro, R.J. (2001). Schooling quality in a cross-section of countries. *Economica,* **68**, 465-488.

Loury, G. (1981). Intergenerational transfers and the distribution of earnings. *Econometrica,* **49**(4), 843-867.

Munandar, H. (2006). *Essays on Economic Integration.* PhD thesis No. 391, Tinbergen Institute (Rotterdam).

Orazem, P. and Tesfatsion, L. (1997). Macrodynamic implications of income-transfer policies for human capital investment and school effort. *Journal of Economic Growth,* **2**, 305-329.

Persson, T. and Tabellini, G. (1994). Is inequality harmful to growth?. *American Economic Review,* **84** (3), 600-621.

Persson, T. and Tabellini, G. (2000). *Political Economics.* Cambridge: MIT Press.

Restuccia, D. and Urrutia, C. (2004). Intergenerational persistence of earnings: the role of early and college education. *American Economic Review*, **94**(5), 1354-1378.

Rubinstein, Y. and Tsiddon, D. (2004). Coping with technological change: the role of ability in making inequality so persistent. *Journal of Economic Growth,* **9**, 305-346.

Shaked, M. and Shanthikumar, J.G. (1994). *Stochastic Orders and Their Applications.* Boston: Academic Press.

Spilimbergo, A., Londono, I. and Szekely, M. (1999). Income distribution, factor endowments and trade openness. *Journal of Development Economics,* **59**, 77-101.

Viaene, J.M. and Zilcha, I. (2002). Capital markets integration, growth and income distribution. *European Economic Review,* **46**, 301-327.

Viaene, J.M. and Zilcha, I. (2003). Human capital formation, income inequality and growth. In T. Eicher and S. Turnovsky (eds.). *Growth and Inequality: Issues and Policy Implications.* Cambridge: MIT Press, 89-118.

Viaene, J.M. and Zilcha, I. (2007). Human capital and inequality dynamics: the role of education technology. The Sapir Center, Working Paper 6-07, Tel Aviv University.

Woessmann, L. (2003). Educational production in East Asia: the impact of family background and schooling policies on student performance. IZA Discussion Paper No. 745.

World Bank (2001). *World Development Report 2000/2001: Attacking Poverty.* New York: Oxford University Press

APPENDIX

Proof of Proposition 1:

Consider the following two equations attained from (15) and (16) :

$$y_{t+1}(\omega) = C_t[h_t^v(\omega) + \tfrac{\beta_2}{\beta_1} e_{gt}\overline{h}_t^{\eta}] \text{ for all } \omega \notin A_t,$$

$$y_{t+1}(\omega) = C_t[\beta_2 e_{gt}\overline{h}_t^{\eta}] \text{ for all } \omega \in A_t.$$

Similarly,

$$y_{t+1}^*(\omega) = C_t^*[h_t^{*v}(\omega) + \tfrac{\beta_2}{\beta_1} e_{gt}\overline{h}_t^{*\eta}] \text{ for all } \omega \notin A_t^*,$$

$$y_{t+1}^*(\omega) = C_t^*[\ \beta_2 e_{gt}\overline{h}_t^{*\eta}] \text{ for all } \omega \in A_t^*,$$

where C_t and C_t^* are some positive constants. Since h_0 and h_0^* are equally distributed, the same holds for $h_0^v(\omega)$ and $[h_0^*(\omega)]^v$, since $v \leq 1$. Moreover, since $\overline{h}_0 < \overline{h}_0^*$ we obtain that $h_1^*(\omega)$ is more equal than $h_1(\omega)$ (see Lemma 1 in Karni and Zilcha, 1995). It is easy to verify from (16) that $h_1(\omega)$ are lower than $h_1^*(\omega)$ for all ω. Note that since $y_1^*(\omega) = C_0^*\ \beta_2 e_{gt}\overline{h}_t^{*\eta}$ for all $\omega \in A_0$ and $y_1(\omega) = C_0\beta_2 e_{gt}\overline{h}_t^{\eta}$ for all $\omega \in A_0^*$ and on these sets $y_1^*(\omega) > y_1(\omega)$ the above argument is not affected by the existence of A_0 and A_0^* with positive measure. In particular we obtain that $[h_1^*(\omega)]^v$ is more equal than $[h_1(\omega)]^v$ (see Theorem 3.A.5 in Shaked and Shanthikumar, 1994). Also we have $[\overline{h}_1]^{\eta} < [\overline{h}_1^*]^{\eta}$. This implies, using (16), that $h_2^*(\omega)$ is more equal than $h_2(\omega)$. It is easy to see that this process can be continued to generalize this to all periods.

Proof of Proposition 2: Let us use the fact that in our model the inequality in incomes originates from the inequality in human capital distribution, since the same wage rate multiplies $h_t(\omega)$ (see (15)). Now the trade and physical capital flow will result in equal wages and rates of interest in

both countries. Moreover, we claim that in such a case there is no effect on the optimal choices of parental investment in their children; namely, that $e_t(\omega)$ will not vary. This can be verified directly from (12), after substituting $y_{t+1}(\omega)$ by (15) : given $h_t(\omega), e_t(\omega)$ and hence $h_{t+1}(\omega)$ will not vary as we change r_{t+1} and w_{t+1}as well. Thus the human capital accumulation process will not vary and the sets A_t as well (see inequality (18)). Now, consider (16) and (17) to verify that the distribution of $h_{t+1}(\omega)$ will not change for $t = 0, 1, 2....$

Proof of Proposition 3: The proof is similar to that of Proposition 6 in Viaene and Zilcha (2002), hence it is omitted.

Proof of Proposition 5: First let us show first that in each generation individuals with a higher level of human capital choose at the optimum higher level of time to be allocated to the private education of their offspring. To see this let us derive from the first order conditions, using some manipulation, the following equation:

$$\text{(A1)} \qquad 1 - [1 + \frac{\beta_1 \alpha_4}{\alpha_3}]e_t(\omega) = \frac{\alpha_4 \beta_2}{\alpha_3} e_{gt} \overline{h}_t^{\eta} [h_t^{-\upsilon}(\omega)] \qquad \text{for } \ e_t(\omega) > 0$$

which demonstrates that higher $h_t(\omega)$ implies higher level of $e_t(\omega)$. Let us show that such a property generates less equality in the distribution of $y_{t+1}(\omega)$ compared to that of $y_t(\omega)$. It is useful however, to apply (15) for this issue. In fact it represents the period $t + 1$ income $y_{t+1}(\omega)$ as a function of the date t income $y_t(\omega)$ via the human capital evolution. Define the function $Q : R \to R$ such that $Q[h_t(\omega)] = h_{t+1}(\omega)$ using (16) whenever $\omega \notin A_t$, and when $\omega \in A_t$ this function is defined by: $Q[h_t(\omega)] = \beta_2 e_{gt} \overline{h}_t^{\eta}$. This function

is monotone nondecreasing and satisfies: $Q(x) > 0$ for any $x > 0$ and $\frac{Q(x)}{x}$ is decreasing in x. Therefore (see Shaked and Shanthikumar, 1994), the human capital distribution $h_{t+1}(\omega)$ is more equal than the distribution in date t, $h_t(\omega)$. This implies that $y_{t+1}(\omega)$ is more equal than $y_t(\omega)$.

As we saw earlier it is sufficient to prove this result under the assumption that $e_t(\omega) > 0$ for all $\omega \in G_t$. When this is not the case, raising e_{gt} entails higher income for all low income individuals $\omega \in A_t$ which only reinforces the claim. Let us consider (1) for $t = 0$. Since $h_0(\omega)$ is given, $h_0^v(\omega)$ and $\overline{h}_0$ are fixed. By raising e_{g0} the distribution of the human capital for generation 1, $h_1(\omega)$ becomes more equal. This follows from Lemma 1 in Karni and Zilcha (1995). Moreover, we claim from (16) that the average human capital in generation 1 increases as well. Increasing e_{g0} will result in higher $h_1(\omega)$ for all ω and higher level of $\overline{h}_1$. Moreover, it also implies that $h_1^v(\omega)$ will have a *more equal* distribution (see Shaked and Shanthikumar, 1994; Theorem 3.A.5).

Now, let us consider $t = 1$. Increasing e_{g1} will imply the following facts: $h_1^v(\omega)$ becomes more equal and $\beta_2 e_{g1}\overline{h}_1^{\eta}$ is larger than its value before we increased the level of public education. Using (16) and the same Lemma as before we obtain that $h_2(\omega)$ becomes more equal. This process can be continued for $t = 3, 4, ...$, which establishes our claim. Now let us consider the set of families with $e_t(\omega) = 0$. To simplify our argument assume that *initially* $e_{g0} = 0$, then as e_{g0} increases $h_1(\omega)$ will be equal or larger than in the private provision case for all $\omega \in G_1$, where $\omega \in A_0$. Namely, we claim

that:

$$\text{(A2)} \qquad \beta_2 e_{g0}\overline{h}_0^{\eta} \geq \beta_1 e_0(\omega) h_0^{\upsilon}(\omega) \qquad \text{for all } \ \omega \in A_0$$

Let us substitute $e_0(\omega)$ and using the upper bound for $h_0^{\upsilon}(\omega)$ from (18), we see that this inequality always holds since, by assumption, $\upsilon \leq \eta$. This fact certainly reinforces the proof of our earlier case since at the lower tail of the distribution of income we *raised* and *equalized* the income for all $\omega \in G_1$, where $\omega \in A_0$. This process can be continued for all generations.

Proof of Proposition 6: Let the initial distribution of human capital $h_0(\omega)$ be given. Compare the following two equilibria from the same initial conditions: One with the human capital formation process given by (1) and another with the same process but β_2 is replaced by a larger coefficient $\beta_2^* > \beta_2$. Clearly, we keep β_1 unchanged. Consider again the following expressions for our individual income:

$$y_{t+1}(\omega) = C_t[h_t^{\upsilon}(\omega) + \tfrac{\beta_2}{\beta_1} e_{gt}\overline{h}_t^{\eta}] \quad \text{for all } \omega \notin A_t$$
$$y_{t+1}(\omega) = C_t[\tfrac{\beta_2}{\beta_1} e_{gt}\overline{h}_t^{\eta}] \quad \text{for all } \omega \in A_t$$
$$y_{t+1}^*(\omega) = C_t^*[h_t^{*\upsilon}(\omega) + \tfrac{\beta_2^*}{\beta_1} e_{gt}\overline{h^*}_t^{\eta}] \quad \text{for all } \omega \notin A_t$$
$$y_{t+1}^*(\omega) = C_t^*[\tfrac{\beta_2^*}{\beta_1} e_{gt}\overline{h^*}_t^{\eta}] \quad \text{for all } \omega \in A_t$$

Since $h_0(\omega)$ is fixed at date $t = 0$ we find (using once again the Lemma from Karni and Zilcha, 1995) that $\frac{\beta_2^*}{\beta_1} > \frac{\beta_2}{\beta_1}$ imply that $y_1^*(\omega)$ is more equal to $y_1(\omega)$. We also derive that $h_1(\omega)$ are lower than $h_1^*(\omega)$ for all ω and, hence, $\overline{h}_1 < \overline{h}_1^*$. This inequality reinforces the result when $\mu(A_0) > 0$. By (16), using the same argument as in the last proof, $h_1^{*\upsilon}(\omega)$ is more equal than $h_1^{\upsilon}(\omega)$ and $\frac{\beta_2^*}{\beta_1} e_{g1}\overline{h}_1^{*\eta} > \frac{\beta_2}{\beta_1} e_{g1}\overline{h}_1^{\eta}$, hence $h_2^*(\omega)$ is more equal than $h_2(\omega)$.

This same argument can be continued for all dates $t = 3, 4, 5, ...$ Also note that $A_t \subset A_t^*$ (where A_t^* is the set of families in G_t who choose $e_t(\omega) = 0$) since $\frac{\beta_2^*}{\beta_1} e_{gt} \overline{h}_t^{*\eta} > \frac{\beta_2}{\beta_1} e_{gt} \overline{h}_t^{\eta}$ for all t. This only contributes to the more equal distribution of $y_{t+1}^*(\omega)$ since the left hand tail has been increased and equalized compared to the $y_{t+1}(\omega)$ case.

To complete the proof of part (a) of this Proposition consider the case where we increase η. When we increase the value of η, keeping all other parameters constant, we are basically increasing the second term in (16), $[\overline{h}_0]^{\eta}$, while $[h_0(\omega)]^v$ remains unchanged. By Lemma 1 in Karni and Zilcha (1995) we obtain that the distribution of $h_1(\omega)$ becomes more equal. Taking into account the families $\omega \in G_1$ who belong to A_0 (i.e., the lower tail of the distribution of income) only reinforces the higher equality since their incomes are uniformly increase to $\beta_2 e_{g1} \overline{h}_0^{*\eta}$, while for all other $\omega \in G_1$, $\omega \notin A_0$ the proportional raise in their income is smaller. This can be continued for $t = 2$ as well since it is easy to verify that $[\overline{h}_1]^{\eta}$ increases while $[h_1(\omega)]^v$ becomes more equal. Now, this process can be extended to $t = 2, 3, ...$, which complete the proof of part (a).

The proof of part (b) follows from the same types of arguments using the fact that if $\beta_1 < \beta_1^*$ then $\frac{\beta_2}{\beta_1} > \frac{\beta_2}{\beta_1^*}$ and, hence, $h_1(\omega)$ is more equal than $h_1^*(\omega)$ and $\overline{h}_1 > \overline{h}_1^*$. This process leads, using similar arguments as before, to $y_t(\omega)$ more equal than $y_t^*(\omega)$ for all periods t.

Claim: *Compare two economies which differ only in the parameter v. The economy with the higher v will have more inequality in the intragenerational income distribution in all periods.*

Since the two economies have the same initial distribution of human cap-

ital $h_0(\omega)$ the process that determines $h_1(\omega)$ differs only in the parameter v. Denote by $v^* < v \leq 1$ the parameters, then it is clear that $[h_0(\omega)]^{v^*}$ is more equal than $[h_0(\omega)]^v$ since it is attained by a strictly concave transformation (see Theorem 3.A.5 in Shaked and Shanthikumar, 1994). Likewise, the human capital distribution $h_1^*(\omega)$ is more equal than the distribution $h_1(\omega)$. This implies that $y_1^*(\omega)$ is more equal than $y_1(\omega)$. Now we can apply the same argument to date 1: the distribution of $[h_1^*(\omega)]^{v^*}$ is more equal than that of $[h_1(\omega)]^v$, hence, using (16) and the above reference, we derive that the distribution of $[h_2^*(\omega)]^{v^*}$ is more equal than that of $[h_2(\omega)]^v$. This process can be continued for all t.

Consider now the claim in part (c). From (16) we see that inequality in the distribution of $h_1(\omega)$ remains unchanged even though all levels of $h_1(\omega)$ increase due to this technological improvement. In particular, $\overline{h}_1$ increases. Now, since inequality of $h_1^v(\omega)$ did not vary but the second term in the RHS of (16) has increased due to the higher value of $\overline{h}_1$, we obtain more equal distribution of $h_2(\omega)$. When $\mu(A_0) > 0$ the higher $\overline{h}_1$results in higher income to all $\omega \in G_1$ who belong to A_0, which only reinforces the more equality in $y_2^*(\omega)$. Now, this argument can be used again at dates 3, 4, ..., which completes the proof.

THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH and
THE SACKLER INSTITUTE FOR ECONOMIC STUDIES
The Eitan Berglas School of Economics
Tel Aviv university

Recent list of Working Papers

1-2008	Limor Hatsor	The Allocation of Public Education Resources
2-2008	Anat Bracha Donald J. Brown	Affective Decision Making: A Behavioral Theory of Choice
3-2008	Saul Lach Gil Shiff Manuel Trajtenberg	Together but Apart: ICT and Productivity Growth in Israel
4-2008	Ariel Rubinstein	Comments on Neuroeconomics
5-2008	Tomer Blumkin Yoram Margalioth Efraim Sadka	Incorporating Affirmative Action into the Welfare State
6-2008	Chemi Gotlibovski Yoram Weiss	Retirement Decisions of Married Couples
7-2008	Ayala Arad	The Tennis Coach Problem: A Game-Theoretic and Experimental Study
8-2008	Victor Lavy M. Daniele Paserman Analia Schlosser	Inside the Black of Box of Ability Peer Effects: Evidence from Variation in Low Achievers in the Classroom
9-2008	Hosny Zoabi	Talent Utilization, a Source of Bias in Measuring TFP
10-2008	Tomer Blumkin Yoram Margalioth Efraim Sadka	The Role of Stigma in the Design of Welfare Programs
11-2008	Jean-Marie Viaene Itzhak Zilcha	Human Capital and Inequality Dynamics:The Role of Education Technology

---

Foerder Institute for Economic Research, Tel-Aviv University, Tel-Aviv, 69978 Israel, Tel: 972-3-640-9255; fax: 972-3-640-5815; e-mail: foerder@post.tau.ac.il
Papers from 2000 onwards (and some from 1998 and1999) can be downloaded from our website as follows:
http://econ.tau.ac.il/research/search_workingPapers.asp