Patents, Imitation and Licensing In an Asymmetric Dynamic R&D Race

By

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In an Asymmetric Dynamic R&D Race

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Abstract

R&D is an inherently dynamic process which involves different intermediate steps that need to be developed before the completion of the final invention. Firms are not necessarily symmetric in their R&D abilities; some may have advantages in early stages of the R&D process while others may have advantages in other stages of the process. The paper uses a simple two-firm asymmetric ability multistage R&D race model to analyze the effect of different types of patent policy regimes and licensing arrangement on the speed of innovation, firm value and consumers' surplus. The paper demonstrates the circumstances under which a weak patent protection regime, which facilitates free imitation of any intermediate technology, may yield a higher overall surplus than a regime that awards patent for the final innovation. This result holds even in cases where the length of the patent is optimally calculated.

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1. Introduction

Patents are designed to provide incentives for innovation. The conventional wisdom is that unless we protect innovators from imitation, there will be less-than-optimal investment in innovation. Recently, however, this rationale has been challenged. There are evidences that the software and computers industries were most innovative in particular during the period of weak patent protection during which these industries experienced rapid innovation (see Bessen and Maskin (2002), Hunt (2004), Gallini (2002), O'Donoghue (1998) and Scotchmer (1996)).[3]

R&D races are inherently dynamic processes that take place over time; during that period, firms may adjust their R&D investments given their assessments about their relative position in the race.[4] The race typically involves development of different intermediate inventions or complementary technologies that may enable the firm to complete the invention. Firms are not necessarily symmetric in their R&D abilities. Some firms may display better abilities in several stages of the R&D race while other firms may exhibit better abilities in other stages of the race.

Multistage races are convenient settings that capture the knowledge accumulated process during the race. In these settings, each firm needs to go through several stages of R&D in order to complete the invention (see Fudenberg et al. (1983), Harris and Vickers (1985, 1987), Grossman and Shapiro (1987), and Lippman and McCardle (1987)). While we adopt the multistage R&D setup, we do not take the patent policy as given. In this respect our paper is closely related to Judd, Schmedders and Yeltekin (2002) who analyze a dynamic multi-stage innovation race and derive the optimal intermediate stage at which patents should be rewarded.

The focus of our paper is on the effect of possible imitation on a multistage asymmetric ability innovation race. In particular we investigate the possibility that weak patent policies that facilitate imitation induce higher value for firms and higher consumers' surplus. We use a simple two-firm multistage R&D race assuming that firms have different abilities at different stages of the race. Abilities are translated into different costs of R&D

[3] See also the survey in Cohen, Nelson and Walsh (2000).

[4] For a dynamic R&D models see for example Reinganum (1981,1982), Judd (1985), Malueg and Tsutsui (1997) and Doraszelski (2003) and the survey Reinganum (1989).

investment or, alternatively, different effectiveness of R&D investment. Profits can be made only after the completion of the innovation.[5] We consider several possible patent policies: (i) a race with a patent at the end (end patent), (ii) a race in which no patent is awarded and the final innovation can be costlessly imitated by competing firms, (iii) a race in which a patent is awarded at the end of the race but only for several periods - the length of the patent is determined optimally to maximize consumers' surplus, (iv) an R&D race in which a patent is rewarded at a pre-specified intermediate stage, (v) a race in which any intermediate technological discovery by one firm can be costlessly imitated by its competitors (hereinafter CTI).[6,7]

We provide a numerical analysis of these R&D races using a variant of the value function algorithm suggested in Pakes and McGuire (1994). We solve the Markov perfect equilibrium of these races repeatedly for a wide range of parameters' value for each one of the above patent regimes. We then compare the speed of innovation, firms' value, consumers' surplus and investment for each type of race. Our main finding is that for a wide range of parameters the weakest patent regime which enables firms to imitate any intermediate technological discovery yields the highest consumers' surplus as well as the highest value for firms. This result depends in particular on the nature of the duopolistic interaction and the type of asymmetry between the firms. In particular we show that such a regime may even yield higher consumers' surplus and firms' value than a patent regime in which the length of the patent is optimally calculated. While in the CTI case the prize at the end of the race is lower as the race always ends up with a duopoly, the R&D process is more efficient. The ability to imitate an intermediate technology is a form of cooperation in which one firm may gain from the success of the other. Moreover, one firm may gain when its discovery is imitated as it provides her with an opportunity to imitate the future discovery of its rival. These advantages are important in a multiple stages and asymmetric abilities races.

The possibility of patenting plus licensing an intermediate technology may also enable the firms to take advantage of their asymmetric abilities and introduce some cooperative

[5] Two types of R&D races are discussed in the literature. The first is a product improvement race in which firms may profit from an existing product and the R&D is aimed to improve the product or reduce its costs. The second type is an innovation race in which firms develop a new innovation/product although they reap no profits during the race itself (unless there is licensing).

[6] Clearly free and costless imitation is not always possible. But we view this as a benchmark case.

[7] Note that in a one-stage race, case (v) is equivalent to case (iii); in our analysis, however, they generate completely different patterns of R&D.

aspect into the R&D race. We consider an R&D race in which firms can license their intermediate technology patents. Licensing occurs whenever it creates a surplus and we assume that the terms of the licensing are determined by an equal sharing rule**.** Licensing may eliminate the firm's leadership position in the race - and indeed in the symmetric case of our model there is no licensing. We compare the outcomes of the CTI type race with the R&D race with licensing of intermediate patents and demonstrate the conditions under which each one of them yields better outcomes.

2. Asymmetric R&D Races

We start by presenting our benchmark multistage R&D race model in which firms are required to complete the development of several stages prior to completion of the invention. In the benchmark case patent is awarded to the first firm that completes this process and imitation is either not feasible or prohibited. We then specify different modifications of the benchmark case allowing for different patent regimes.

2.1 Benchmark Model: A Multistage R&D Race with End Patents

Our benchmark model is a simple asymmetric multistage R&D race in which patent is awarded to the first firm that completes the development of all relevant stages. An innovation is the outcome of N stages of intermediate technologies denoted by $\{k_1,..,k_N\}$. Moving from step k_n to step k_{n+1} is a stochastic process depending on the firm's investment. Letting $x_i \geq 0$ be firm i's investment or effort, we assume that the probability of success (i.e., moving from stage k_n to stage k_{n+1}), denoted by $p(x_i)$, is increasing with x_i. If a firm is unsuccessful in moving from k_n to k_{n+1} in one period, it can try again in the next period.[8]

The firms that participate in the race are not symmetric in their R&D abilities. We capture this asymmetry by allowing for different investment cost profiles, that is, the cost of investment may differ between firms and across stages. Let $c^i = (c_1^i,...,c_N^i)$ be firm i's cost profile, which captures the firm's abilities at different stages of the R&D process. If firm i invest x_i in moving from stage k_n to stage k_{n+1}, then the cost of such investment would be $c^i_n x_i$.

[8] An alternative formulation would be to consider such a race with learning such that any attempt to move from one stage to the other provides information on the likelihood of having at the end a successful innovation, see Malueg and Tsutsui (1997).

Thus, a lower $c^i{}_n$ implies a greater ability at stage n of the race. We allow for different levels of ability asymmetries between the firms such that one firm has greater advantages at some stages while the other has advantages at other stages.

An R&D race will be defined as $R \equiv \{\eta, N, L, (c^1,...,c^n), M(\{1,...,\eta\}^2)\}$ such that η is the number of firms, N is the number of stages in the race, L is the stage at which a patent is rewarded where $L \leqslant N$; $(c^1,...,c^n)$ are the firms' costs or abilities; $M(\{1,..., \eta\}^2)$, the prize that the firms get depending on the set of firms that complete the invention (since we consider also races with imitation and licensing, there might be several firms that have the final innovation).

For tractability we simplify our framework and consider only a two-firm race. The final product is sold in the market but no profits can be made from any intermediate technology (unless there is licensing of technology). If a single firm holds a patent, then its reward is π^M, the monopolistic profits, while the other firm makes zero profit. In a duopolistic market, each firm will obtain the profits, π^D; $2\pi^D < \pi^M$. When the two firms reach the patent stage at the same time, we assume that each will obtain the patent with probability 0.5. Firms maximize discounted payoffs and we let β be the firms' common discount factor.

Following most of the literature, we consider the Markov Perfect Equilibrium (MPE) of the race (see Maskin and Tirole (2001)). At every period, firms need to decide - simultaneously - on their investment level given the state of the race, (k_l, k_m), and their respective cost structures. Investment strategies are defined for every state of the race regardless of how this state has been reached. To simplify notation we take (l, m) to mean that firm 1 is at stage k_l and firm 2 at stage k_m.

A *Markov Perfect Equilibrium* for a two-firm race game *R* is defined by

- Investment strategies $x_i^*(l,m)$ for i=1,2 and very possible *(l,m)*.
- Value functions $V_i(l,m)$ for i=1,2 and very possible *(l,m)* (for convenience, we suppress dependence of the value function on the firms' cost structures and the different prizes).

Such that:

(i) The strategies $x_i^*(l,m)$ are optimal given the value functions $V_i(l,m)$.

(ii) For every state *(l,m)*, the value functions describe the present value of profits realized when both firms play the equilibrium strategies $x_i^*(l,m)$.

In calculating the value functions $V_1(l,m)$ and $V_2(l,m)$, we make repeated use of the following Bellman equation:

$$(1)\ V_1(l,m)=\begin{cases}\pi^M & l=N, m<N\\ 0.5\pi^M & l=N, m=N\\ \max\limits_{x_1\geq 0}\left\{-c_1(l)x_1+\beta\sum\limits_{l',m'}V_1(l',m')p(l'\mid x_1)p(m'\mid l,m)\right\} & l<N, m<N\\ 0 & l<N, m=N\end{cases}$$

Where l' is k_l+1 with probability $p(x_1)$ and k_l with probability $1\text{-}p(x_1)$ (m' is defined in a similar manner).

2.2 Patent Policy, Imitation and R&D Races

While the end patent (with limited patent length) is the most familiar form of patent regime, one can think of different plausible variations. We consider in this paper several different patent protection regimes. The list is clearly not exhaustive and there are many possible variations of these regimes.

1. R&D Race with an End Patent (hereafter E-Pat): This is our benchmark model described in Section 2.1.

2. R&D Race with No Patent Protection for the Final Product (hereafter E-Imit): This is an R&D race in which there is no patent protection and firms can freely imitate the final innovation of other firms. Imitation occurs only at the last stage of the R&D race and is followed by a duopolistic market. Formally, the race resembles our benchmark case but we need to modify the Bellman equation (1) by letting the value function be $V(l,N)=V(N,l)=V(N,N)=\pi^D$ for all $l \leq N$.[9]

3. R&D Race with End Patent of Optimal Length (hereinafter Opt-Pat): In the E-Pat case patent protection can be too strong while in the E-Imit case it may be too weak. We thus consider a case in which a patent is awarded to the first firm that completes the invention but the patent has a limited length. Patent will be awarded for *n* periods which will be followed by free imitation. The length of the patent protection will be calculated such that it would maximize consumers' surplus.[10] The optimal length, *n*, may vary with the parameters of the R&D race.

4. R&D Race with Intermediate Patenting (hereafter Mid-Pat): An alternative patent regime would award patent to intermediate technology rather than to the final innovation. After obtaining the intermediate patent, the race is over but the winning firm has to carry its R&D efforts to complete all the remaining stages before enjoying any profits. The firm may do so however, at a slower pace because competition is over. We assume that the patentable intermediate technology is at stage L (where $L<N$). Judd, Schmedders and Yeltekin (2002) consider the optimal intermediate stage at which patent should be awarded. Note that when costs are asymmetric, having an intermediate patent may provide advantages to the firm that enjoyed greater ability (or lower cost) at earlier stages of the race.

Formally, the analysis of the intermediate patenting case is similar to our benchmark E-Pat case. The first modification is the stage at which patent is awarded, L instead of N. The second modification is the prize itself. In the E-Pat case the prize is the monopolistic profits while in the Mid-Pat case the prize is the expected profits of the continuation single firm race.

5. R&D Race with Complete Technology Imitation (hereinafter CTI): Finally consider an R&D race in which each development of intermediate technology is observable and there is no patent protection for such technologies which implies an immediate imitation of any R&D

[9] While we assume in this case the possibility of immediate imitation, it is easily possible to generalize the analysis to situations in which imitation is delayed or is costly. We only need to modify the prizes to account for these cases.

[10] Clearly we can repeat the analysis by adopting a rule of maximizing total surplus.

advances. In the CTI case firms may imitate, without any delay or cost, any technology invented by their competitors.[11] Among all the cases examined in this paper, this case probably represent the weakest patent protection scenario. Invention does not provide exclusive rights and imitation is free at any stage – implying that whatever the firm does, it cannot lead the race nor ever enjoy monopolistic profits.[12]

Note that not all the above cases are relevant in all R&D races. Imitation, for example, is not always feasible (sometimes only at a cost). In other cases the firms may not observe the (intermediate) technology available to their rival and thus may be unable to condition their R&D effort on the state of the race etc.

3. Results

3.1 Details of the Numerical Analysis

We provide a numerical analysis of our R&D race using a variant of the value function algorithm suggested in Pakes and McGuire (1994).[13] We solve the race repeatedly for a wide range of parameters' value for each one of the above patent regimes.

We consider a two-firm race and set the number of innovation steps, N, at 6. We let the probability of success $p(x_i)$ be defined as $p(x_i) \equiv ax_i/(1+ax_i)$ and consider the cost structure $c^1= (1,1,1,\gamma,\gamma,\gamma)$; $c^2= (\gamma,\gamma,\gamma,1,1,1)$ where $\gamma>1$. This cost structure implies that the first firm has an advantage at the first three stages of the R&D while the second firm has an advantage at the last three stages.[14]

When the innovation process is over, the winning firm obtains the monopolist prize π^M. This value is the net present value of the monopolistic profits in a market with a linear inverse demand function, p=D-q, and no production costs. When there are two firms producing the good the duopolistic profits are assumed to be $\pi^D=\mu\pi^M$. When μ=0.5 the firms

[11] For simplicity, we focus on the extreme case of free imitation. Clearly, there are cases in which imitation is costly and requires some reverse engineering.
[12] The CTI race and the E-Imit race are equivalent in a one-stage race model. However, the two regimes display an entirely different pace of R&D in the multistage case.
[13] For a different numerical algorithm see Judd (1985), Judd, Schedders and Yeltekin (2002) and Doraszelski (2003).
[14] While we have considered different variations of the cost structure, this structure provides the basic intuition in the most straightforward way.

share the monopolistic payoffs and when $\mu=0$ there is a Bertrand type price competition with zero equilibrium profits. However we will typically assume Cournot equilibrium payoffs but discuss different possible variations.

We consider the effect of three different variables on the outcome of the race. (i) α - market size (the market prizes will be $\alpha\pi^M$ and $\alpha\pi^D$, and consumers' surplus is adjusted accordingly). (ii) γ - which describes the cost asymmetry. (iii) μ - the intensity of the market competition i.e., the portion of the monopolistic payoffs that is captured in a duopolistic competition.

We adopt Pakes and McGuire (1994) algorithm to calculate the Markov Perfect equilibrium of the above race. For each state, the algorithm calculates the value functions by using the corresponding duopolistic and monopolistic profits and the Bellman equation (Eq. 1). Note, however, that our setting is simpler than the one presented in Pakes and McGuire. While Pakes and McGuire analyze an infinite horizon game, we study a finite race with a known prize at the end. Consequently, we know all firms' values at the end of the game, and can then use these values together with the iterative algorithm and backward induction to calculate the value of firms at other states of the race.

The algorithm is iterative and works as follows: First, the algorithm initiates the value function $V^0(l,m)$ and investment level $x^0(l,m)$ for states (l,m) with $max\{l,m\}=N$. $V^0(l,m)$ is initiated with the corresponding monopolistic and duopolistic profits based on the patent regime, and $x^0(l,m)$ is set to zero. States (l,m) where $l,m<N$ are initiated with an arbitrary value function $V^0(l,m)$ and investment level $x^0(l,m)$. The algorithm then works iteratively. To move from iteration k to iteration $k+1$, the algorithm takes the value function $V^k(.)$ and policy function $x^k(.)$ as its input and uses the Bellman equation (Eq. 1) to generate updated value and policy functions, separately for each firm. In each iteration, the algorithm first uses $V_1^k(.)$ and $x_2^k(.)$ from memory and solves equation (1) to calculate firm 1's investment strategy, $x_1^{k+1}(.)$. It then takes the calculated $x_1^{k+1}(.)$ and computes firm 1's value function, $V_1^{k+1}(.)$. The same calculations are then done to compute firm 2's $\{V_2^{k+1}(.), x_2^{k+1}(.)\}$. The algorithm iterates over the value functions and the investment strategies, and stops when $\{V^k(.), V^{k+1}(.)\}$; and $\{x^k(.), x^{k+1}(.)\}$ are very close point-wise between iterations.[15]

[15] Our stopping criteria is $\varepsilon=10^{-6}$

The equilibrium value functions and investment strategies (i.e., $V^*(.)$,$x^*(.)$) are then used to simulate the innovation process. We simulate different patent policies and compare their descriptive statistics. The descriptive statistics presented are the averages of the corresponding values over 100,000 simulations (for each parameters values).

For our benchmark case we will consider the case with $\pi^M = 900$ and for the duopolistic payoffs, π^D, we take the net present value of the Cournot duopolistic profits, $\pi^D = 400$.[16] Whenever the outcome of the race is such that one firm enjoys monopolistic payoffs for a number of periods, after which it enjoys duopolistic payoffs, we make the necessary modifications. The per-period monopolistic payoff is $r\pi^M$; the per period duopolistic payoff is $r\pi^D$ where r is the discount rate. Consumers' surplus is derived from the same assumed demand function. For our benchmark case the net present value of consumers' surplus is CS^M=450 whenever there is a single producer, and CS^D=800 for a duopolistic (Cournot type) market. We make the necessary modifications if market structure changes. For the benchmark case we further let a=0.1 and set the discount factor $\beta = 0.92$.

3.2 The Performance of the different Patent Regimes.

We first provide a comparison of the performance of the different patent policy regimes without getting into details regarding the firms' investment strategies. We compare the consumers' surplus, the value for firms and the duration of the race until invention. The comparisons are based on our simulated races.[17] We vary the values of the parameters α, γ and μ to obtain a better understanding of the race characteristics.

3.2.1 The market size effect:

Figure 1a,b,c presents the performance of the different patent regimes as a function of α. The parameter α is a market multiplier. It changes the size of the market (an alternative interpretation is a proportional reduction of investment cost). We maintain for this presentation the cost asymmetry at γ=1.2, and assume a regular Cournot profits whenever the market became duopolistic.

[16] We investigate different levels of payoffs by investigating the race under different levels of α.
[17] For each set of parameters we simulated the race 100K times and used the relevant summary statistics.

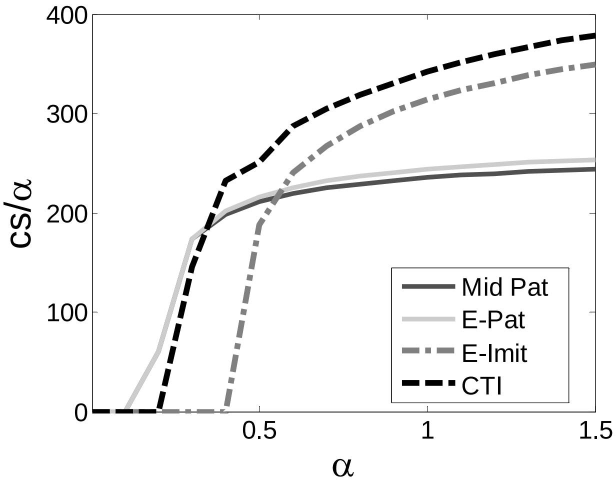

Figure 1a

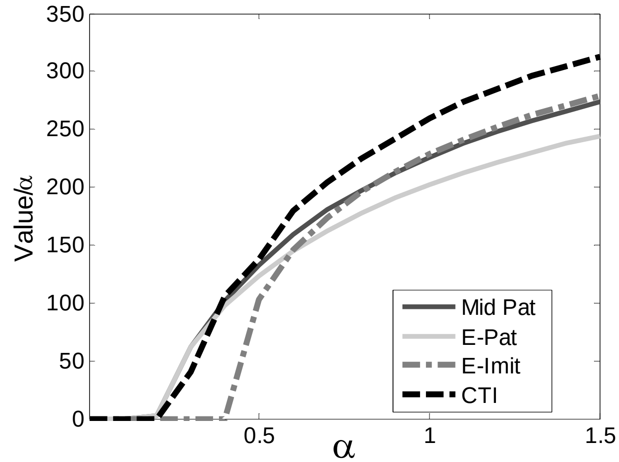

Figure 1b

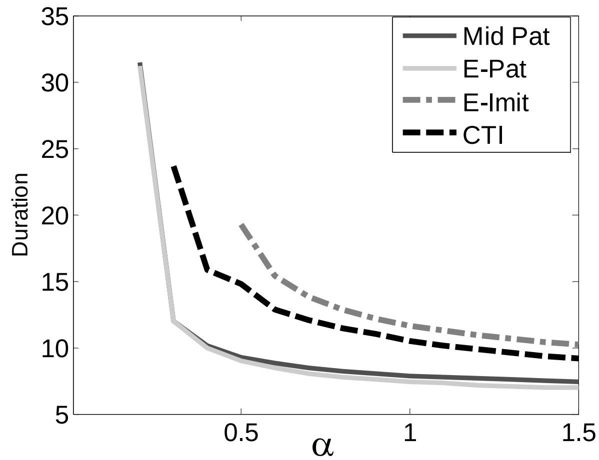

Figure 1c

Figure 1a illustrates the standard argument that justifies the need for patent protection. At a low range of α the CTI and E-Imit regimes do not provide sufficient incentives for the firms to invest in R&D. Note that for these levels of α, R&D investment is socially optimal and will be carried out under regimes with patent protection (like in the E-Pat and Mid-Pat regimes).

For larger values of α, Figure 1a illustrate the debate about the need to limit the monopoly power provided to patent holders (or actually to limit the length of patents). For large levels of α, CS is determined mainly by the resultant market structure. The regimes that end up with a duopoly yield higher CS than the regimes that provide patent protection and end up in a monopolistic market (even though these regimes lead to speedier innovations - see

Figure 1c). Consequently, both the CTI and E-Imit regimes do better, in terms of CS, than the regimes with patent protection.

Figure 1b depicts the value of the race for firms. For low levels of α, the E-Pat and Mid-Pat regimes provide higher values for firms than the regimes without patent protection. Nevertheless for most levels of α, the CTI regime provides the highest value for firms, while the E-Pat regime gives the lowest value. This result is somewhat surprising as the CTI regime ends up with a duopolistic market, and the E-Pat ends up with a monopolistic market. The intuition behind this result is as follows: the E-Pat regime provides high profits for the winning firm but zero profits for the second firm. This triggers high investment dissipating the innovation prize. This is also evident from Figure 1c – the E-Pat regimes results in faster inventions. While consumers benefit from the fast innovation, firms value is reduced because of the over-investment effect. For example, for $\alpha=1.5$ the average total investment in the E-Pat case is four times higher than the average total investment in the CTI case.[18]

3.2.2 The effect of R&D cost asymmetry.

We now turn to compare the performance of the different patent regimes when we vary the cost asymmetry parameter γ. The firms' R&D costs are defined as $(1,1,1,\gamma,\gamma,\gamma)$; $(\gamma, \gamma,\gamma,1,1,1)$. A larger γ implies an overall greater cost of R&D as well as greater cost asymmetry. The simulation presented in Figure 2 are done letting $\alpha=1$ and assuming Cournot duopoly profits.

[18] So far we consider only the average value for firms. Since we consider in this paper an asymmetric race, as will be discussed in the next section, firms may have different preferences over the different patent regimes.

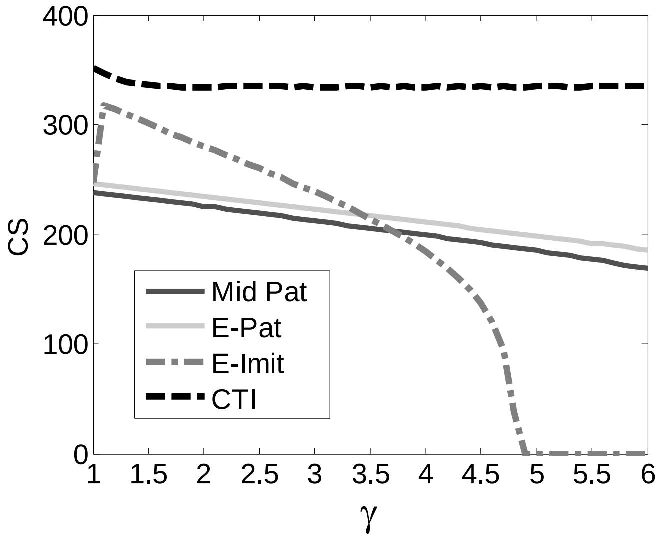

Figure 2a

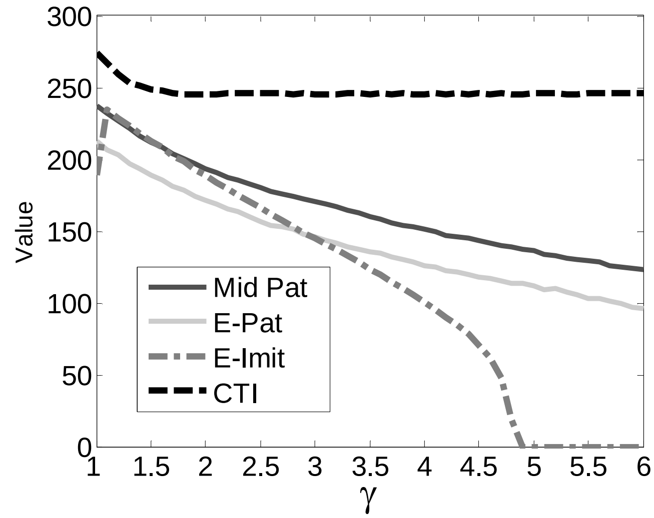

Figure 2b

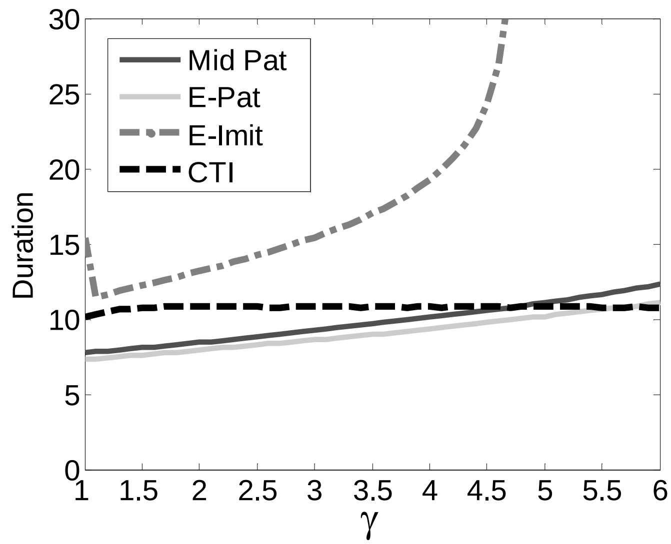

Figure 2c

Figures 2a-c show that when there is a significant cost asymmetry the CTI regime facilitates a complete specialization; each firm invests only in developing the technologies it has an advantage in. Thus, changing γ does not affect the firms' investment strategies. The CTI regime yields a higher CS and a higher firms' value than the other regimes. Furthermore, since in the E-Pat, Mid-Pat and E-Imit cases CS and firms' value decline with γ, increasing γ only increases the advantage of the CTI regime.

The E-Imit regime exhibits an interesting non-monotonic pattern. Moving from a symmetric cost structure, i.e., γ =1, to non-symmetric cost structure, i.e., γ>1, Figure 2b illustrates a sharp increase in both CS and firms' value and a sharp decline in the duration of the race despite the fact that there is a cost increase. The explanation of this phenomenon is that in the E-Imit case there is a strong second mover advantage. When the costs are identical, i.e., $\gamma = 1$, firms do not invest much at the beginning of the race hoping that their competitor will be lucky (and at the end they will be able to imitate his success). Once we move to the asymmetric case it is clear which firm should do the R&D investment and there is no reason to delay investment at the early stage.[19]

At a high γ (i.e., $\gamma > 5$) there is no investment in the E-Imit case. This situation is similar to the low α case and demonstrates once again the standard argument advocating for patent protection and prohibiting imitation.

3.2.3 The effect of market competition

We now turn to discuss the effect of μ, the intensity of the market competition, on the performance of R&D race under the different patent regimes. We let the duopolistic profits be $\pi^D = \mu\pi^M$. The Cournot equilibrium profits case, that was assumed so far, is equivalent to assuming $\mu = 0.44$. A low value of μ implies tougher market competition and lower duopolistic payoffs, which reduces the firms' incentives to innovate whenever an innovation is imitated and only duopolistic profits are realized. On the other hand when μ is close to 0.5, firms share the monopolistic profits which support high R&D investment even when innovation is easily imitated.

In Figure 3a,b,c we present the performance of the different patent regimes as a function of μ. For this comparison we fix the other parameters at $\alpha = 1$ and $\gamma = 1.2$.

[19] See more details in our numerical analysis in the next section.

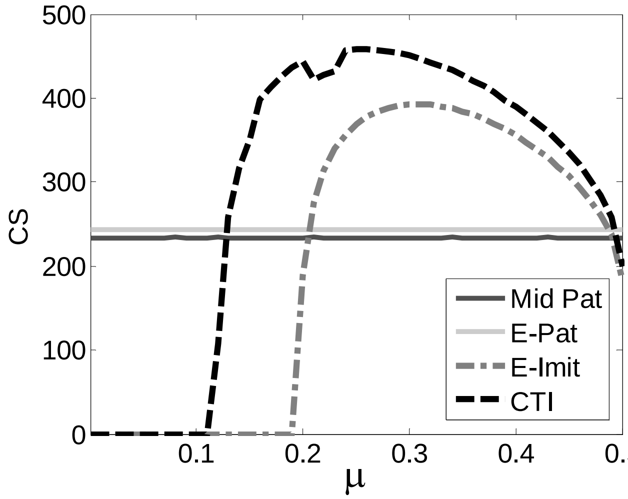

Figure 3a

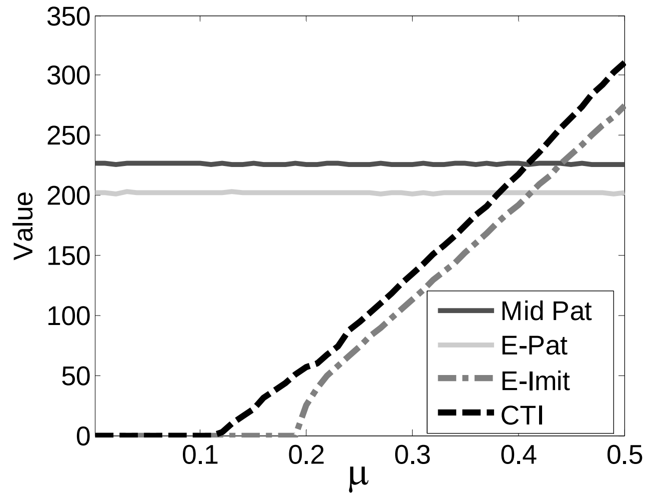

Figure 3b

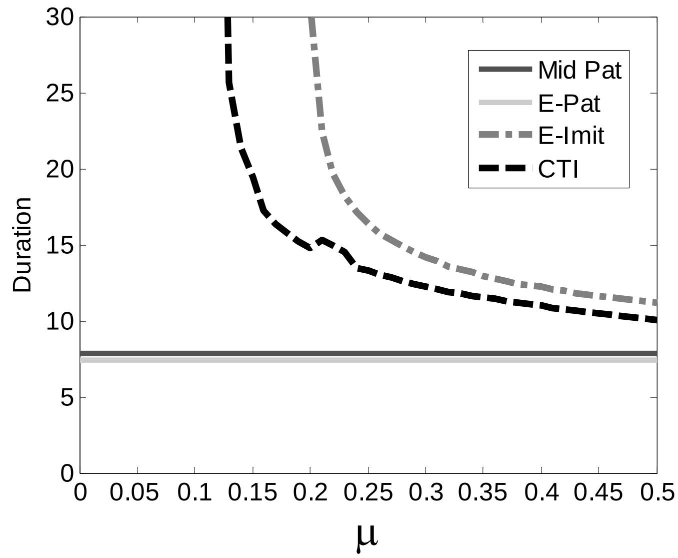

Figure 3c

The E-Pat and Mid-Pat races are not affected by μ as they never end up with a duopolistic market. Duopolistic market, in our setting occurs only as a result of imitation. For low levels of μ the possibility of imitation in the E-Imit and the CTI cases imply that firms do not have incentives to invest in R&D. In this case we have again the standard argument in favor of patent protection.

In medium levels of μ, the CTI regime yields a higher consumer surplus (but lower value for firms) than the regimes which provide patent protection. But when μ approaches 0.5 this effect is reversed. In this range, the duopolistic market structure does not contribute to consumers' surplus as the market outcome is close to the monopoly outcome. However, the possibility of imitation implies slower pace of innovation; resulting in a lower consumers' surplus.

The value for firms in the CTI and E-Imit cases are monotonically increasing with μ. In these cases the market structure is always duopolistic and a larger μ implies higher profits. For large μ the value for firms in the CTI and E-Imit cases are larger than in the E-Pat and Mid-Pat cases as firms do not lose much from having a duopolistic market but gain from a more efficient R&D process. Duration, as expected, is shorter whenever there are patents.

3.3 Optimal patent length

The patent regimes that we considered so far either allow for end (or mid) patents or assume free imitation. Most patent regimes however award patents for a fixed number of years. The length of a patent affects the firms' incentives to innovate but also limits the period in which the firm can exploit its monopolistic power. Finding the optimal patent length is the balancing of these two forces.[20]

We now consider an R&D race in which patent is provided for n periods, followed by a costless imitation by a second firm. The patentable innovation grants n periods of monopolistic market which follows by a duopolistic market.[21] We choose n optimally to maximize consumers' surplus and thus the optimal n depends on the parameters of the race.

In Figure 4a-c we compare the performance of the CTI with the optimal-patent regime. Note that for the optimal patent case, we choose for the different parameters values different n – the one that maximizes CS. In Appendix B we describe the optimal n as a function of the parameters of the race.

[20] For a discussion on optimal patent policy see Nordhaus (1969), Klemperer (1990), Gilbert and Shapiro (1990) and Denicolo (1999,2000).

[21] Note that the regimes of E-Pat and E-Imit are special cases where $n=\infty$ and 0 respectively.

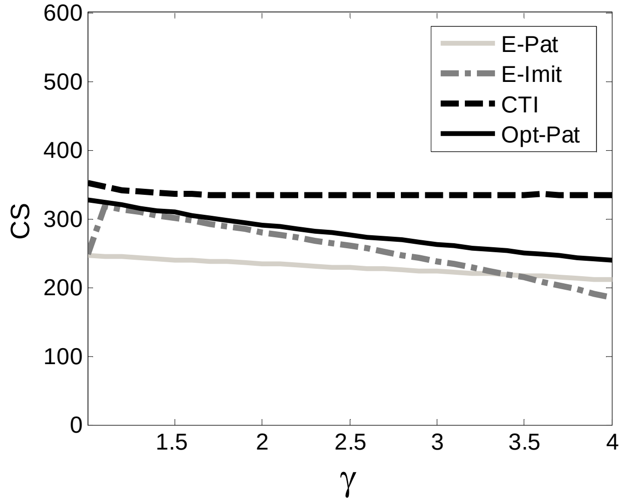

Figure 4a

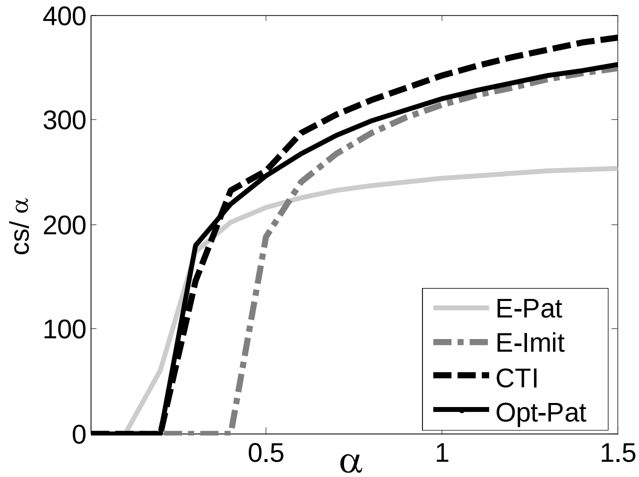

Figure 4b

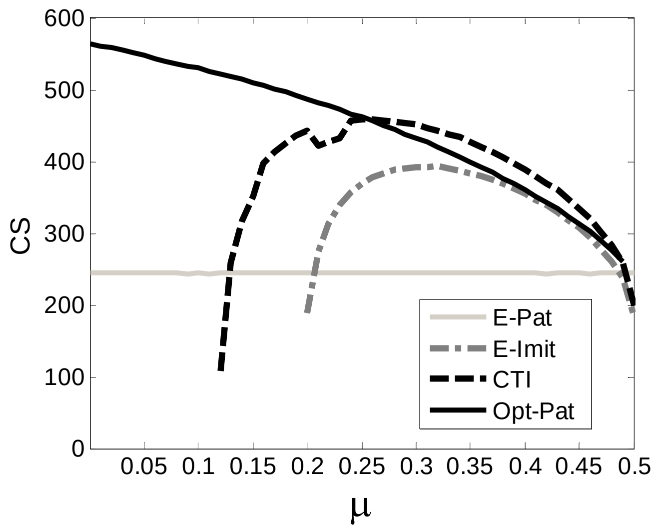

Figure 4c

The most striking conclusion of the above figures is that the CTI patent regime yields a higher CS than the Opt-Pat regime providing that μ is not too small or not too large.[22] Thus the advantage of the CTI regime over the E-Pat regime does not stem from the E-Pat regimes providing too much monopolistic protection or too little incentives to innovate. Rather, the fact that CTI is superior for a wide range of parameters is derived from the more efficient innovation process that it induces (see Section 4 for more details).

[22] Obviously, being special cases of the optimal patent length, the End-Pat and E-Imit regimes yield lower CS than the optimal patent length regime.

4. A Numerical Analysis of an Asymmetric R&D Race

So far we presented the summary performance of the R&D race under different patent regimes without going into details regarding the strategic interaction between the firms, and their investment levels at different stages of the race. In order to illustrate the reasons that lead to the superiority of the CTI regime over other patent regimes, we analyze in details a race with specific set of parameters.

We consider a multistage race with the cost structure of $\{c_1=(1,1,1,\gamma,\gamma,\gamma)$, $c_2=(\gamma,\gamma,\gamma,1,1,1)\}$. We set $\gamma=5$ and $\alpha=5$.[23,24] We ran this race 100K times and in Table 1 we present the summary statistics for this race. Inv1 and Inv2 are the investment of each of the two firms; TotInv is the firms' total investment. Dur_Prod1 is the average duration of the race prior to first production while Dur_Race is the number of periods until the race ends. When there is a patent and no licensing, Dur_Prod1=Dur_Race; otherwise, the difference is the number of periods until the second firm starts producing. The lines "Value" and "CS" give the average total value of the firms and the average consumers' surplus. Prob_1 and Prob_2 show the probability that firm 1 or 2 is the first firm to complete the invention while V1 and V2 give the value of the two firms in the race.

	Mid-Pat	E-Pat	E-Imit	CTI	Opt-Pat
Tot_Inv	904.3	1202.2	301.85	188.9	443.6
Inv1	448.4	567.5	301.85	103.5	241.4
Inv2	455.9	634.7	0	85.4	202.2
Dur_Prod1	7.25	6.7	9.3	7.9	8.1
Dur_Race	7.25	6.7	9.3	7.9	11.1
Tot_Val	1569.1	1382.9	1570.1	1902.4	1674.7
CS	1236.7	1292.6	1871.9	2091.3	1861.7
prob_1	0.65	0.53	1	0	0.6
prob_2	0.35	0.47	0	0	0.4
V1	1134.6	783.2	634.1	942.1	863.97
V2	434.5	599.7	935.96	960.3	810.76

Table 1: Asymmetric R&D race, α=5.

[23] We keep our benchmark case with π^M = 900 and the Cournot duopolistic payoffs π^D=400. The per-period monopolistic and duopolistic payoffs are $r\pi^M$ and $r\pi^D$ respectively. Consumers' surplus is derived from the same assumed demand function. Letting the net present value of consumers' surplus be CS^M=450 for the monopolistic case and CS^D=800 for a duopolistic (Cournot type) market. We make the necessary modifications if market structure changes. For the benchmark case we further let a=0.1 and set the discount rate factor β = 0.92.

[24] α and γ were chosen so that the results highlight the important attributes of each patent regime.

The advantages of the CTI regime: In the above race it is the CTI regime that leads to the highest level of CS and firms' value. The relevant levels are higher even than those in the Opt-Pat race. In order to explain this phenomenon we provide the details of the equilibrium investment strategies and value functions for the E-Pat, CTI and Opt-Pat cases (see Tables A1-A6 in the appendix).

The advantage of the CTI regime is derived mainly from the efficient investment process it induces. In the E-Pat race, each firm needs to develop all the six stages of innovation and gets a huge prize if it succeeds to do it before the other firm. As presented in Table A2, both firms invest heavily from the beginning even in technologies that they have disadvantage in. For example the investment at (l,m)=(0,0) is (71,155) in the E-Pat case, (25,46) in the Opt-Pat case and only (27,0) in the CTI case. On the other hand in the CTI case both firms benefit from any success in the innovation process. The value function increases any time there was a successful innovation regardless which firm did it (see Table A3). Thus, while the firms compete at the end in the product market the possibility to imitate at every stage of the race induces a type of "cooperative" innovation process even though there is no explicit cooperation or coordination between the firms.

As table A4 indicates, in the CTI case firms share the innovation process. Each firm invests only at the stages it has an advantage in; thus avoiding duplication and inefficient investment (as in the other patent regimes).[25] The cost asymmetry has an interesting role in such a race – as it allocates the investment effort at each stage only to the firm that is efficient for that technology. There is no need for a coordination device or RJV in such a case as in MPE effort is allocated according to the firms' technological abilities.

The advantage of the CTI regime exists only in races where the cost of innovation is substantial relative to the prize itself. When the gains from the efficient investment process is sufficiently large to offset the lower value derived from the duopolistic market structure, firms are indeed better off with the CTI regime. In the race reported in Table 1, overall cost of investment in the E-Pat race is 1202 while for the CTI case the cost is 188. The difference is

[25] Note that in the E-Imit case there is no duplication in the investment process but yet the process is not efficient as investment is not necessarily done by the most efficient firm. Clearly in a symmetric race this disadvantage disappears.

larger than the difference between the discounted monopolistic and duopolistic profits. Hence, both firms are better off with the CTI despite the fact that it leads to a duopolistic market.

The race with Opt-Pat is interesting as it shares some of the properties of both CTI and E-Pat races. After all, the succeeding firm gains a monopolistic power for couple of periods which follows by imitation and duopolistic market. The value function of each firm in such a race (Table A5) is not necessarily monotonic in the success of its rival. We can see from Table A5 that a success of a rival firm first reduces the value of the firm but then a further success only increases the value of the firm. There are two prizes in this race – monopoly profits for the winner and delayed duopolistic profits for the second firm. A success by one firm reduces the probability of its rival to gain the first prize but at the same time it prompts getting the second prize. Changes of the firm's value function are determined by these two effects which can be either positive or negative depending on the state of the race. However, given that there are less powered incentives to innovate, the level of investment is lower which result in higher value than in the E-Pat case.

The effect of market competitiveness: Table 1 assumes a Cournot type competition in the duopolistic market (for which μ=0.44). The question is whether the advantage of the CTI remains under more competitive duopolistic markets. Table 2 below provides the market statistics for μ=0.1 (i.e. lower duopolistic profits).

	Mid-Pat	E-Pat	E-Imit	CTI	Opt-Pat
Tot_Inv	904.7	1201.3	87.3	94.3	362.8
Inv1	447.3	567.2	87.3	53.6	197.3
Inv2	457.3	634.0	0.0	40.7	165.5
Dur_Prod1	7.2	6.7	20.9	10.5	9.0
Dur_Race	7.2	6.7	20.9	10.5	12.0
Tot_Val	1569.6	1383.1	100.0	291.9	455.0
CS	1237.1	1292.2	1680.0	3465.0	3270.5
Prob_1	0.7	0.5	1.0	0.0	0.6
Prob_2	0.3	0.5	0.0	0.0	0.4
V1	1128.6	784.6	6.3	139.5	260.8
V2	441.0	598.5	93.6	152.4	194.2

Table 2: Asymmetric R&D race with competitive duopolistic market

Reducing μ to 0.1 implies that the CTI still yields a higher consumers' surplus but the values for firms are lower than in most of the other regimes. Lower duopolistic profits reduce the incentives of firms to innovate and thus slow down the innovation process (which in turn reduces consumers' surplus). A further reduction of μ leads to no innovation as there are not sufficient incentives to invest in R&D (see also Figure 3a). But the lower cost of innovation does not compensate in such a case for the lower duopolistic profits and implies a lower value for firms.

Early or Late Advantage: Our investment cost structure implies that the first firm is more efficient in the first three stages while the second firm is more efficient in the last three stages of the race. However, all these stages are necessary for the completion of the invention. Does this asymmetry imply an advantage to one of the firms? Table 1 indicates that the answer to this question depends on the patent regimes.

In E-Pat, Mid-Pat, or Opt-Pat races - the firm that has better abilities at the first three steps of the race has an advantage in the entire race, i.e., it has a greater value than the second firm and it has a higher probability to be the first to complete the innovation. On the other hand in the CTI regime, it is the second firm - the firm with the better abilities in the last three stages of the race - which has the advantage in the entire race i.e., $V2 > V1$.

In the E-Imit race it is again the second firm that enjoys the advantage in the entire race. This race is characterized by a second mover advantage. In the symmetric case, this second mover advantage implies very low investments at the early stages as both firms are waiting for the success of their rival firm. Cost asymmetry "solves" this waiting game and identifies the first mover; the firm that has an advantage at early stages.

5. Licensing and Asymmetric R&D Race

The possibility of licensing a patent clearly affects the firms' incentives to invest and consequently the speed of innovation. The focus of the literature on optimal patent licensing is mostly on licensing final innovations patents. After the R&D is over the firm that obtains the patent may license it and the question is what the procedure of licensing that will maximize revenues is. Optimal licensing affects the prize at the end of the R&D race and thus the firms' incentives to innovate. In our setting there are only two firms that compete in the R&D race as well as in the product market. If one of the firms obtains a patent for the final innovation it does not have any incentives to license it to the other firm. Exclusive licensing does not generate any additional gains as we assume identical production costs for the two firms. If the licensing is without exclusivity then licensing is followed by a duopolistic market and lower overall profits. We thus focus on licensing of intermediate technologies.

5.1 Licensing of intermediate technology patents.

One can distinguish between licensing of an intermediate technology patent and licensing (or actually selling) of intermediate technology even without holding a patent for it. In both cases such licensing reduces the probability that the firm will win the race but it guarantees some payoffs at intermediate stage. When there is a patent for an intermediate technology licensing facilitates the continuation of the race. On the other hand when there is no patent for intermediate technology and the leading firm refrains from licensing its technology, the race continues. Hence, the terms of licensing would be different in the two cases, reflecting the firms' different outside option. We limit our discussion to licensing of intermediate patents. Yet, the setup can easily be extended to the case of selling of intermediate technologies.

Licensing of intermediate patents may occur at different stages of the race. For instance, a firm that has a patent at stage L of the invention might license this patent only after it completes stage L+k of the race and guarantees itself a head start even following the licensing stage. This will be clearly reflected by the licensing price. However, for simplicity, we focus on the case where licensing occurs only after one of the firms reaches stage L of the race. Failing to license at this stage leads to a monopolistic race.

In Sections 3 and 4 we demonstrated the advantage of the CTI regimes over other patent regimes. This advantage was mainly derived from efficient allocation of R&D investment. We now turn and analyze whether a system in which we allow for the patenting of intermediate technologies and the licensing of this patent can obtain the same advantages.

5.2 A Multistage R&D Race with Intermediate Technology Patent Licensing

We consider an R&D race in which patent may be awarded to the intermediate technology L; L<N. For simplicity we assume that there is only one stage of the innovation development that is patentable. We allow firms to (voluntary) license patents to their competitors. Once the patent is licensed, firms become engaged in a competitive R&D race with no final patents. Once one of the firms reaches the final stage, it obtains monopoly profits for several periods until the second firm catches up; both firms then enjoy duopoly profits. If there is no licensing of the intermediate invention, the firm that has the intermediate patent starts to enjoy monopoly profits after completing the N stages of the innovation process while the other firm is out of the race.

We now turn to define the appropriate value function for this race. We let $\widetilde{V}_1(k,l)$; $k,l \geq L$, be the value of the race when both firms have already obtained technology L and continue onto stage N yet are unable to patent their final invention. If one of the firms obtains an intermediate patent and there is no licensing, the value of the race for this firm is $V^M(k)$; $k \geq L$. This firm is the only firm in the race and it needs to develop the remaining technologies. $V^M(.)$ is the firm's expected monopolistic prize minus expected investment. The value of the other firm is zero as it is out of the race.

Assume that firm 1 obtains the intermediate patent. If it licenses this patent to firm 2, it will lose its monopoly position and thus become engaged in a competitive race which implies that firm 1 loses $V_1^M(L) - \widetilde{V}_1(L,L)$ if it licenses. The gain of firm 2 from the licensing is $\widetilde{V}_2(L,L)$ as without the licensing its value is zero whereas after licensing, it participates in the R&D race.

Since licensing is voluntary, licensing would occur whenever it creates a surplus that can be divided between firms, that is, whenever $\widetilde{V}_2(L,L) - (V_1^M(L) - \widetilde{V}_1(L,L)) > 0$. We assume that once the surplus is positive, a licensing agreement is concluded and the firms share the licensing surplus equally between them. Let $T_{1,2}$ be the amount paid by firm 2 to firm 1 for the license of intermediate technology. Then, the licensing term is:

(3) $$T_{1,2} = (V_1^M(L) - \widetilde{V}_1(L,L)) + 1/2[\widetilde{V}_2(L,L) - (V_1^M(L) - \widetilde{V}_1(L,L))].$$

Given the parameters of each race, we can use the continuation values $\widetilde{V}_2(L,L)$, $\widetilde{V}_1(L,L)$ and $V_1^M(L)$ to find out if firm *i* would license the intermediate patent to firm *j* and the licensing terms (i.e., $T_{i,j}$) — should licensing occurs. When there is no licensing we reassume the situation described in Section 3 (i.e., a regular R&D race with Mid-Pat regime). If licensing occurs we need to modify the value functions of the race at its early stages—before the intermediate patents is granted—to take into account the possibility of such licensing and the transfers that accompany it. We should distinguish between cases in which licensing will occur by both firms and when only one of the firms finds it profitable to license. For each case we need to calculate the appropriate equilibrium strategies and value functions. For example, consider the case in which the analysis of the second part of the game indicates that licensing occurs in both directions - when firm 1 achieves stage L first, and when firm 2 achieves stage L first - and the terms of license are T_{12} or T_{21}, respectively. In such cases the value function $V_i(k,l)$ and the Bellman equation will have the following form.

(4)

$$V_1(l,m)=\begin{cases}\widetilde{V}_1(l,m) & l>L,m>L\\ T_{1,2}+\widetilde{V}_1(L,L) & l=L,m<L\\ \max\limits_{x_1\ge 0}\left\{-c_1(l)x_1+\beta\sum\limits_{k'}V_i(l',m')p(l'|x_1)p(m'|x_2^*(l,m)\right\} & l<L,m<L\\ -T_{2,1}+\widetilde{V}_1(L,L) & l<L,m=L\\ 1/2*V_1(l,L-1)+1/2*V_1(L-1,m) & l=L,m=L\end{cases}$$

The value function $V_2(l,m)$ is similarly defined.

5.3 R&D Races with Licensing: Numerical Analysis

5.3.1 Summary Statistics:

For the numerical analysis we maintain the value of the parameters as in our benchmark case and assume that L=3.[26] We assume L=3 in order to facilitate a complete

[26] As in Tables 1&2 we keep our benchmark case with γ=5, π^M = 900 and the Cournot duopolistic payoffs π^D=400. The per-period monopolistic and duopolistic payoffs are $r\pi^M$ and $r\pi^D$ respectively. Consumers' surplus is derived from the same assumed demand function. Letting the net present value of consumers' surplus be CS^M=450 for the monopolistic case and CS^D=800 for a duopolistic (Cournot type) market. We make the necessary modifications if market structure changes. For the benchmark case we further let a=0.1 and set the discount rate factor β = 0.92.

specialization in R&D process and to see if such a behavior indeed emerges as part of the MPE. Table 3 presents the summary statistics for the R&D race with licensing in comparison to the previous patent regimes that we analyzed. There are two immediate conclusions from this analysis:

(i) There is no intermediate licensing in the symmetric case.[27]

(ii) In the asymmetric case licensing occurs only when the prizes are not excessively large relative to costs (given our parameters, licensing would occur only if $\alpha<2$).

When firms are symmetric in their abilities there are no gains from licensing of the intermediate technology as licensing transforms the market to a duopolistic market while there are no technological advantages from such licensing - as both firms have the same abilities. Even if there are different R&D abilities, when α is large, firms will not license the intermediate technology as the gains from a more efficient innovation process will not be sufficient to cover the loss from having a duopolistic market.

	α=0.5[28]				α=1.5				
	Mid Pat	E-Pat	CTI	License	Mid Pat	E-Pat	CTI	License	Opt-Pat
TotInv	87.32	87.20	59.97	76.68	351.44	440.24	108.19	364.72	243.73
Inv1	87.32	87.20	35.45	40.01	237.91	225.62	60.95	199.33	158.69
Inv2	0	0	24.52	36.67	113.53	214.62	47.25	165.39	85.04
DurProd1	20.86	20.89	13.96	11.85	9.32	8.52	9.71	7.89	10.61
DurRace	20.86	20.89	13.96	11.85	9.32	8.52	9.71	11.01	18.61
Value	6.34	6.09	72.72	97.64	281.99	233.64	436.42	292.93	297.63
CS	46.83	46.64	132.69	87.16	316.71	336.94	544.61	519.23	401.63
License	--	--	--	100K	--	--	--	81332	--
T12	--	--	--	154.82	--	--	--	448.47	--
T21	--	--	--	--	--	--	--	--	--
Prob_1	1	1	0	0	0.81	0.60	0	0	0.72
Prob_2	0	0	0	1	0.19	0.40	0	0.19	0.28
V1	6.34	6.09	30.90	49.67	257.72	164.75	211.36	264.37	165.87
V2	0	0	41.82	47.97	24.26	68.89	225.06	28.57	131.76

Table 3: Asymmetric R&D race with Licensing (γ=5). [29]

[27] All the simulations with the symmetric case ended with no licensing. We do not present these simulations here but they can be obtained upon request from the authors.

[28] The optimal length of patent, n, for this case is larger than 30. We thus do not include for this case the Opt-Pat case as it is very similar to the E-Pat case.

The main conclusion from Table 3 is that, in the cases we studied, the CTI regime provides a higher overall surplus than the race with licensing. When α=0.5, the possibility of licensing results in complete specialization of the R&D investment like in the CTI case. Firm 1 carries out the R&D until stage 3, obtains the patent, licenses it out to firm 2 and then exits the race. Firm 2 then completes the invention to become a monopolist in the market. Note that the market ends up as a monopoly even though there is no patent that blocks firm 1 from continuing its R&D and joining the market. Licensing implies faster innovation than the CTI regime but results in a monopolistic market structure and lower CS.

When α=1.5 both firms begin the race and invest in the early stages of the R&D. In 81% of the races, firm 1 uses its technological advantage and obtains the intermediate patent first; but it always licenses it to firm 2. Licensing does not induce exit of the first firm which continues the race to the completion of the invention. In 19% of the cases it is the second firm which obtains the patent. In such a case there is no licensing. The second firm completes the innovation and becomes the monopolistic producer. While the possibility of licensing implies higher overall surplus than in the mid-Pat and Opt-Pat cases it still falls short of the race with the CTI regime. Both consumers' surplus and firms' value are higher under the CTI than in the race with licensing. The race with licensing ends up with monopoly in 19% of the case. Moreover.the R&D investment in the licensing case is less efficient as firms tempt to invest in R&D even in technologies they have relative disadvantage (as a result, R&D cost in our example is more than three times higher in the licensing case than in the CTI case).

5.3.2 Comparison between the different patent regimes.

We now turn to discuss the effect of different R&D parameters on the comparison between the CTI and the licensing cases. We will first consider the effect of different level of α (maintaining γ=5).[30]

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[29] Table 3 provides additional information than in the previous tables. The row *license* gives the percentages of the races that actually end up with a license. T_{12} (and respectively T_{21}) indicates the transfer (price) of licensing the mid patent from firm 1 to 2.

[30] Since the graphs of the duration of the race do not provide additional information, we do not present them here. These graphs can be obtained upon request from the authors.

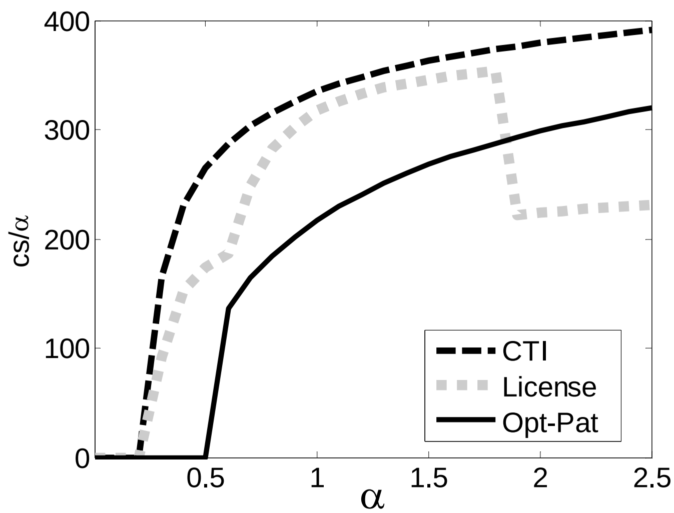

Figure 5a

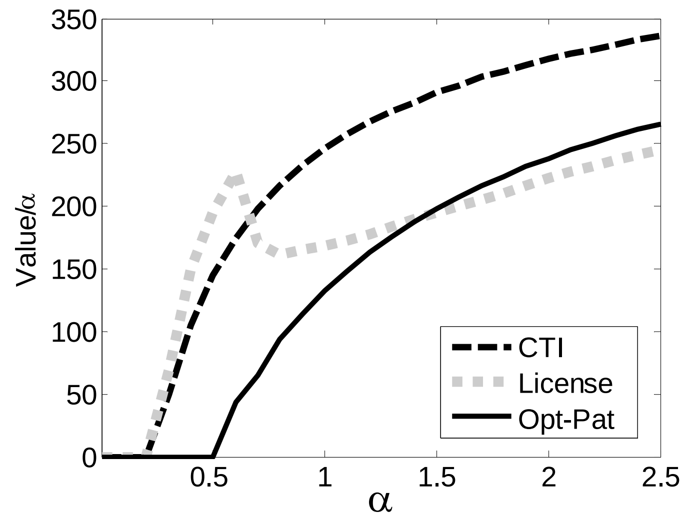

Figure 5b

The main observation from Figure 5a is that for all the range of α the CTI regime yields higher consumer surplus than the R&D race with licensing of intermediate technology.

For low values of α; (i.e., α<0.5) the race with licensing ends up with a monopolistic market. Firm 1 gets the mid patent, licenses it to firm 2 and exits the race. For medium range of α, both CTI and the race with licensing end up with a duopolistic market (and therefore CS is similar in both cases). The CTI yields a slightly higher CS since the two regimes differ in the timing in which the market switches from monopoly to duopoly. In the CTI regime the duration of the race (until first innovation) is longer, but the duopolistic market is created immediately. As α increases there is no incentive to license the intermediate technology and indeed we observe a sharp decline in CS. The race in this case becomes identical to the race with mid patent (and no licensing).

Figure 5b describes the firms' value as a function of α. When α is small the race with licensing results with monopoly and specialization in the R&D process and therefore yields a higher value for firms. As α increases, at first firm 1 still licenses its patent but without exclusion—resulting in a duopolistic market. For higher levels of α there is no licensing at all. In both cases value for firms is higher in the CTI regime than in the licensing case

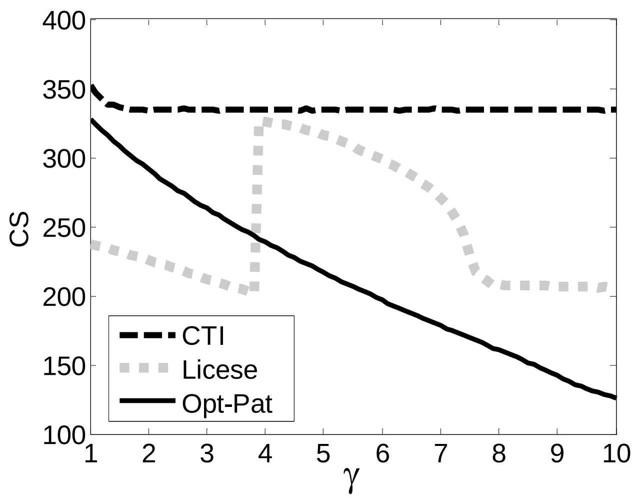

Figure 6a

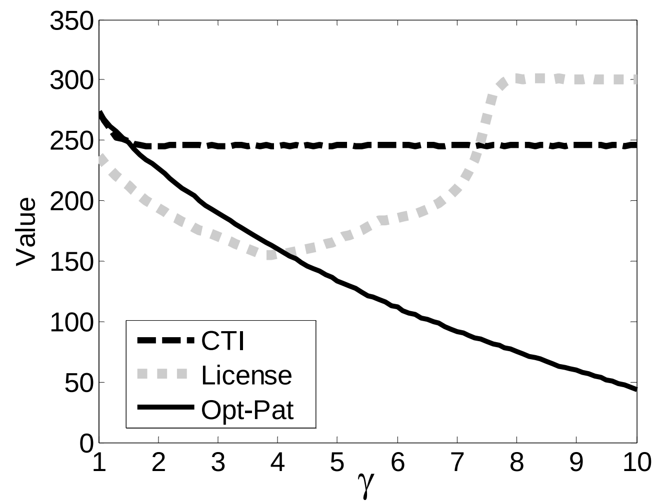

Figure 6b

Figures 6a and 6b describe CS and firms' value as a function of γ – the degree of cost asymmetry between the firms (in all these figures we hold α as α=1). For all the range of γ the CTI yields higher consumers' surplus than with other patent regimes. For low levels of γ there is no licensing at all. For medium range of γ there is licensing without exclusion resulting in a high consumers' surplus and low value for firms. For high levels of γ there is a licensing with exclusion, i.e., firm 1 license the intermediate patent and exit the race. For this range CS is lower than for the CTI case but the value for firms is higher.

6. Concluding Remark

There is always a question what one can learn from a numerical analysis. After all, the conclusions are limited to the set of parameters assumed in the analysis. What can be generalized beyond our numerical observation? This is a standard and debated question whenever a numerical analysis is presented. Clearly a general claim about the appropriate optimal patent policy is not what we tried to derive. The numerical analysis of a more complex model allows us to examine the intuition derived from simple analytical tractable models. In our case, we show that the CTI regime with no patent protection and perfect imitation opportunity may yield a better outcome for both consumers and firms than the standard end patent regime even when the length of the patent is optimally calculated. This conclusion depends very much on the parameters of the race. Nevertheless, it suggests that the analysis of optimal patent policy is not a straightforward task and there are situations in which no protection at all gives far better results than standard patent protection.

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Appendix A:

Investment Strategies and value functions for the R&D race presented in Section 4.

	0	1	2	3	4	5	6
0	(784,600)	(135,1795)	(4,2633)	(0,3181)	(0,3578)	(0,4017)	(0,4500)
1	(1832,62)	(757,939)	(92,2363)	(0,3181)	(0,3578)	(0,4017)	(0,4500)
2	(2420,0)	(1950,151)	(742,1366)	(33,3035)	(0,3578)	(0,4017)	(0,4500)
3	(2737,0)	(2682,5)	(2115,290)	(729,1890)	(17,3482)	(0,4017)	(0,4500)
4	(3256,0)	(3256,0)	(3201,7)	(2687,325)	(1114,1989)	(23,3933)	(0,4500)
5	(3842,0)	(3842,0)	(3842,0)	(3794,14)	(3510,194)	(1614,2110)	(0,4500)
6	(4500,0)	(4500,0)	(4500,0)	(4500,0)	(4500,0)	(4500,0)	(2250,2250)

Table A1: value function, α=5, γ=5, E-Pat

Table A2:

	0	1	2	3	4	5	6
0	(71,155)	(33,193)	(5,122)	(0,50)	(0,54)	(0,57)	(0,0)
1	(81,53)	(74,189)	(28,208)	(0,50)	(0,54)	(0,57)	(0,0)
2	(44,0)	(92,82)	(77,222)	(17,96)	(0,54)	(0,57)	(0,0)
3	(105,0)	(127,13)	(228,109)	(182,113)	(28,88)	(0,57)	(0,0)
4	(114,0)	(114,0)	(137,16)	(267,52)	(226,123)	(32,99)	(0,0)
5	(124,0)	(124,0)	(124,0)	(148,10)	(290,41)	(275,135)	(0,0)
6	(0,0)	(0,0)	(0,0)	(0.0,0.0)	(0,0)	(0,0)	(0,0)

investment, α=5, γ=5, E-Pat

	(V_1,V_2)
0	(942,960)
1	(1095,1074)
2	(1265,1199)
3	(1455,1336)
4	(1620,1535)
5	(1802,1755)
6	(2000, 2000)

Table A3: value function, α=5, γ=5, CTI

	$(\mathrm{Inv}_1,\mathrm{Inv}_2)$
0	(27,0)
1	(30,0)
2	(32,0)
3	(0,32)
4	(0,35)
5	(0,37)
6	(0,0)

Table A4: investment, α=5, γ=5, CTI

	0	1	2	3	4	5	6
0	(864,811)	(884,1081)	(998,1402)	(1142,1741)	(1268,1984)	(1406,2254)	(1557,2553)
1	(1037,847)	(965,1017)	(1007,1384)	(1142,1741)	(1268,1984)	(1406,2254)	(1557,2553)
2	(1208,944)	(1163,978)	(1075,1289)	(1142,1741)	(1268,1984)	(1406,2254)	(1557,2553)
3	(1392,1052)	(13912,1052)	(1297,1168)	(1188,1624)	(1268,1984)	(1406,2254)	(1557,2553)
4	(1730,1205)	(1730,1205)	(1725,1207)	(1600,1357)	(1402,1761)	(1406,2254)	(1557,2553)
5	(2115,1373)	(2115,1373)	(2115,1373)	(2102,1382)	(2024,1462)	(1688,1903)	(1557,2553)
6	(2553,1557)	(2553,1557)	(2553,1557)	(2553,1557)	(2553,1557)	(2553,1557)	(2055,2055)

Table A5: value function, α=5, γ=5, optimal patent

	0	1	2	3	4	5	6
0	(25,46)	(9,76)	(0,75)	(0,37)	(0,40)	(0,42)	(0,0)
1	(30,6)	(23,73)	(5,84)	(0,37)	(0,40)	(0,42)	(0,0)
2	(31,0)	(36,35)	(21,95)	(0,37)	(0,40)	(0,42)	(0,0)
3	(75,0)	(75,0)	(89,59)	(41,49)	(0,40)	(0,42)	(0,0)
4	(83,0)	(83,0)	(85,3)	(115,30)	(74,55)	(0,42)	(0,0)
5	(92,0)	(92,0)	(92,0)	(99,6)	(137,23)	(109,61)	(0,0)
6	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

Table A6: investment, α=5, γ=5, optimal patent

Appendix B: Optimal Patent length.

Figure 5a and 5b describe the optimal patent length n as a function of γ and μ.

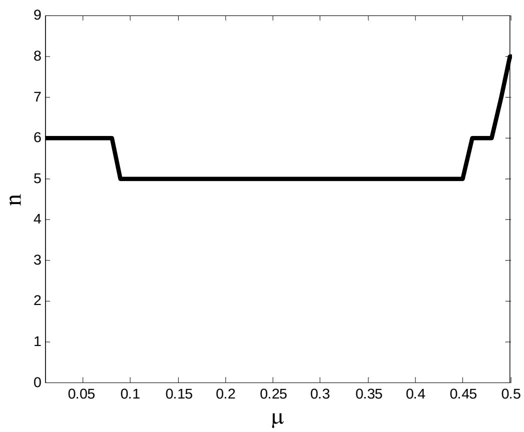

Figure 7a

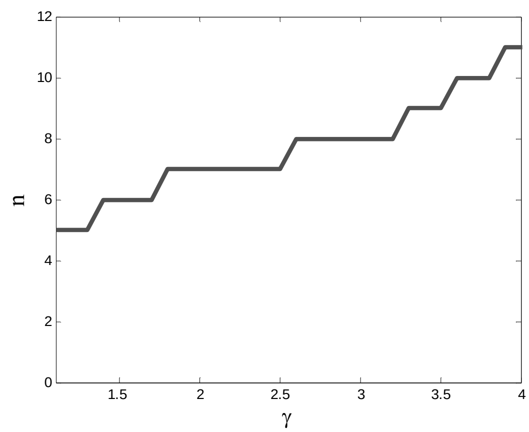

Figure 7b

The optimal patent length, n, is increasing in γ and is a non monotonic function of μ (see Figure 7a,b). As γ increases the overall R&D cost increase, decreasing the incentives to innovate. On the other hand the optimal n is not a monotonic function of μ. There are two conflicting forces that can explain the changes in the optimal n. Lowering μ implies lower duopolistic profits reducing the incentives to innovate. This in turn requires more periods of monopolistic payoffs—a higher n. On the other hand, for a very large μ consumers' surplus is similar in the monopolistic and duopolistic market. In this case CS is more affected by the timing of innovation than from the resultant market structure. Increasing n increases the incentives to innovate and yields a speedy innovation.