Worker Matching and Firm Value

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Abstract

This paper studies the hiring and firing decisions of firms and their effects on firm value. This is done in an environment where the productivity of workers depends on how well they match with their co-workers and the firm acts as a coordinating device. Match quality derives from a production technology whereby workers are randomly located on the Salop circle, and depends negatively on the distance between the workers. It is shown that a worker's contribution in a given firm changes over time in a nontrivial way as co-workers are replaced with new workers.

The paper derives optimal hiring and replacement policies, including an optimal stopping rule, and characterizes the resulting equilibrium in terms of employment, firm output and the distribution of firm values. The paper stresses the role of horizontal differences in worker productivity, as opposed to vertical, assortative matching issues. Simulations of the model reveal a rich pattern of worker turnover dynamics and their connections to the resulting firm value and age distributions.

Key words: firm value, complementarity, worker value, Salop circle, hiring, firing, match quality, optimal stopping.

JEL codes: E23, J62, J63.

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1 Introduction

How does the value of the firm depend on the value of its workers? When one considers firms that have little physical capital – such as IT firms, software development firms, investment banks and the like – the neoclassical model does not seem to provide a reasonable answer. The firm has some value that is not manifest in physical capital. Rather, Prescott and Visscher's (1980) 'organization capital' may be a more relevant concept in this context. One aspect of the latter form of capital, discussed in that paper, is the formation of teams and this is the issue taken up in the current paper. We ask how workers affect each other in production and how this interaction affects firm value. The paper studies the value of firms and their hiring and firing decisions in an environment where the productivity of the workers depends on how well they match with their co-workers and the firm acts as a coordinating device. This role of the firm is what generates value.

In the model, match quality derives from a production technology whereby workers are randomly located on the Salop (1979) circle and depends negatively on the distance between them. It is shown that a worker's contribution in a given firm changes over time in a nontrivial way as co-workers are replaced with new workers. The paper derives optimal hiring and replacement policies, including an optimal stopping rule, and characterizes the resulting equilibrium in terms of employment and the distribution of firm values.

Key results are the derivation of an optimal worker replacement strategy, based on a productivity threshold that is defined relative to other employees. This strategy, interacted with exogenous worker separation and firm exit shocks, generates rich turnover dynamics. The resulting firm value distributions are found to be – using simulations – non-normal, with negative skewness and negative excess kurtosis. This shape reflects the fact that, as firms mature, there is a process of forming good teams on the one hand and the effects of negative separation and exit shocks on the other hand.

The paper proceeds as follows: in Section 2 we discuss the model in the context of the literature. In Section 3 we outline the model. We describe the

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set up and delineate the interaction between workers. In Section 4 we derive the optimal hiring and firing policy and study the implications for firm value. In Section 5 we allow for exogenous worker separation. Section 6 presents simulations of the model, exploring the mechanisms inherent in it. Section 7 discusses the key assumptions in light of the results and Section 8 concludes.

2 The Model in the Context of the Literature

The paper has points of contact with papers in the search literature. We exploit the idea of optimal stopping, as in McCall (1970) and a rich strand of search literature which followed (see McCall and McCall (2008), in particular chapters 3 and 4, for a comprehensive treatment). The existing literature does not cater, however, for the key issue examined here, namely that of worker complementarities. We capture this by assuming that the firm hires three workers and that match quality between all worker-pairs matter. Conceptually this is an important distinction, and it allows us to analyze team formation in detail. Technically it also gives rise to new challenges. Total match quality (or output) depends on two variables that are stochastic *ex ante*, the distances from the best placed worker to each of her two co-workers. At the same time the firm only replaces one worker at a time. This creates a new dimension to the optimal stopping problem, which, in contrast to most earlier studies, now depends on a state variable (the distance between the two closest workers who are not replaced in this round). Furthermore, optimal stopping behaviour depends on this state variable in a non-trivial way, and it is not even obvious from the outset that a simple optimal stopping rule exists.

Our paper also shares some features with the search model of Jovanovic (1979a,b): there is heterogeneity in match productivity and imperfect information ex-ante (before match creation) about it; these features lead to worker turnover, with good matches lasting longer.[2] Burdett, Imai and Wright (2004) analyze models where agents search for partners to form relationships and may or may not continue searching for different partners while matched. Both unmatched and matched agents have reservation match qualities.

The paper bears limited similarity to Kremer's (1993) O-ring production function model. The similarity pertains to the importance attributed to the idea of workers working well together. In that model firms employ workers

[2] Pissarides (2000, Chapter 6) incorporates this kind of model into the standard DMP search and matching framework, keeping the matching function and Nash bargaining ingredients, and postulating a reservation wage and reservation productivity for the worker and for the firm, respectively.

of the same skill and pay them the same wage. In this set-up quantity cannot substitute for quality. But the models differ in their treatment of the matching of workers: in Kremer (1983) there is a multiplicative production function in workers/tasks and this underlies their complementarity. In the current paper there is explicit modelling of the match between workers, formalized as random state variables, which realization elicits the firm's optimal worker replacement policy.

The paper stresses the role of horizontal differences in worker productivity, as opposed to vertical, assortative matching issues. The literature on the latter – see, for example, the prominent contributions by Teulings and Gautier (2004), Shimer (2005), Shimer and Smith (2000), Eeckhout and Kircher (2010, 2011) and Gautier, Teulings and Van Vuuren (2010) – deals with the matching of workers of different types. Key importance is given to the vertical or hierarchical ranking of types. These models are defined by assumptions on the information available to agents about types, the transfer of utility among workers (or other mating agents), and the particular specification of complementarity in production (such as supermodularity of the joint production function). In the current paper, workers are ex-ante homogenous, there is no prior knowledge about their complementarity with other workers before joining the firm, and there are no direct transfers between them.

3 The Model

In this section we first describe the set-up of the firm and the production process (3.1). We then define worker interactions and the emerging state variables (3.2). We subsequently provide stylized facts supporting this way of modelling (3.3).

3.1 The Set-Up

A firm enters the market by sinking an entry cost K. The firm starts off with three workers with given productivity. Workers are located on the Salop (1979) circle, with their placement randomly and independently drawn from a uniform distribution. Any new worker placement will be drawn independently from the same distribution. The worker's contribution to the firm's output depends negatively on the distance between her and the other two workers. Each period the firm faces an exogenous exit probability.

In each period the firm can replace at most one worker. It does so by first firing one of the existing workers without recall, and then sampling – from outside the firm – one worker. Thus, we do not allow the firm to compare the existing and the sampled worker and hire the more productive one. We rationalize this by assuming that it takes a period to learn a worker's productivity. Replacing a worker is costly. Furthermore, the model is static, in the sense that wages and productivity distributions are time independent.

3.2 Workers' Productivity and Interactions

We now turn to a formal description. The three workers are located on the unit circle. The one in the middle (out of the three) is the j worker who satisfies

$$\min_j \sum_{i=1}^{3} d_{ij} \tag{1}$$

where d_{ij} is the distance between worker i and j, and $d_{ii} = 0$. We shall define two state variables δ_1, δ_2 as follows:

$$\begin{aligned} \delta_1 &= \min_{i,j} d_{ij} && (2) \\ \delta_2 &= \min_j d_{kj} \quad , k \neq i^*, j^* \quad i^*, j^* = \arg\min_{i,j} d_{ij} && (3) \end{aligned}$$

The first state variable δ_1 expresses the distance between the two closest workers. The second state variable δ_2 expresses the distance between the third worker and the closest of the two others.

The following figure illustrates:

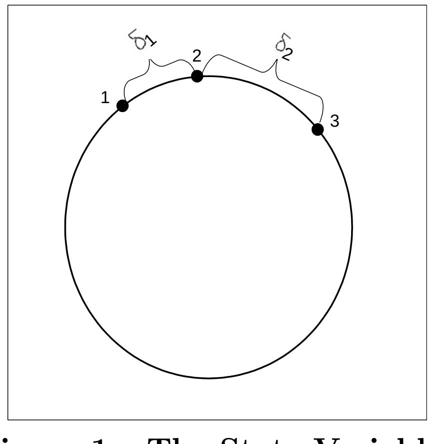

Figure 1: The State Variables

The firm's task is to find what we refer to as a common ground for the three workers; in what follows we assume that the firm chooses the middle worker as the focal point and all distances are measured going via the middle worker (see the discussion in Section 7 below for remarks on this assumption).

Every period, each worker works together with both co-workers to produce output. Production y_{ij} is negatively related to the distance d_{ij} :

$$y_{ij} = \frac{\widetilde{y}}{3} - d_{ij} \tag{4}$$

The firm's total output is then given by the linear additive function:

$$\begin{aligned} Y &= y_{12} + y_{13} + y_{23} \qquad (5) \\ &= \widetilde{y} - \sum_{i=1}^{3} d_{ij} \\ &= \widetilde{y} - 2(\delta_1 + \delta_2) \end{aligned}$$

We assume that wages are independent of match quality. This is consistent with a competitive market where firms bid for *ex ante* identical workers prior to knowing the match quality. The profits (π) of the firm are then given by:

$$\begin{aligned} \pi &= Y - W \qquad (6) \\ &= \widetilde{y} - 2(\delta_1 + \delta_2) - W \\ &= y - 2(\delta_1 + \delta_2) \end{aligned}$$

where W is the total wage bill and y is production net of wages ($\widetilde{y} - W$).

Within a period, the firm cannot fire the workers. Hence it will produce as long as output is positive ($\widetilde{y} - 2(\delta_1 + \delta_2) > 0$). We will assume that this is always the case. Furthermore, the firm may want to exit the market endogenously if δ_1 is sufficiently high. In what follows we rule this out by assumption. Below we show that in equilibrium it will never be optimal to exit the market or halt production after a bad draw if $K > 4/(3r')$, where $r' = r/(1 + r)$. Allowing for firm exit after a bad draw is trivial, though cumbersome, and does not add interesting new results.

As already mentioned, the firm can replace up to one worker each period, at a cost c. It replaces the worker who is further away from the middle worker. The new values δ_1' and δ_2' are random draws from a distribution that depends on δ_1. We write $(\delta_1', \delta_2') = \Gamma\delta_1$. Figure 2 illustrates, how, without loss of generality, workers 1 and 2, who are not replaced, are situated symmetrically around the north pole:

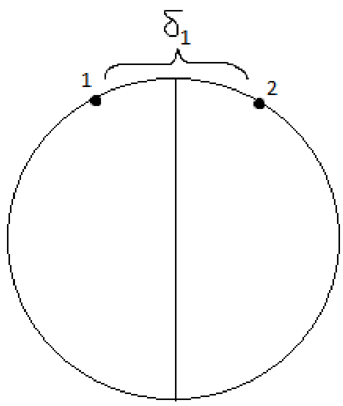

Figure 2: Incumbent Workers

From Figure 2 it follows that Γ can be characterized as follows:

1. With probability $1 - 3\delta_1$, $\delta_1' = \delta_1$ and $\delta_2' \sim unif[\delta_1, \frac{1-\delta_1}{2}]$
2. With probability $2\delta_1$, $\delta_1' \sim unif[0, \delta_1]$ and $\delta_2' = \delta_1$
3. With probability δ_1, $\delta_1' \sim unif[0, \delta_1/2]$ and $\delta_2' = \delta_1 - \delta_1'$

Note that the transition probabilities, and hence continuation values when replacing, are a function of δ_1 and thus are independent of δ_2. Hence δ_2 only influences continuation values in states where the firm is not replacing. That is, as follows from the definition of profits (equation 6), the continuation value of inaction is a function of $(\delta_1 + \delta_2)$.

3.3 Microeconomic Stylized Facts

The afore-going set-up aims at capturing properties that have been found in empirical micro-studies of team production. Hamilton, Nickerson and Owan (2003) find that teamwork benefits from collaborative skills involving communication, leadership, and flexibility to rotate through multiple jobs. Team production may expand production possibilities by utilizing collaborative skills. Turnover declined after the introduction of teams. Using evidence from professional baseball teams in the U.S., Gould and Winter (2009) find evidence in favor of the idea that workers adjust their effort in a rational way that is dependent on the technology of team production. For example, they increase effort in response to increased efforts by workers which they complement. Mas and Moretti (2009) use high-frequency data on worker productivity from a large supermarket chain in the U.S. They find strong evidence of positive productivity spillovers from the introduction of highly productive personnel into a shift. A worker effort is positively related to the productivity of workers who see him, but not to workers who do not see him. Additionally, workers respond more to the presence of co-workers with whom they frequently interact.

A very recent study, undertaken by MIT's Human Dynamics Laboratory, collected data from electronic badges on individual communications behavior in teams from diverse industries. The study, reported in Pentland (2012), stresses the huge importance of communications between members for team productivity. In describing the results of how team members contribute to a team as a whole, the report actually uses a diagram of a circle (see Pentland (2012, page 64)), with the workers placed near each other contributing the most. The findings state that face to face interactions are the most valuable form of communications, much more than email and texting, thereby emphasizing the role of physical distance.

4 Optimal Hiring and Firing

Our aim in this section is to derive an optimal stopping rule for replacement. We show that an optimal stopping rule can be expressed in terms of the state variable δ_1, independently of δ_2.

4.1 The Optimal Stopping Rule

In this section we will be looking for a rule of the form "stop searching if $\delta_2 \leq \overline{\delta}_2(\delta_1)$." To gain intuition, take a pair δ_1,δ_2 for which, w.l.o.g, the rule says "do not replace", i.e., the continuation value of inaction is higher than the continuation value of replacement including costs. Start moving the furthest worker even further away. The value of replacement does not change, whereas the value of inaction goes down (the sum of the distances goes up). When we continue this exercise, at some point, the optimal decision turns to "replace," the value of inaction becomes lower than the value of replacement (provided that the replacement cost is not too high). As the value of inaction decreases monotonically in our exercise, there must be some point, that is, a particular $\overline{\delta}_2$, for which the values are equal and the firm is indifferent between replacing or not. Therefore, for each δ_1 we have a rule saying replace if $\delta_2 > \overline{\delta}_2$, and not replace otherwise. This is a cutoff rule for each δ_1.

Note that the cutoff, which applies to the longer distance in the team, by definition must satisfy $\overline{\delta}_2(\delta_1) \geq \delta_1$. Below, we will show analytically that this inequality defines δ_1^* above which replacement takes place in any case, regardless of the second distance δ_2. These are teams that are so bad, in the sense that the distance between workers in the best pair δ_1 is high, so that replacement is optimal, no matter how well the second-best pair collaborates.

An important issue here is the slope of the cutoff function. We will look for the cutoff function $\overline{\delta}_2(\delta_1)$ that is decreasing in δ_1 and prove that such a function exists by characterizing it analytically. A negative slope of the cutoff function implies that whenever δ_1 declines as a result of replacement, the firm will stop replacing. Under an increasing cutoff function, a smaller δ_1 will make the firm even more demanding with respect to δ_2, and it might be the case that the firm will keep replacing and improving its team infinitely, and we want to rule this out. We will show that with this formulation, the incentives to replace is weaker the lower is δ_1.[3]

Lemma 1 *Suppose $\overline{\delta}_2(\delta_1)$ is strictly decreasing in δ_1 in the region below δ_1^*. Then, in the region below δ_1^*, whenever the smaller distance declines after replacement, i.e., $\delta_1' < \delta_1$, the firm will stop.*

The proof is instructive, so it is included as follows.

Proof. If the new δ_1' is below δ_1, then a decreasing cutoff requires $\overline{\delta}_2(\delta_1') > \overline{\delta}_2(\delta_1)$. But the smallest distance can decline only in two cases: (i) the new position of the worker fell between the two incumbents, in which case the new $\delta_2' < \delta_1 < \overline{\delta}_2(\delta_1) < \overline{\delta}_2(\delta_1')$ and the firm stops, or (ii) the new position of the worker fell close to one of the incumbents, but outside the arc between them, in which case $\delta_2' = \delta_1 < \overline{\delta}_2(\delta_1) < \overline{\delta}_2(\delta_1')$, and the firm stops. This completes the proof. ■

Consequently, if after replacement the firm finds it optimal to replace again, it means that the smallest distance has not declined, it stayed the same, and with it, the expected value from replacement.

In the next sub-section we characterize the cutoff analytically, for the region where it is applicable, that is, where $\delta_1 < \delta_1^*$.

4.2 Characterizing the Stopping Rule

Suppose the firms behave according to a stopping rule $\overline{\delta}_2(\delta_1)$ which is decreasing in δ_1. Let $\beta = \frac{1}{1+r}$ denote the discount factor and r the discount rate of the firm. In the simulations below we let r include a stationary probability of exiting the market, after which the value of the firm is zero. Denote the value function of the firm by $V(\delta_1, \delta_2)$, and let $\overline{V}(\delta_1) \equiv EV(\delta_1', \delta_2')|\delta_1$. In what follows, let $E^{|z}$ denote the conditional expectation given z. Now

[3] If we formulate the stopping rule in terms of total distance $X = 2(\delta_1 + \delta_2)$ instead of δ_2, one can show that the cut-off $\overline{X}(\delta_1)$ is strictly *increasing* in δ_1. This is why we have formulated the stopping rule in terms of δ_2.

$$
\begin{aligned}
V(\delta_1,\delta_2) &= \pi(\delta_1,\delta_2)+\beta\max[V(\delta_1,\delta_2),\overline{V}(\delta_1)-c] \qquad (7)\\
&= y-2(\delta_1+\delta_2)+\max[\frac{y-2(\delta_1+\delta_2)}{r},\frac{\overline{V}(\delta_1)-c}{1+r}]
\end{aligned}
$$

It follows directly from proposition 4 in Stokey and Lucas (1989, p.522) that the value function exists. By definition the optimal stopping rule must satisfy

$$
V(\delta_1,\overline{\delta}_2(\delta_1))=\overline{V}(\delta_1)-c
$$

Or (from 7)

$$
\frac{y-2(\delta_1+\overline{\delta}_2(\delta_1))}{r}=\frac{\overline{V}(\delta_1)-c}{1+r} \qquad (8)
$$

Intuitively, the expected value of replacement, $\overline{V}(\delta_1)$, is given by:

$$
\begin{aligned}
\overline{V}(\delta_1) = \; & \underbrace{y-2\cdot E^{|\delta_1}(\delta_1'+\delta_2')}_{\substack{(1)\,:\,\text{expected flow output}\\ \text{after replacement}}} \qquad (9)\\
& + \underbrace{\Pr(\delta_2'\le\overline{\delta}_2(\delta_1'))}_{\substack{(2)\,:\,\text{probability of}\\ \text{stopping}}} \cdot \underbrace{\frac{y-2\cdot E^{|\delta_1,\delta_2'\le\overline{\delta}_2(\delta_1')}(\delta_1'+\delta_2')}{r}}_{\substack{(3)\,:\,\text{expected discounted value}\\ \text{if stopped after replacement}}} +\\
& + \underbrace{\Pr(\delta_2'>\overline{\delta}_2(\delta_1))}_{\substack{(4)\,:\,\text{probability of}\\ \text{replacing again}}} \cdot \underbrace{\frac{\overline{V}(\delta_1)-c}{1+r}}_{\substack{(5)\,:\,\text{expected discounted value}\\ \text{if replacing again}}}
\end{aligned}
$$

There are two important points about this equation:

(i) The probability of stopping (2) includes the possibility that the smallest distance δ_1 has changed to δ_1', and the expected value if stopped (3) takes this into account.

(ii) The probability of replacing again (4) and the expected discounted value if replacing again (5) build on the fact that repeated replacement can occur iff the smallest distance between the workers remained the same (follows from Lemma 1 in the previous section).

We will show that equation (9) can be expressed as

$$\begin{aligned}\overline{V}(\delta_1) &= y-(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2}) \\ &+\frac{(\delta_1+2\overline{\delta}_2)y-2\overline{\delta}_2(2\delta_1+\overline{\delta}_2)-2\delta_1^2}{r} \\ &+(1-\delta_1-2\overline{\delta}_2)\frac{\overline{V}(\delta_1)-c}{1+r}\end{aligned} \tag{10}$$

1. First we show that expected flow output (1) from equation 9 is $y-2\cdot E^{|\delta_1}(\delta_1'+\delta_2')=y-(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2})$.

- Consider Figure 2. With probability $2\cdot\left(\frac{1}{2}-\frac{\delta_1}{2}\right)$ the new worker falls outside the arc between the two incumbents (to the left or to the right), and the expected sum of distances between all workers in this case will be $2\cdot\left(\delta_1+\frac{1}{2}\cdot\left(\frac{1}{2}-\frac{\delta_1}{2}\right)\right)$
- With probability δ_1 the new worker will fall between the two incumbents, and the total sum of distances between all workers will be $2\delta_1$

Summing up, the total expected sum of distances between all workers after replacement is:

$$\begin{aligned}2\cdot E^{|\delta_1}(\delta_1'+\delta_2') &= 2\cdot\left(\frac{1}{2}-\frac{\delta_1}{2}\right)\cdot 2\cdot\left(\delta_1+\frac{1}{2}\cdot\left(\frac{1}{2}-\frac{\delta_1}{2}\right)\right)+\delta_1\cdot 2\delta_1= \\ &= \frac{1}{2}+\delta_1+\frac{\delta_1^2}{2}\end{aligned}$$

2. Then we show that the probability of stopping (2) and the expected discounted value if stopped (3) in equation 9 above is:

$$\Pr(\delta_2'\leq\overline{\delta}_2(\delta_1'))\cdot\frac{y-2\cdot E^{|\delta_1,\delta_2'\leq\overline{\delta}_2(\delta_1')}(\delta_1'+\delta_2')}{r}=\frac{(\delta_1+2\overline{\delta}_2)y-2\overline{\delta}_2(2\delta_1+\overline{\delta}_2)-2\delta_1^2}{r}$$

- With probability δ_1 the new worker will fall between the two incumbents, in which case the smallest distance will fall, and by Lemma 1, the firm will stop. The total sum of distances between the workers in this case will be $2\delta_1$. The expected discounted value in this case will be $\frac{y-2\delta_1}{r}$

- With probability $2\overline{\delta}_2$ the new worker falls outside the two incumbents and below the threshold, and the firm will stop. The expected distance between the new worker and the closest incumbent is $\frac{\overline{\delta}_2}{2}$, so that the expected total sum of distances between the workers in this case will be $2 \cdot \left(\delta_1 + \frac{\overline{\delta}_2}{2}\right)$ .The expected discounted value in this case will be $\frac{y-2\delta_1-\overline{\delta}_2}{r}$

Summing up:

$$\begin{aligned}
\Pr(\delta_2' &\leq \overline{\delta}_2(\delta_1')) \cdot \frac{y - 2 \cdot E^{|\delta_1, \delta_2' \leq \overline{\delta}_2(\delta_1')}(\delta_1' + \delta_2')}{r} \\
&= \delta_1 \cdot \frac{y - 2\delta_1}{r} + 2\overline{\delta}_2 \cdot \frac{y - 2\delta_1 - \overline{\delta}_2}{r} \\
&= \frac{(\delta_1 + 2\overline{\delta}_2)y - 2\overline{\delta}_2(2\delta_1 + \overline{\delta}_2) - 2\delta_1^2}{r}
\end{aligned}$$

3. Finally we show that

$$\Pr(\delta_2' > \overline{\delta}_2(\delta_1))\frac{\overline{V}(\delta_1) - c}{1+r} = (1 - \delta_1 - 2\overline{\delta}_2)\frac{\overline{V}(\delta_1) - c}{1+r}$$

This comes from the fact that with probability $(1 - \delta_1 - 2\overline{\delta}_2)$ the new worker is above the $\overline{\delta}_2$ threshold. The firm will keep replacing and pay the cost c again.
We have thus fully derived equation (10).

Let us write:

$$\begin{aligned}
& (\delta_1 + 2\overline{\delta}_2)y - 2\overline{\delta}_2(2\delta_1 + \overline{\delta}_2) - 2\delta_1^2 \\
= \; & (\delta_1 + 2\overline{\delta}_2)(y - 2(\delta_1 + \overline{\delta}_2)) + 2\overline{\delta}_2^2 + 2\delta_1\overline{\delta}_2
\end{aligned}$$

Hence we can re-write (10) as follows:

$$\begin{aligned}
\overline{V}(\delta_1) &= y - (\frac{1}{2} + \delta_1 + \frac{\delta_1^2}{2}) \\
&\quad + \frac{(\delta_1 + 2\overline{\delta}_2)(y - 2(\delta_1 + \overline{\delta_2})) + 2\overline{\delta}_2^2 + 2\delta_1\overline{\delta}_2}{r} \\
&\quad + (1 - \delta_1 - 2\overline{\delta}_2)\frac{\overline{V}(\delta_1) - c}{1+r}
\end{aligned} \tag{11}$$

Substituting out $\overline{V}(\delta_1)$ and using (8), gives the rule (see Appendix A for details):

$$c+\frac{1}{2}+\frac{\delta_1^2}{2}-\delta_1-2\overline{\delta}_2=\frac{2\delta_1\overline{\delta}_2+2\overline{\delta}_2^2}{r} \tag{12}$$

This cut-off rule has a very intuitive interpretation:

The LHS of (12) represents net costs of replacing, evaluated at the threshold ($\overline{\delta}_2$). If not replacing the worker, the total distance is given by $2(\delta_1+\overline{\delta}_2)$.When replacing the worker, the firm expects to have a distance of $\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2}$,(see derivation of equation 10 above). The firm pays c when replacing the worker. So the net costs are $c+$ the expected total distance with replacement less the total distance without replacement. The net costs are thus

$$c+\frac{1}{2}+\frac{\delta_1^2}{2}+\delta_1-2(\delta_1+\overline{\delta}_2)=c+\frac{1}{2}+\frac{\delta_1^2}{2}-\delta_1-2\overline{\delta}_2$$

which is the LHS of (12).

The RHS of (12) represents the gains from replacement associated with lower costs in all future periods if the draw is good.

With probability δ_1 the new worker will be between the two existing workers who have a distance of δ_1 between them. The total distance between the three workers is $2\delta_1$. Existing total distance is $2(\delta_1+\overline{\delta}_2)$, and the savings in distance is thus $2\overline{\delta}_2$. Multiplying this with the probability of the event, δ_1, gives the first term in the nominator of the RHS of (12).

With probability $2\overline{\delta}_2$ the worker is not between the existing workers but within a distance of $\overline{\delta}_2$ from one of them. The expected distance of the new worker to the nearest existing worker is $\overline{\delta}_2/2$ and to the other existing worker it is $\delta_1+\overline{\delta}_2/2$. The per period cost savings is thus

$$2(\delta_1+\overline{\delta}_2)-[\delta_1+\frac{\overline{\delta}_2}{2}+(\delta_1+\frac{\overline{\delta}_2}{2})]=\overline{\delta}_2$$

Multiplying this with the probability of the event $2\overline{\delta}_2$ gives the second term of the RHS of (12).

We see from equation (12) that an increase in δ_1 reduces the net cost of replacing (reduces the left-hand side) and increases the gain of replacement (the right-hand side) This means that the higher is δ_1 the worse is the team and the more the firm is willing to replace. Thus $\overline{\delta}_2(\delta_1)$ is declining. Note also that

$$\frac{\partial\overline{V}(\delta_1)}{\partial\delta_1}=-\frac{1+r}{r}\left[1+\frac{\delta_1(1+r)}{\delta_1+2\overline{\delta}_2+r}\right]<0 \tag{13}$$

Proposition 2 *For $\delta_1 \leq \delta^*$, the optimal stopping rule $\overline{\delta}_2(\delta_1)$ is uniquely defined by (12), where δ^* solves (12) for $\delta_1 = \overline{\delta}_2(\delta_1)$.*

The proof is given in Appendix B.

The intuition for optimal behavior is simple. The gain from replacing is higher the higher is δ_1 (for a given δ_2), as the higher is the probability that an improvement will take place, and the higher is the expected gain given that an improvement takes place.

4.3 Turnover Dynamics With Optimal Stopping

The following figure illustrates this optimal behavior:

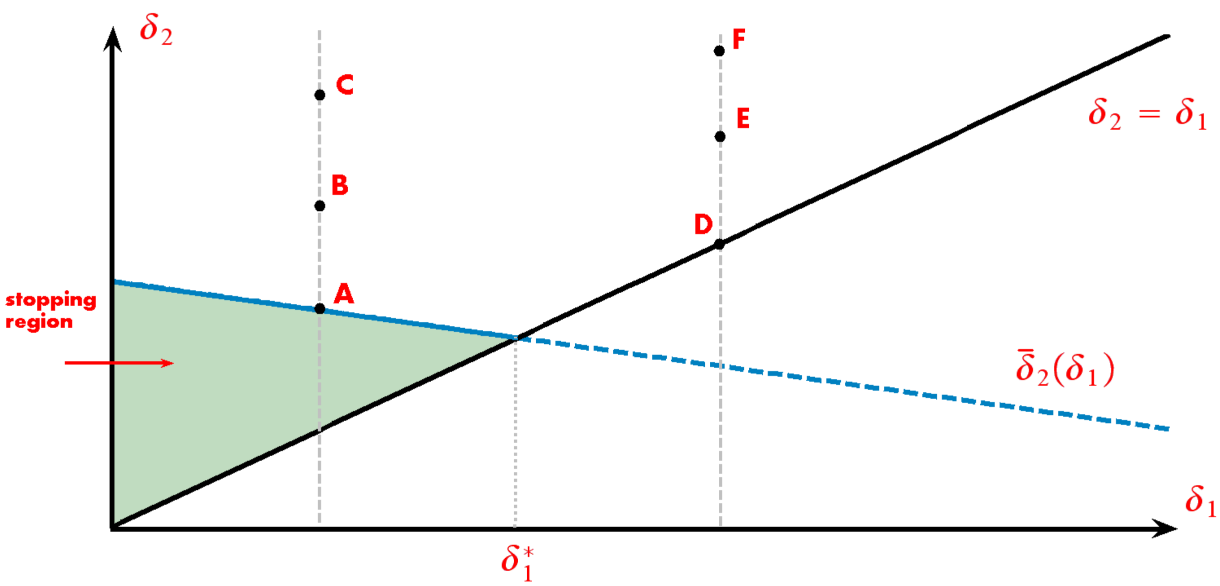

Figure 3: Optimal Policy

The space of the figure is that of the two state variables, δ_1 and δ_2. The feasible region is above the 45 degree as $\delta_2 \geq \delta_1$ by definition. The downward sloping line shows the optimal replacement threshold $\overline{\delta}_2$ as a function of δ_1.

With the replacement of a worker, the firm may move up and down a vertical line for any given value of δ_1 (such as movement between A, B and C or between D, E and F). If the replacement implies a lower value of δ_1, this vertical line moves to the left. This is what happens till the firm gets into the absorbing state of no further replacement in the shaded triangle formed by the $\delta_1^* = \overline{\delta}_2(\delta_1^*)$ point, the intersection of $\overline{\delta}_2(\delta_1)$ line with the vertical axis, and the origin ($\delta_1 = \delta_2 = 0$).

The following properties of turnover dynamics emerge from this figure and analysis:

(i) At the NE part of the $\delta_1 - \delta_2$ space, δ_1, δ_2 are relatively high, output is low, and the firm value is low. Hence the firm keeps replacing and there is high turnover. Note that some workers may stay for more than one period in the firm when in this region. The dynamics are leftwards, with δ_1 declining, but δ_2 may move up and down.

(ii) Above the $\overline{\delta}_2(\delta_1)$ threshold, left of δ_1^*, newcomers may still be replaced, but veteran workers are kept.

(iii) In the stopping region there is concentration at a location which is random, with a flavor of New Economic Geography agglomeration models. Thus firms specialize in the sense of having similar workers. There is no turnover, and output and firm values are high.

(iv) Policy may affect the regions in $\delta_1 - \delta_2$ space via its effect on c. The discount rate affects the regions as well.

(v) These replacement dynamics imply that the degree of complementarity between existing workers may change. This feature is unlike the contributions to the match of the agents in the assortative matching literature, where they are of fixed types.

4.4 Closing the Model

Finally, the model is closed by imposing a zero profit condition on firms. There are costs $K \geq 3c$ to open a firm. A zero profit condition pins down the wage ($w = \frac{W}{3}$):

$$E^{|\delta_1 \delta_2} V(\delta_1, \delta_2; w; \widetilde{y}, c) = K \tag{14}$$

As we have seen, the hiring rule is independent of w (since it is independent of y). If y is sufficiently large relative to K, we know that $E^{|\delta_1 \delta_2} V(\delta_1, \delta_2; w; \widetilde{y}, c) > K$, and there exists a wage w^* that satisfies (14). A formal proof of existence, as well as sufficient conditions on the parameters that ensure existence and production in each period, is given in Appendix C.

5 Exogenous Replacement

We now allow, with probability λ, for one worker to be thrown out of the relationship at the end of every period. If the worker is thrown out, the firm is forced to search in the next period.[4] Thus, if the replacement shock hits,

[4] With minor adjustments of the model, replacement can be interpreted as a change of position on the circle of one worker, due to learning to work better with other workers or,

one of the workers, chosen at random, has to be replaced. The firm can only hire one worker in any period, and hence will not voluntarily replace a second worker if hit by a replacement shock. If the shock does not hit, the firm may choose to replace one of its workers or not.

Suppose one worker is replaced by the firm as above. The transition probability for (δ_1, δ_2) was denoted by $\Gamma(\delta_1)$, and depends only on δ_1. We refer to this as the basic transition probability.

The forced transition probabilities are the transition probabilities which occur when one worker is forced to leave, to be denoted by $\Gamma^F(\delta_1, \delta_2)$. Which of the three incumbent workers leaves is random: with probability 1/3 the least well located worker leaves, in which case the transition probability is $\Gamma(\delta_1)$; with probability 1/3, the second best located worker leaves, in which case the transition probability is $\Gamma(\delta_2)$; with probability 1/3, the best located worker leaves, in which case the distance between the two remaining workers is $\min[\delta_1 + \delta_2, 1 - \delta_1 - \delta_2]$. It follows that the forced transition probabilities can be written as

$$\Gamma^F(\delta_1, \delta_2) = \frac{1}{3}\Gamma(\delta_1) + \frac{1}{3}\Gamma(\delta_2) + \frac{1}{3}\Gamma(\min[\delta_1 + \delta_2, 1 - \delta_1 - \delta_2]) \tag{15}$$

The Bellman equation now reads:

$$\begin{aligned} V(\delta_1.\delta_2) &= \pi(\delta_1, \delta_2) + \beta[\lambda E^{\Gamma^F} V_1(\delta_1', \delta_2') - c] \\ &\quad +(1-\lambda)E\max[V(\delta_1, \delta_2), EV^{\Gamma}(\delta_1^{'}, \delta_2^{'}) - c]] \end{aligned} \tag{16}$$

The first term in the bracket shows the expected NPV of the firm if the firm is hit by a replacement shock. The second term in the bracket shows the expected NPV if the firm is not hit by a replacement shock. It follows directly from Proposition 4 in Stokey and Lucas (1989, p. 522) that the value function exists.

With exogenous replacements, it is impossible to obtain a closed form solution for V. At this point we therefore turn to simulations.

6 Simulations: Exploring the Mechanisms

We undertake simulations in order to explore the mechanisms inherent in the model. This gives a sense of the model's implications for worker turnover, firm age, firm value and the connections between them. In particular, we examine the properties of the resulting firm value distributions and relate them to turnover policy.

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the opposite, the "souring" of relations.

6.1 The Set-Up

When simulating the model we look at the full model, with both endogenous and exogenous replacement and allowing for exogenous firm exit. Exogenous worker replacement occurs with a probability of λ. If the latter does not occur there is a decision on voluntary replacement. Both occur with a cost c. The firm exit shock occurs at the end of each period, after production has taken place, at a given rate s. Let $\overline{\beta}$ denote the pure time preference factor. When a firm is hit by this shock it stops to exist and its value in the next period is zero. Free-entry guarantees that in the next period this firm will be replaced by a new firm, and the latter will pay an entry cost K in order to get its first random triple of workers and start production. As long as the shock does not hit, the firm goes through periods of inaction and voluntary or forced replacement. Thus, in a given period, there coexist young and old firms.

The value function is:

$$\begin{aligned} V(\delta_1.\delta_2) &= \pi(\delta_1, \delta_2) + \beta[\lambda \cdot \left[E^{\Gamma^F} V(\delta_1', \delta_2') - c\right] \\ &\quad +(1-\lambda) \cdot \max[V(\delta_1, \delta_2), E^{\Gamma} V(\delta_1^{'}, \delta_2^{'}) - c]] \end{aligned}$$

where $\beta = \overline{\beta}(1-s)$. This value function can be found by a fixed point algorithm. Appendix D provides full details.

When simulating firms over time, we use the value function formulated above, and subtract from it $K = 3c$ in case a firm is new-born in a particular period.

We simulate 1000 firms over 30 periods, and repeat it 100 times to eliminate run-specific effects. In the benchmark case, we set: $y = 1, c = 0.01, \overline{r} = 0.04$ (the pure discount rate), $\lambda = 0.1, s = 0.1, K = 0.03$.

6.2 Firms Turnover Dynamics Over Time

In each period, depending upon the realization of the shocks and the optimal hiring decision, a firm might be in one of four states: inactive (there was no exogenous separation or firm exit shock, and no voluntary replacement); replacing voluntarily (there was no exogenous separation or firm exit shock and the firm chooses to replace); replacing while forced (there was no firm exit shock but there was an exogenous separation shock); doomed (there is a firm exit shock and in the next period a new triple is drawn with a cost K paid). The share of firms in each of above states by periods is shown in the following figure.

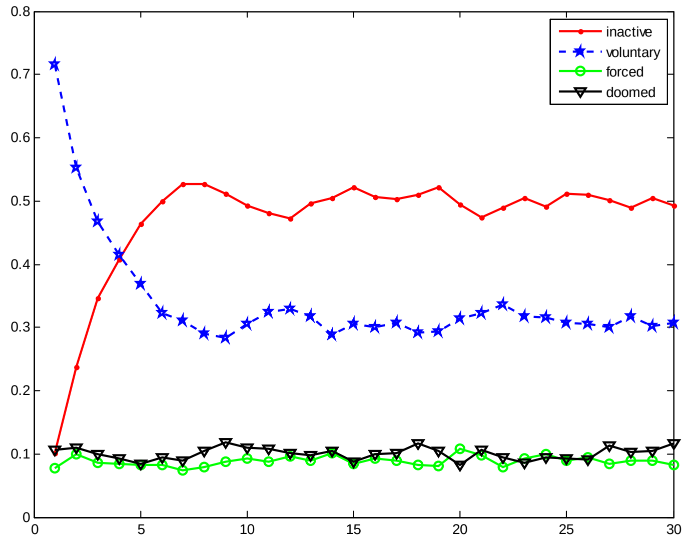

Figure 4: Shares of firms in different states

Figure 4 shows that it takes about 10 periods for the simulated sample to arrive at a regime in which the distribution of firms by states is relatively stable. Before that, there is a reduction in the share of firms engaged in voluntary replacement and an increase in the share of inactive ones, which reflects (temporary) arrival into the absorbing state. After period 10, when almost all firms have already experienced a re-start, as a result of the exit shock occurring at a 10% rate, turnover becomes more stable. The figure embodies two main forces that are in action: the process of convergence into the stopping region (disrupted from time to time by worker exits), and the perpetual entry of new firms with new triples drawn randomly. These turnover dynamics of the model are very much in line with the findings in Haltiwanger, Jarmin and Miranda (2010), whereby, for U.S. firms, both job creation and job destruction are high for young firms and decline as firms mature.

6.3 The Distribution of Firm Values

The following figures describe the cross-sectional distributions of log firm values, in selected periods, and the evolution of the moments of these distributions over time.

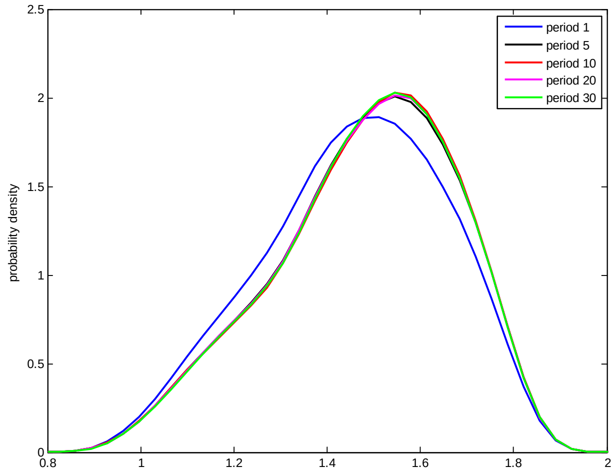

Figure 5a: Cross-sectional log firms values, by periods

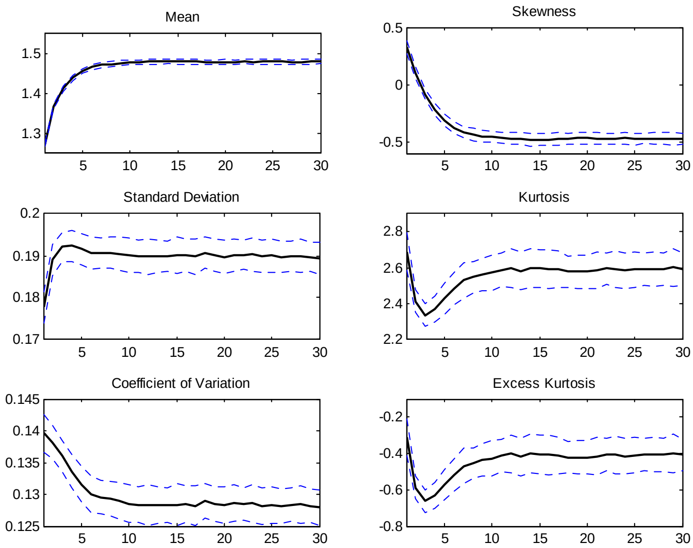

Figure 5b: Moments of cross-sectional log firm values, by periods

Figures 5a and 5b indicate that mean firm value rises and volatility rises in the early periods.[5] Not much changes after period 10. This is consistent with the movement of firms towards the SW corner of the state space $\delta_1 - \delta_2$ in Figure 3, and the constant inflow of new-born firms with all kinds of

[5]The confidence bands in the figures are the standard deviations of the moments, calculated over 100 runs.

teams. With time, the sample becomes more polarized: a group of firms is concentrated in the stopping region and low-value entrants flow in. As a result, extreme values become more frequent and excess kurtosis goes up (though it remains negative). Along the same lines, skewness turns more strongly negative, so that the left tail becomes thinner and more spread out.

6.4 Firm Value and Age

We repeat the computation of the distributions and their moments but now define them over firm age rather than over time. To construct the distributions of firm value by age we looked for all periods and all firms, when each particular age was observed. For example, due to the firm exit shock and the entry of new firms, age 1 will be observed not only for all firms in the first period, but also in all cases when a firm exogenously left and was replaced by a new entrant. In this manner we gathered observations of values for all ages, from 1 to 30, and built the corresponding distributions.

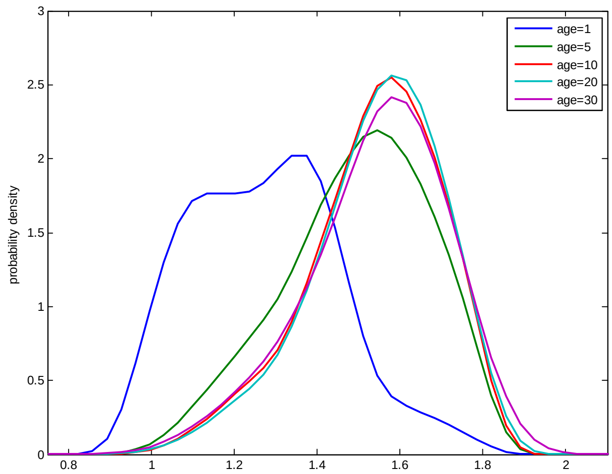

Figure 6a: Cross-sectional log firm values, by age

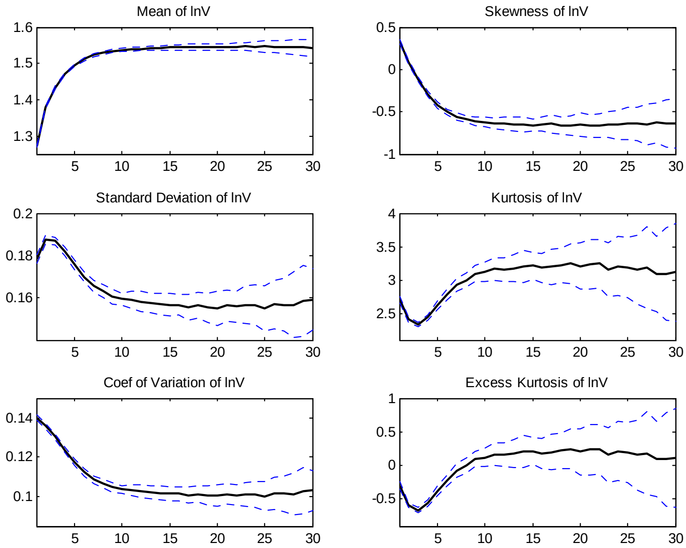

Figure 6b: Moments of cross-sectional log firms value, by age

The patterns are essentially the same as in Figures 5a and b which have related to time, not age, but here they reflect the pure process of convergence, disrupted from time to time by workers' exogenous exits, without the entry of new-born firms. The value of the firm grows with age as a result of team quality improvements, while the standard deviation is rather stable. As firms mature, more of them enter the absorbing state, with relatively high values, and at the same time there are always unlucky firms that do not manage to improve their teams sufficiently, or which have been hit by a forced separation shock. Therefore the distribution becomes more and more skewed over time. Excess kurtosis fluctuates.

Table 1 below presents further results on the connection between firm value and firm age. Here we look only at a simulated subsample of firms which have survived until the 30th period. There have been 45 such firms in our simulation. The estimated equation is:

$$\overline{\ln(V)}_t = c_0 + c_1 * \ln(t) \tag{17}$$

where $\overline{\ln(V)}_t$ is the average logged value of firms at age t, $t = 1, 2, ..., 30$.

Table 1

The Relation Between Firm Value and Age

Regression Results of Simulated Values, 1000 firms

c_1	0.05
	(0.01)
c_0	1.37
	(0.02)
R^2	0.62

The coefficients are highly significant and imply a positive relation, illustrated below:

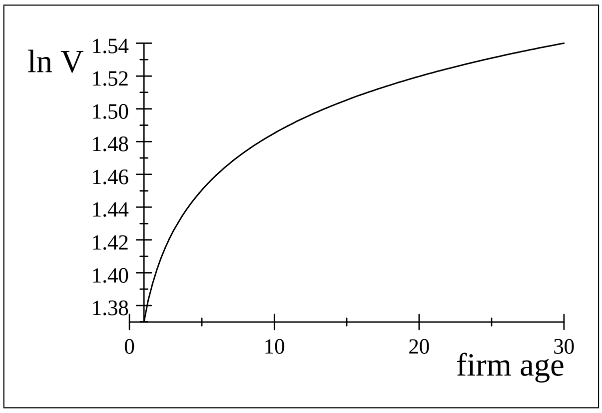

Figure 7: Predicted firm value (logs) and firm age

Figure 7 shows that overall, despite exogenous separation shocks, firms tend to increase in value as they mature, due to the improvement of their teams' quality.

6.5 The Role of Model Parameters

The core parameters of the model at the benchmark are the worker replacement cost, $c = 0.01$, the annual rate of interest, $r = 0.04$, the exogenous worker replacement rate, $\lambda = 0.1$, and the exogenous firm destruction rate, $s = 0.1$. In addition, we set the cost of entry at $K = 3c$ and the flow output at $y = 1$. Changes in these parameters affect the values of the firms both directly, through the value function and exogenous random events, and indirectly, through adjustments in the optimal hiring decisions. In what follows we analyze changes in these core parameters. Table E1 in Appendix E presents the moments of the log firm value distributions for given changes in the parameters relative to their benchmark values.

The following patterns emerge:

Increases in the cost of replacement c or in the interest rate r are illustrated in Figure 8a (and reported in rows 2-6 of Table E1):

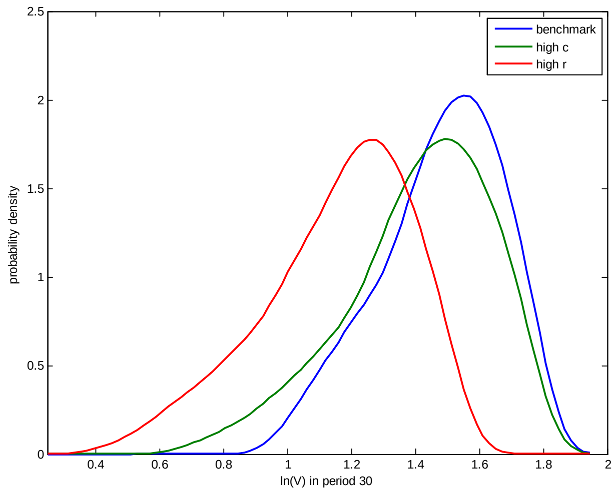

Figure 8a: effects of c and r

These two different increases affect the values distribution similarly: the mean value goes down, the coefficient of variation goes up, skewness becomes more negative and excess kurtosis goes up from negative to positive. Both higher costs of replacement and costs of time make the firms retain their teams rather than improve them; firms enter the stopping region more quickly, with worse teams than before and the mean value goes down. As firms tend to stay with their current, randomly-drawn, teams, firm values become more dispersed. Along the same lines, extreme values become relatively more frequent and excess kurtosis goes up. As inaction becomes optimal for so many firms, firms values become more concentrated above the mean. At the same time, in any period there are always unlucky firms, which have just obtained a very bad team as a result of the λ or s shock. Hence skewness becomes more negative. The sensitivity to the interest rate is higher than to changes in replacement costs. As described, under higher c or higher r the distribution has a longer left tail, lower mean, and fatter and longer tails relative to the benchmark.

Increases in the exogenous worker separation rate λ are illustrated in Figure 8b (and reported in rows 7-9 of Table E1).

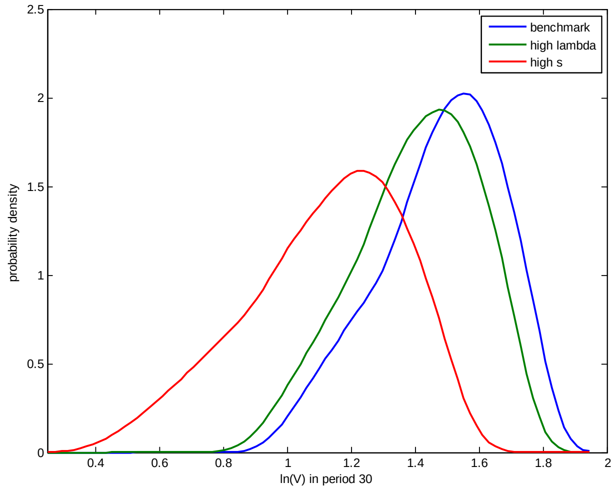

Figure 8b: effects of λ and s

Increased separation depresses the mean value, slightly increases the coefficient of variation, make the skewness less negative and kurtosis more negative. The possibility of a worker's exogenous exit is a burden on the firms, limiting their control over teams and the possibility to improve them. Hence the decrease in mean value. With optimization repeatedly disrupted by the shock, less firms are able to achieve the high-value steady state in each given period, there are less values concentrated above the mean, and skewness becomes less negative. Kurtosis becomes more negative as λ grows, implying that the bulk of the dispersion now comes from moderate deviations from the mean. Such a separation shock may hit any firm, occasionally throwing some firms out of the stopping region, or bringing other firms into it; the sample becomes more homogenous in terms of values, with extreme deviations from the mean less frequent, hence the negative excess kurtosis.

The simulated increases in the exogenous firm destruction rate s, also shown in Figure 8b, as well as in rows 10-12 in Table E1, bring the mean value down more than threefold, the coefficient of variation jumps more than tenfold, skewness becomes more negative and kurtosis becomes less negative. As there is a positive probability for any firm of being closed down in the next period, and due to the constant inflow of new-born firms which have not yet started to improve their teams, the mean value in the simulated cross-section goes down as s goes up. The inflow of random worker triples increases dispersion drastically, so the coefficient of variation goes up. As there are less firms in the stopping region and extreme values become more frequent, excess kurtosis goes up. The inflow of new firms with all kinds of values, including extremely low ones, makes the left tail of the distribution longer and the skewness more negative.

Going the other way and *shutting down exogenous worker separation and firm destruction*, $\lambda = s = 0$, presented in row 13 of Table E1, has firms just smoothly converge to the stopping region. Removing exogenous uncertainty improves the mean value drastically and it is higher than in any other specification. The coefficient of variation is low, as a result of massive convergence. Likewise, excess kurtosis is substantially negative. Skewness is slightly negative as there is no drag on value as a result of some unlucky firms being hit by a shock or replaced, with all the firms allowed to converge (and they do so by period 30).

To sum up, each of the parameters above has an impact on the process of convergence into the stopping region. The factors that facilitate stopping, such as high c and r or low λ produce higher concentration of firms in the stopping region and therefore make skewness more negative. The replacement of old firms by new ones does not impact the process of convergence directly. It adds new triples everywhere, thereby lengthening the left tail of the distribution and adding more extreme values – skewness becomes more negative and excess kurtosis goes up. The factors that impede firms, namely high c, high r, high λ or high s decrease the mean value. The factors that make the firms stop quickly wherever they are (high c or r), or add new triples exogenously, such as high s, make the values more dispersed, distribution tails fatter and excess kurtosis higher.

6.6 Comparing the Simulated Model to the Data

In order to see how the simulated value distributions compare to the data, we take the 2010 Compustat data sample of 4,293 U.S. firms. We use SIC codes to select firms belonging to industries in which human capital is more important in creating value than physical capital, namely firms in financial services, health care and IT. This leaves us with 1,440 firms. We divide firm value by employment and take logs in order to be able to compare the data to log firm values in our model, where all firms have the same number of employees.

The properties of the value distributions in the data vary greatly with the size of the firms. The sample is highly heterogenous in terms of employment: there are lots of small firms, with employment less than 100 workers, and there are huge firms, with employment above 400,000, so that mean employment is around 6,000 workers. The (approximate) distribution of firms in the sample by employment and the corresponding moments of their value distribution are given in Table 2a.

Table 2a
Firms Values by Employment Quantiles

No. employees	Quantile	mean	c.o.v.	skewness	ex. kurtosis
< 100	1	14.32	0.11	−0.01	0.18
100 – 350	2	13.44	0.10	0.00	−0.33
350 –1100	3	13.25	0.10	−0.22	−0.33
1100 – 4500	4	12.92	0.11	−0.24	−0.21
> 4500	5	12.36	0.13	−0.56	0.15
All firms		13.25	0.12	−0.16	0.49

As seen in Table 2a, mean value per worker declines with the size of the firms, the coefficient of variation goes up, and skewness becomes more negative. Excess kurtosis is positive in the smallest and in the biggest firms, and negative in firms in the three middle quantiles. The full sample is moderately left-skewed and has clear positive excess kurtosis.

The table suggests that the distribution of value per worker in the firms in the middle of the size range is rather close to what most specifications of the model predict (see Figure 5b above). In addition, the simulated distribution under high costs (see row 3 in Table E1) is close to the distribution of values in the upper size quantile. When we merge the three middle quantiles, that is, cut out the 20% smallest and 20% biggest firms, we get moments close to the simulated ones. Even more similarities emerge when we cut 33% from both ends. Table 2b report these comparisons.

Table 2b
Truncated Data Sample and the Simulated Model

Subsample	c.o.v.	skewness	excess kurtosis
without 20% biggest and 20% smallest	0.10	−0.18	−0.20
without 33% biggest and 33% smallest	0.10	−0.24	−0.34
benchmark model[6]	0.13	−0.47	−0.40
low interest rate	0.10	−0.39	−0.53
low exogenous worker replacement rate	0.12	−0.58	−0.27

In Figure 9 we present the value distributions for two cross-sections of the data (33% truncation, second row in Table 2b, and biggest firms, the upper quantile in Table 2a) and two simulated distributions (rows 1 and 3 in Table E1):

[6] "benchmark" refers to the specification of row 1 in Table E1, "low interest rate" refers to row 4 in Table E1, "low exogenous worker replacement rate" refers to row 8 in Table E1.

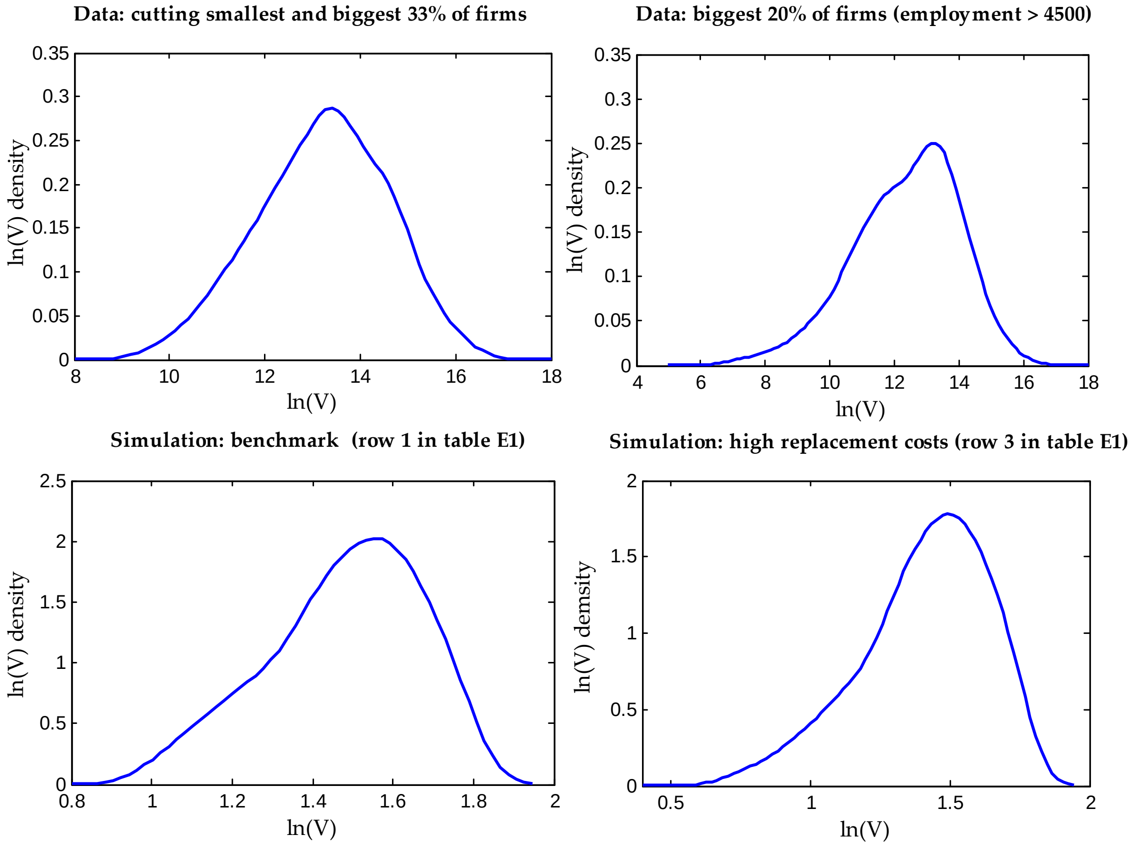

Figure 9: Distribution of ln V – simulation and data

Overall, the same general pattern prevails: the simulation is more negatively skewed than the data and has a lower (i.e., more negative) excess kurtosis, meaning that extreme values are less frequent than in the data. The two main mechanisms in the model, namely the convergence of firms into the stopping region on the one hand and the perpetual inflow of random new-born firms on the other, generate these differences. When firms converge to the stopping region, and new-born firms appear in the left tail of the distribution, skewness becomes more negative. In this respect, in order to get closer to the data and to moderate skewness, we would like firms to converge less (higher λ) and the left tail to be less long and fat, compared to the group in the stopping region (low s). On the other hand, in order to achieve higher excess kurtosis we would like the simulated sample to become less bounded, with extreme values being more frequent. This means more firms in the absorbing state in each period and a longer and fatter left tail, which requires lower λ and higher s.

That said, our goal is not to fit the data but rather to show that despite being highly stylized, the model has driving forces which produce distributions of firm values that are not far off the empirical ones. Indeed, the figures and the tables above suggest that when parameters are set at plausible levels, the model works quite well for middle-sized firms.

7 Discussion

Our model builds on several strong assumptions regarding technology, wage determination, search behaviour, etc. We turn now to a brief discussion of these assumptions in light of the analysis.

One important underlying assumption is that workers are horizontally but not vertically differentiated. From an *ex ante* perspective, workers are identical, while *ex post* the workers may work more or less well together. Our assumption reflects a view that an interesting part of team formation is related to horizontal differences, i.e., finding workers that work particularly well together. Of course finding the correct mix of workers with respect to productivity (ability, "types") may also be important. However, we would expect that workers of different types segregate more easily in the labor market, searching in different markets and requiring different wages.

Our second assumption is the use of the Salop circle. We choose the Salop circle because it easily captures the notion that if A works well with B and B with C, then A and C are also likely to work well together. There may exist other stochastic structures that capture the same type of regularities, but the Salop structure does so in a particularly nice and tractable way. Note that we could alternatively let output depend positively on the difference between the workers, in order to capture a love of variation. To some extent this may be a matter of interpretation of what a good match is.

A third assumption we make is that distances between the agents are measured going via the middle worker. Above we motivated this by arguing that the firm chooses the middle worker as the focal point. For instance, if location refers to physical location, the firm is placed at the location of the middle worker. In this context the results of the afore-cited MIT study results, reported in Pentland (2012), justify this modelling choice. If the firm has to choose a modus operandi, which may be represented as a location on the circle, it also chooses the location of the middle worker.[7] Furthermore,

[7] In these cases one may argue that it is only the sum of the distances from the two peripheral to the middle worker that matters, not the distance between the two peripheral workers. However, this is not important, as it is only a matter of scaling. If only the distance to the middle worker matters, total distance is $\delta_1 + \delta_2$, while in our case it is $2(\delta_1 + \delta_2)$.

without this assumption, the shortest distance between the two peripheral workers will sometimes not go through the middleman, and in these cases total distance is independent of the agents' position on the circle. This is against the spirit of our paper, and in addition it is analytically cumbersome. Note also that if $\overline{\delta}_2 < 1/4$, the shortest distance between the peripheral workers will always go via the middle worker in the region close to the stopping region.

We assume that wages are independent of match quality. As mentioned above, this is consistent with a competitive market where firms bid for *ex ante* identical workers prior to knowing the match quality. An alternative formulation would be to allow for bargaining, in which case part of the surplus from a good match would be allocated to the worker. This may give rise to an interesting hold-up problem, if the firm pays the entire cost of replacing the worker and only gets a fraction less than one of the return in terms of a better match. In addition, the workers may receive rents, which may be dissipated through unemployment. This would alter the nature of the equilibrium. Furthermore, in the present version of the model, workers have no incentives to do on-the-job search, as wages are the same across firms. With wage bargaining, workers may have an incentive to search for a new job, and bargaining may therefore lead to on-the-job search.

Throughout we have assumed that the efficiency of a given team stays constant over time. Although a natural assumption as a starting point, one may think that the quality of a team may develop over time. As the employees get to know each other better, their ability to communicate and collaborate may improve. On the other hand, good relationships may get sour over time. Introducing dynamics of team quality may lead to interesting hiring patterns. For instance, a firm that has been passive for a while may start a replacement frenzy if the relationship suddenly sours. This is on our agenda for future research.

8 Conclusions

The paper has characterized the firm in its role as a coordinating device. Thus, output depends on the interactions between workers. The paper has derived optimal policy, using a threshold on a state variable and allowing for endogenous hiring and firing. Firm value emerges from optimal coordination done in this manner and fluctuates as the quality of the interaction between the workers changes. Simulations of the model generate non-normal firm value distributions, with negative skewness and negative excess kurtosis. These moments reflect worker turnover dynamics, whereby a large mass of firms is inactive in replacement, having attained good team formation, while exogenous replacement and firm exit induce dispersion of firms in the region of lower value. Future work will examine alternative production functions, learning and training processes, and wage-setting mechanisms within this set-up.

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Appendix A. Derivation of Equation (12)

Substituting (8) into (11) gives

$$\begin{aligned}\frac{y-2(\delta_1+\overline{\delta}_2(\delta_1))}{r}(1+r)+c &= y-(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2}) \\ &+\frac{(\delta_1+2\overline{\delta}_2)(y-2(\delta_1+\overline{\delta_2}))+2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2}{r} \\ &+(1-\delta_1-2\overline{\delta}_2)\frac{y-2(\delta_1+\overline{\delta}_2(\delta_1))}{r}\end{aligned} \tag{18}$$

Collecting all terms containing $y-2(\delta_1+\overline{\delta}_2(\delta_1))$ on the left-hand side gives

$$\begin{aligned}&\frac{y-2(\delta_1+\overline{\delta}_2(\delta_1))}{r}[1+r-(\delta_1+2\overline{\delta}_2)-(1-(\delta_1+2\overline{\delta}_2))]+c-y \\ = \quad & -(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2})+\frac{2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2}{r}\end{aligned}$$

which simplifies to

$$-2(\delta_1+\overline{\delta}_2(\delta_1))+c=-(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2})+\frac{2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2}{r}$$

Collecting terms give

$$\frac{1}{2}+\frac{\delta_1^2}{2}-\delta_1-2\overline{\delta}_2(\delta_1)=\frac{2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2}{r}$$

which is (12).

Appendix B. Proof of Proposition 2

We repeat the cut-off equation for convenience

$$c + \frac{1}{2} + \frac{\delta_1^2}{2} - \delta_1 - 2\overline{\delta}_2 = \frac{2\delta_1\overline{\delta}_2 + 2\overline{\delta}_2^2}{r} \tag{19}$$

If $\overline{\delta}_2 = 0$, the left-hand side of (19) is strictly positive while the right-hand side is zero (since $\delta_1 \leq 1/3$ by construction). As $\overline{\delta}_2 \to \infty$, the left-hand side goes to minus infinity and the right-hand side to plus infinity. Hence we know that the equation has a solution. Since the left-hand side is strictly decreasing and the right-hand side strictly increasing in $\overline{\delta}_2$, we know that the solution is unique.

In the text we have already shown that $\overline{\delta}_2(\delta_1)$, if it exists, is decreasing in δ_1. It follows that δ^* can be obtained by inserting $\overline{\delta}_2 = \delta_1 = \delta^*$ in (19). This gives

$$c + \frac{1}{2} + \frac{\delta^{*2}}{2} - \delta^* - 2\delta^* = \frac{2\delta^*\delta^* + 2\delta^{*2}}{r}$$

Hence δ^* is the unique positive root to the second order equation

$$c + \frac{1}{2} - \delta^{*2}\frac{8 - r}{2r} - 3\delta^* = 0$$

Appendix C. Proof of Existence of Equilibrium

Define

$$\overline{V} \equiv E^{|\delta_1\delta_2} V(\delta_1, \delta_2; 0; \widetilde{y}, c)$$

Given our assumption that the firm always produces until it is destroyed, it follows that

$$E^{|\delta_1\delta_2} V(\delta_1, \delta_2; w; \widetilde{y}, c) = \overline{V} - \frac{W}{r'}$$

where $r' = r/(1+r)$ and where, as above, $W = 3w$. By assumption, $\overline{V} > 0$ (see below). It follows that there exists a unique W that solves the zero-profit condition given by

$$\overline{V} - \frac{W}{r'} = K$$

The solution is given by $W = r'(\overline{V} - K)$.

We will give conditions on parameters that ensure that $\overline{V} > 0$,and that firms, if entering, will produce even after the worst possible draws. The supremum of per-period output is $\widetilde{y}$ (obtained with $\delta_1 = \delta_2 = 0$). It follows that

$$\overline{V} < \frac{\widetilde{y}}{r'}$$

Suppose

$$K > \frac{4}{3}\frac{1}{r'} \tag{20}$$

From the zero profit condition it then follows that

$$W = r'(\overline{V} - K) < \widetilde{y} - 4/3 \tag{21}$$

The infimum of per period profit is $\pi^{\inf} = \widetilde{y} - 4/3 - W$ (obtained when $\delta_1 = \delta_2 = 1/3$). From (21) it follows that

$$\pi^{\inf} = \widetilde{y} - 4/3 - W > 0$$

Hence a sufficient condition for firms to operate after the lowest possible draws is that (20) is satisfied.

We assume that the lower bound on wages is that $W \geq 0$. To ensure that $\overline{V} > K$, note that

$$\overline{V} > \frac{\widetilde{y} - 4/3}{r'}$$

since $\widetilde{y} - 4/3$ is the lowest per period output and a firm can always choose not to replace. Entry occurs in equilibrium if and only if it is profitable to enter when $W = 0$. Hence a sufficient condition for entry to occur i is that $\frac{\widetilde{y}-4/3}{r'} > K$ or that $\widetilde{y} \geq r'K + 4/3$ (tighter bounds can also be found).

Appendix D. The Simulation Methodology

The entire simulation is run in Matlab with 100 iterations. In order to account for the variability of simulation output from iteration to iteration, we report the average and the standard deviation of the moments and the probability density functions, as obtained in 100 iterations.

Calculating the Value Function

We find the value function V numerically for the discretized space (δ_1, δ_2), using a fixed-point procedure. First we guess the initial value for V in each and every point of this two-dimensional space; we then mechanically go over all possible events (exit, in which case the value turns zero, forced or voluntary separation, with the subsequent draw of the third worker) to calculate the expected value in the next period, derive the optimal decision at each point (δ_1, δ_2), given the initial guess V, and thus compute the RHS of the value function equation below:

$$V(\delta_1.\delta_2) = \pi(\delta_1, \delta_2) + \beta \left[s \cdot 0 + (1-s) \cdot \left(\begin{array}{c} \lambda \cdot \left[E^{\Gamma^F} V(\delta_1', \delta_2') - c \right] \\ +(1-\lambda) \cdot E \max[V(\delta_1, \delta_2), E^{\Gamma} V(\delta_1^{'}, \delta_2^{'}) - c] \end{array} \right) \right] \tag{22}$$

Next, we define the RHS found above as our new V and repeat the calculations above. We iterate on this procedure till the stage when the discrepancy between the V on the LHS and the RHS is less than the pre-set tolerance level.

The mechanical steps of the program are the following:

1. We assume that each of δ_1, δ_2 can take only a finite number of values between 0 and 1. We call this number of values BINS_NUMBER and it may be changed in the program.

2. However, not all the pairs (δ_1, δ_2) are possible, as by definition $\delta_2 \geq \delta_1$ and $\delta_2 \leq \frac{1}{2} - \frac{\delta_1}{2}$ (the latter ensures that the distances are measured "correctly" along the circle). We impose the above restriction on the pairs constructed earlier, and so obtain a smaller number of pairs, all of which are feasible. Note that all the distances in the pairs are proportionate to 1/BINS_NUMBER

3. In fact, the expected value of forced and voluntary replacement, $E^{q^F} V(\delta_1', \delta_2')$ and $EV^q(\delta_1^{'}, \delta_2^{'})$, differ in only one respect: when the replacement is voluntary, two remaining workers are those with δ_1 between them, whereas when the replacement is forced, it might be any of the three: δ_1, δ_2 or $\min((\ \delta_1 + \delta_2), 1 - (\ \delta_1 + \delta_2))$, with equal probabilities. In the general case, if there are two workers at a distance δ, and the third worker is drawn

randomly, possible pairs in the following period may be of the following three types: (i) δ turns out to be the smaller distance (the third worker falls relatively far outside the arch), (ii) δ turns out to be the bigger distance (the third worker falls outside the arch, but relatively close) (iii) the third worker falls inside the arch, in which case the sum of the distances in the next period is δ. In the simulation we go over all possible pairs to identify the pairs that conform with (i)-(iii). Note that because all the distances are proportionate to 1/BINS_NUMBER, it is easy to identify the pairs of the type (iii) described above. This can be done for any δ, whether it is δ_1, δ_2 or $\min((\delta_1 + \delta_2), 1 - (\delta_1 + \delta_2))$

4. Having the guess V, and given that all possible pairs are equally probable, we are then able to calculate the expected values of the firm when currently there are two workers at a distance δ. Call this value $EV\left(\delta\right)$. Then, if there is a firm with three workers with distances $(\delta_1 . \delta_2)$, the expected value of voluntary replacement is $EV\left(\delta_1\right)$, and expected value of forced replacement is $1/3 \cdot EV\left(\delta_1\right) + 1/3 \cdot EV\left(\delta_2\right) + 1/3 \cdot EV\left(\min((\delta_1+\delta_2), 1-(\delta_1+\delta_2))\right).$ Thus we are able to calculate the RHS of equation (22) above and compare it to the initial guess V.

We iterate the process till the biggest quadratic difference in the values of LHS and RHS, over the pairs (δ_1, δ_2), of equation (22) is less than the tolerance level, which was set at 0.0000001.

Dynamic Simulations

Once the value function is found for all possible points on the grid, the simulation is run as follows.

1. The number of firms (N) and the number of periods (T) is defined. We use $N = 1000, T = 30$.

2. For each firm, three numbers are drawn randomly from a uniform distribution $U[0,1]$ using the Matlab function *unifrnd.*

3. The distances between the numbers are calculated, the middle worker is defined, and as a result, for each firm a vector (δ_1, δ_2) is found.

4. For each firm, the actual vector (δ_1, δ_2) is replaced by the closest point on the grid found above $\left(\widetilde{\delta}_1, \widetilde{\delta}_2\right)$.

5. According to $\left(\widetilde{\delta}_1, \widetilde{\delta}_2\right)$, using the calculations from previous section, we assign to each firm the value and the optimal decision in the current period.

6. It is determined whether an exit shock hits. If it does, instead of the current distances of the firm, a new triple is drawn in the next period. If it does not, it is determined whether a forced separation shock λ hits. If λ hits, a corresponding worker is replaced by a new draw and distances are recalculated in the next period. If it does not, and it is optimal not to replace, the distances are preserved for the firm in the next period, as well as the value. If it is optimal to replace, the worst worker is replaced by a new one, distances are re-calculated in the next period, together with the value.

Steps 4-6 are repeated for each firm over all periods.

As a result, we have a T by N matrix of firm values. The whole process is iterated 100 times to eliminate run-specific effects. We also record the events history, in a T by N matrix which assigns a value of 0 if a particular firm was inactive in a particular period,1 if it replaced voluntarily, 2 if it was forced to replace, and 3 if it was hit by an exit shock and ceased to exist from the next period on. We use this matrix to differentiate firms by states and to calculate firms' ages.

9 Appendix E. Changes in Parameters

Table E1
The Effects of Changes in Parameters

	Parameters				**Moments of ln(V) in period 30**			
	c	r	λ	s	mean	coef. of var.	skewness	excess kurtosis
1	0.01	0.04	0.1	0.1	1.46	0.13	−0.47	−0.40
2	0.05	–[8]	–	–	1.45	0.14	−0.55	−0.28
3	0.10	–	–	–	1.44	0.16	−0.68	0.06
4	–	0.01	–	–	1.60	0.10	−0.39	−0.53
5	–	0.04	–	–	1.46	0.13	−0.47	−0.40
6	–	0.10	–	–	1.15	0.20	−0.72	0.02
7	–	–	0	–	1.73	0.11	−0.67	−0.04
8	–	–	0.05	–	1.58	0.12	−0.58	−0.27
9	–	–	0.15	–	1.46	0.13	−0.41	−0.48
10	–	–	–	0	2.82	0.02	−0.21	−0.52
11	–	–	–	0.05	1.86	0.07	−0.41	−0.40
12	–	–	–	0.15	1.09	0.22	−0.53	−0.32
13	–	–	0	0	3.11	0.02	−0.12	−0.49

The implications of these changes are discussed in Sub-section 6.5.

[8] As in the benchmark, row 1.