**Research Department** **Bank of Israel**

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# A Real Model of the Israeli Economy

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**Abstract**

This paper presents a real open-economy model of the Israeli economy. The model is designed to perform quarterly macroeconomic forecasts and fiscal policy experiments. This is illustrated by the analyses of pre-announced tax cuts and lowering the public debt target – policies which were adopted during the fiscal consolidation process in Israel during the 2000s.
The model is neoclassical in nature, but with some non-standard features such as a liquidity effect on nondurable consumption and on durable goods purchases. It includes dynamic optimization of households and firms, and a government which determines expenditures given exogenous tax rates and a public debt target.

The model is estimated using the sample of the last decade. The quantitative results include the following: (i) Announcements of future income tax cuts have a simultaneous expansionary effect via consumer demand, but, at the time of implementation, the necessary cuts in government expenditure have a contractionary effect. (ii) The fiscal multiplier is relatively low, 0.4, a result that stems from the openness of the economy and the substitution between GDP and imports. (iii) The economy is sensitive to external world trade shocks that affect not only exports but also private consumption through the wealth and liquidity effects.

# מודל מאקרו כלכלי ריאלי של המשק הישראלי

**צבי הרקוביץ ועמית פרידמן**

**תקציר**

מאמר זה מציג מודל של משק פתוח עבור המשק הישראלי, אשר משמש לעריכת תחזיות רבעוניות עבור המשתנים המאקרו-כלכליים הריאליים המרכזיים, וככלי להערכת ההשפעות של שינויים במדיניות הפיסקאלית. כהדגמה, מוצגות ההשפעות של הפחתות מיסים ידועות מראש, והפחתה של יעד יחס החוב לתוצר בטווח הארוך. שינויי מדיניות אלו אפיינו את המדיניות הפיסקאלית בישראל בשנות האלפיים.
המודל הוא ניאו-קלסי בעיקרו, אבל כולל כמה אלמנטים ייחודיים, כגון השפעת נזילות על הצריכה הפרטית ורכיביה – הצריכה השוטפת ורכישת מוצרים בני-קיימא. המודל מורכב מפירמות ומשקי בית המבצעים אופטימיזציה דינאמית, וממשלה הקובעת את הצריכה הציבורית בהינתן יעד של יחס חוב לתוצר ושיעורי מס. תחת משטר פיסקאלי כזה, הכרזה על הפחתת מסי הכנסה בעתיד פועלת לגידול הפעילות הכלכלית באופן מידי, אך כאשר ההפחתה תבוצע, היא תלווה בקיצוץ הצריכה הציבורית המקזזת חלקית את ההשפעה המרחיבה של הפחתת המס.
המודל נאמד תוך שימוש בנתוני המשק בעשור האחרון. התוצאות מצביעות על כך שהמכפיל הפיסקאלי בישראל הוא 0.4 בלבד – נמוך יחסית – תוצאה הנובעת מפתיחותו של המשק והתחלופה בין יבוא ותוצר. עוד נמצא כי המשק רגיש מאד לזעזועים בסחר העולמי, שמשפיעים לא רק על היצוא אלא על הצריכה הפרטית דרך אפקט ההכנסה.

## Contents

## 1. Introduction

This paper presents a real, small-scale and quantitative open-economy model. Application of the model to the Israeli economy serves two purposes. One is to perform short and medium-term quarterly forecasts of real economic activity—the main national accounting aggregates, employment, and key relative prices. The other purpose is to conduct fiscal policy experiments. Specifically, we use the model to study the macroeconomic effects of the Israeli fiscal consolidation between 2003 and 2008, a period during which the government share in GDP was reduced, debt to GDP ratio declined, and a pre-announced tax-cut plan was implemented.

The model is built as an additional supporting tool for policy assessment at the Bank of Israel. While the typical "workhorse" model in central banks is New Keynesian, here we present a neoclassical model, with non-standard elements such as a liquidity effect on household decisions.

The model has the following general features. Identical households and firms perform dynamic optimization within a competitive environment. The government determines its expenditure level according to a rule described below. The interest rate is exogenously given from abroad, and hence the economy is small and open for the world capital markets. The goods market and the labor market are endogenously solved in the model, along with the prices of imports and labor relative to that of output.

The goods market can be summarized as follows. Aggregate supply is based on a production function for final output, rather than the commonly used production function for domestic product. The production function includes three factors: capital, labor, and imports—all of which are treated as intermediate products. The productive role of imports implies that the resulting aggregate supply function is decreasing in the relative price of imports.

Aggregate demand includes private and public consumption, investment and exports. The economy's output does not have a perfect substitute abroad; hence, as an exporter, the economy is not price taker. The world demand for the economy's exports is an increasing function of the price of a substitute abroad relative to domestic output. We assume that the price of that substitute relative to imports is exogenous. Therefore, given that exogenous relative price, the demand for exports is an increasing function of the relative price of imports. Because the economy exports the domestic output, the relative price of imports coincides with the terms of trade.

The relative price of imports is determined so as to clear the goods market: it equates the aggregate demand to the aggregate supply of domestic output. This equilibrium concept is based on Bruno and Sachs (1985). In this model, the current account is balanced only in the long run.

In the present setup, given that all imports are treated as intermediate products, the degree of openness of the economy, i.e., the ratio of imports and exports to GDP is determined by the structural parameters of the production function—thus, it is a function of technology.

Labor market outcomes are determined as follows. The labor market equilibrates only in the long run, when the real wage is at the intersection of labor demand with an inelastic labor supply curve. In the short run, however, employment deviates from its equilibrium level due to a sticky real wage process: employment is determined by labor demand given the real wage, which gradually adjusts to the long-run market-clearing level.

This modeling strategy captures standard short-run labor market dynamics—e.g., a labor demand shock increases simultaneously employment and the real wage—even though the model does not include a short-run labor supply decision.

Specific features of private consumption are the following. First, households obtain utility from the consumption of a nondurable good and the stock of a durable good. This distinction is relevant for addressing the observed higher volatility of durable purchases than nondurable consumption. Another feature, less standard in macroeconomic models, is a liquidity type of effect. We model this effect by an exogenous target level of net assets, deviations from which involve a cost—interpreted as reflecting too much or too little liquidity. To illustrate the liquidity effect, suppose a temporary increase in income. Increasing net assets to smooth consumption generates too much liquidity and thus the consumption response is larger than predicted under permanent income theory. This is the "excess sensitivity" property of consumption (Flavin, 1985). In contrast, when income goes up permanently, the upward adjustment of the durable goods stock requires borrowing—or reducing net assets level below target. This liquidity shortage generates a gradual adjustment of durable and nondurable goods to the new optimal level—rather than an immediate jump as under permanent income theory. This is the "excess smoothness" of consumption (Deaton, 1987).[1] This modeling strategy results in a consumption behavior that is also consistent with the empiri-

[1]Moreover, in our model, a permanent income increase in the future may even result in an on-impact decrease in private consumption. The known increase in income requires an increase in investment, which triggers a liquidity shortage.

cal findings of Lavi (2004), who found that consumption in Israel is well described by a Campbell-Mankiw consumption equation, that is, a mix of permanent income agents and liquidity constrained agents, and that the fraction of liquidity constraints agents is sizable.

An additional implication of introducing an assets target is that the model possesses a steady state. Otherwise, given the exogenous foreign interest rate, assets would follow a random walk, and, thus, the long-run variances of assets and other variables would be infinite.

In contrast with what is generally assumed, government expenditure in this model are endogenous to the debt path and expected tax revenue. The public debt/output ratio target, the rate of convergence to that target, and the tax rates are exogenous. This setup seems relevant for the modelling of fiscal policy in Israel since 2003. In that year, a multi-year tax-cut program was announced, and a commitment to reduce the debt to GDP ratio was made. Between the years 2003 and 2008, the tax-cut program was implemented fully, and the debt to GDP ratio decreased from 100 to 76 percent. Accordingly, government consumption dropped from 28 percent of GDP in 2003 to 24 percent in 2008. In December 2009, in spite of the global economic crisis, the government renewed its commitment to cut tax rates and to further reduce the public debt/GDP ratio to 60 percent—the Maastricht benchmark—in about a decade. This setup stands in contrast to Barro's (1979) tax smoothing framework, where the tax rate is adjusted to exogenous changes in government expenditure, and the optimal debt is a random walk.

As mentioned above, all imports are treated as intermediate inputs in the production of domestic output. This treatment of imports is supported by the following observations: (1) Raw materials for production account for about 50 percent of goods imports. (2) The domestic market prices of the remaining imports—investment and consumption goods—include a large domestic value added share, composed of importers' services, domestic transportation and taxation. Market prices of imported investment goods may also include installation costs.

The model is partially calibrated and partially estimated. The parameters determining the steady-state levels are calibrated using long-run average ratios, while the parameters affecting the dynamics are estimated using quarterly data for the period between 1999 and 2010.

The paper proceeds as follows. Section 2 presents the production technology and the representative firm's optimization problem, and Section 3 does likewise for preferences and the representative household's optimization problem. Sections

4 and 5 describe government behavior and the role of the rest of the world in the model. The equilibrium conditions are addressed in Section 6, and Section 7 presents the system of equations governing the model's dynamics. Section 8 reports the calibration and estimation of the model's parameters. The parameterized model is implemented and discussed in Section 9 with impulse responses to the model's shocks and in Section 10 with the analysis of fiscal policy changes. Concluding remarks and possible extensions are given in Section 11.

## 2. Firms

In period $t$, the representative firm produces output $Q_t$ according to the Cobb-Douglas technology

$$Q_t = Y_t^{\gamma} M_t^{1-\gamma}, \qquad 0 < \gamma < 1, \tag{2.1}$$

where $Y_t$ is domestic value added and $M_t$ is imports of intermediate products. All imports are considered intermediate products.

Value added or GDP, $Y_t$, is produced with capital, $K$, and labor input $L$:

$$Y_t = K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{2.2}$$

where $A_t$ is stochastic labor-augmenting technology.

Substituting (2.2) into (2.1), we get

$$Q_t = A_t^{(1-\alpha)\gamma} K_t^{\alpha\gamma} L_t^{(1-\alpha)\gamma} M_t^{1-\gamma}. \tag{2.3}$$

The capital stock evolves according to

$$K_{t+1} = K_t(1-\delta^k) + I_t, \quad 0 < \delta^k < 1, \tag{2.4}$$

where $I_t$ is gross investment and $\delta^k$ is the depreciation rate of capital.

Changing the capital stock involves the adjustment cost $J_t^k$, of the form

$$J_t^k = \frac{\omega^k}{2} \left(K_{t+1} \exp(\upsilon_t^k) - K_t\right)^2, \quad \omega^k > 0, \tag{2.5}$$

where $\omega^k$ is a parameter governing the magnitude of these costs, and $\upsilon_t^k$ is a stochastic shock. This shock increases the adjustment costs, and it turns out to work as a negative shock to investment.[2]

[2]If the shock multiplies $\left(K_{t+1} - K_t\right)^2$, rather than $K_{t+1}$, the coefficient of the linear term on

The firm takes prices as given. In terms of domestic output these prices are the wage, $W_t$, and the price of imports $P_t^m$.

The after tax dividend paid by the firm to the shareholders in period $t$ is

$$\Pi_t = (1-\tau_t^c)\left[A_t^{(1-\alpha)\gamma}K_t^{\alpha\gamma}L_t^{(1-\alpha)\gamma}M_t^{1-\gamma} - W_tL_t - P_t^m M_t\right] - J_t^k - K_{t+1} + K_t(1-\delta^k), \tag{2.6}$$

where $\tau_t^c$ is the corporate tax rate. Note that in this case, the depreciation of the capital stock and the cost of adjustment of the stock are not tax deductible. We also assume that firms are fully owned by the local households.

## 2.1. The Firm's Optimization Problem

The firm maximizes the expected sum of discounted dividends

$$E_0\left\{\Pi_t + \frac{\Pi_{t+1}}{1+r_t} + \frac{\Pi_{t+2}}{(1+r_t)(1+r_{t+1})} + ...\right\}$$

where $r_t$ is the real interest rate in period $t$.

The first-order conditions are:

$$1+\omega^k\left(K_{t+1}\exp(\upsilon_t^k) - K_t\right)\exp(\upsilon_t^k) = \frac{1}{1+r_t}E_t\left[\begin{array}{c}\left(1-\tau_{t+1}^c\right)\alpha\gamma E_t Q_{t+1}/K_{t+1} + 1 - \delta^k \\ +\omega^k\left(K_{t+2}\exp(\upsilon_{t+1}^k) - K_{t+1}\right)\exp(\upsilon_{t+1}^k)\end{array}\right]. \tag{2.7}$$

$$W_t = (1-\alpha)\gamma A_t^{(1-\alpha)\gamma}K_t^{\alpha\gamma}L_t^{(1-\alpha)\gamma-1}M_t^{1-\gamma}, \tag{2.8}$$

$$P_t^m = (1-\gamma)A_t^{(1-\alpha)\gamma}K_t^{\alpha\gamma}L_t^{(1-\alpha)\gamma}M_t^{-\gamma}. \tag{2.9}$$

In the absence of adjustment costs, (2.7) reduces to the familiar equality

$$\left(1-\tau_{t+1}^c\right)E_t\alpha\gamma\left(A_{t+1}^{(1-\alpha)\gamma}K_{t+1}^{\alpha\gamma-1}L_{t+1}^{(1-\alpha)\gamma}M_{t+1}^{1-\gamma}\right) = r_t + \delta^k,$$

of the marginal productivity of capital to the relevant marginal cost, and (2.8), (2.9) equate the marginal productivities of labor and intermediate inputs to their relative prices.

the shock $\upsilon_t^k$ is zero, and thus this shock does not appear in a linearized-based solution.

## 3. Households

Preferences of the representative household are described by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\theta) \ln C_t^n + \theta \ln D_t \right\}, \quad 0 < \beta < 1, \quad 0 < \theta < 1, \tag{3.1}$$

where $C_t^n$ is nondurable consumption and $D_t$ is the stock of durable goods.

As with productive capital, changes in the stock of durable goods involve adjustment costs of the form

$$J_t^d = \frac{\omega^d}{2} \left( D_{t+1} \exp(\upsilon_t^d) - D_t \right)^2 . \tag{3.2}$$

This formulation makes it possible to capture the long cycles of durable goods purchases evident in the data. This is also consistent with the findings in Caballero (1990), who found strong support for slow adjustment of durable goods in the US.

We deviate from the standard Permanent Income Hypothesis by allowing for a mechanism capturing liquidity effects. Households have an exogenous target level of assets. Let us denote net financial assets at the beginning of period $t$ with $F_t$, and the target with $F^*$. The cost $J_t^f$ of deviating from the target is

$$J_t^f = \frac{\omega^f}{2} \left( F_{t+1} - F^* \right)^2 , \ \omega^f > 0, \tag{3.3}$$

reflecting too much or too little liquidity. Symmetry is required for tractability. We return below to the implications of this specification.

Only foreign assets are included in $F$ because ownership of firms and physical capital is already captured by the dividends $\Pi_t$ received from the firms. Government bonds are not included assuming Ricardian beliefs; government debt will be assumed to be held abroad.

The household is endowed with one unit of labor which is supplied inelastically in the long run. In the short run, however, the household supplies the labor input $L_t$ demanded by the firms. Recall that there is no disutility from working, so the household is indifferent about hours worked. The household receives income from labor, dividends and transfers from the government, $T_t$. For the household, the relevant tax rates are: $\tau_t^l$ on labor income, $\tau_t^n$ on nondurable consumption, and $\tau_t^d$

on durable purchases. The one-period household's budget constraint is given by

$$(1+\tau_t^n)\,C_t^n + \left(1+\tau_t^d\right)C_t^d + J_t^d = (1-\tau_t^l)W_tL_t + \Pi_t + T_t + (1+r_{t-1})F_t - F_{t+1} - J_t^f, \tag{3.4}$$

where $C_t^d = D_{t+1} - \left(1-\delta^d\right)D_t$ denotes purchase of durable goods, and $0 < \delta^d < 1$ is the depreciation rate of the stock.

### 3.1. The Household's Optimization Problem

The household chooses sequences of $C_t^n$, $D_{t+1}$ and $F_{t+1}$ to maximize the utility function in (3.1) subject to the sequences of constraints in (3.4), and the initial stocks $F_0$ and $D_0$. The first-order optimality conditions are

$$\frac{(1-\theta)}{C_t^n\,(1+\tau_t^n)}\left[1+\tau_t^d+\omega^d\left(D_{t+1}\exp(\upsilon_t^d)-D_t\right)\exp(\upsilon_t^d)\right] =$$

$$\beta\frac{\theta}{D_{t+1}}+\beta E_t\left[\frac{(1-\theta)}{C_{t+1}^n\left(1+\tau_{t+1}^n\right)}\left(\left(1-\delta^d\right)\left(1+\tau_{t+1}^d\right)+\omega^d\left(D_{t+2}\exp(\upsilon_{t+1}^d)-D_{t+1}\right)\exp(\upsilon_{t+1}^d)\right)\right], \tag{3.5}$$

$$\frac{1}{(1+\tau_t^n)}\frac{1}{C_t^n}\left[1+\omega^f\left(F_{t+1}-F^*\right)\right] = \beta(1+r_t)\frac{1}{\left(1+\tau_{t+1}^n\right)}E_t\left[\frac{1}{C_{t+1}^n}\right]. \tag{3.6}$$

Equation (3.5) equates the purchasing cost of the durable good—which includes the marginal adjustment cost—to its marginal utility plus its discounted resale value, including savings in future marginal adjustment costs. Equation (3.6) is the Euler equation, which leads to consumption smoothing in the standard case where $\omega^f = 0$, $\tau_t^n = \tau_{t+1}^n$, and $\beta(1+r_t) = 1$.

The cost of deviating from target assets generates a steady state in the model. Without this cost, assets and thus other variables follow random walks. Additionally, this feature produces realistic deviations from permanent income behavior: excess sensitivity to temporary income changes, as in Flavin (1985), and excess smoothness to permanent income changes, as in Deaton (1987).[3]

To illustrate this, consider first a temporary increase in the wage or dividends. The desire to save to smooth consumption over time increases $F_{t+1}$. According to (3.6), this implies a larger response of current consumption than predicted by permanent income theory. This is the excess sensitivity property.

[3]Campbell and Hercowitz (2009) show that both phenomena can be explained by liquidity constraints.

Consider now a permanent increase in wage or dividends. The desire to upscale the durable stock to the new optimal level requires borrowing—a reduction of $F_{t+1}$—counting on future income. This, in (3.6) implies a lower $C_t^n$ than with permanent income, and hence the adjustment of $C_t^n$ upwards is gradual. This is the excess smoothness property.[4]

To facilitate the intuition about the decision about durable goods in (3.5), let us assume here that the tax rates on the two goods are the same and fixed over time and no adjustment costs. In this case, we can express this condition in a more intuitive form by substituting

$$\left[1+\omega^f\left(F_{t+1}-F^*\right)\right] = \beta(1+r_t)E_t\left[\frac{C_t^n}{C_{t+1}^n}\right],$$

$$E_t\left[\frac{C_t^n}{C_{t+1}^n}\right] = \left[\frac{1+\omega^f\left(F_{t+1}-F^*\right)}{\beta(1+r_t)}\right],$$

from (3.6) into (3.5) to get

$$\frac{\beta\theta}{1-\theta}\frac{C_t^n}{D_{t+1}} = \frac{\delta^d+r_t}{1+r_t} - \frac{\left(1-\delta^d\right)\omega^f\left(F_{t+1}-F^*\right)}{(1+r_t)}. \tag{3.7}$$

Here, the marginal rate of substitution between durable and nondurable consumption is equated to the "user cost" of durable goods, the first term on the right hand side, minus the second term term, which depends on the assets level. According to the second term, when assets are higher than target, $D_{t+1}$ relative to $C_t^n$ is higher. This can be rationalized by the durability of $D$. Because saving via $F$ has a cost, the household uses durable goods as a substitute.

Equation (3.7) can also be used to rationalize the high volatility of durable purchases. Given the interest rate and $F_{t+1}$, income changes affect nondurable

[4] Another interpretation of the costs of deviating from $F^*$ is the introduction of an "effective" interest rate, which depends on the level of assets (or debt). The Euler equation (3.6) can be rewritten as

$$\frac{1}{(1+\tau_t^n)}\frac{1}{C_t^n} = \beta(1+r_t^e)\frac{1}{\left(1+\tau_{t+1}^n\right)}E_t\left[\frac{1}{C_{t+1}^n}\right],$$

where

$$r_t^e \equiv \frac{1+r_t}{1+\omega^f\left(F_{t+1}-F^*\right)} - 1$$

can be interpreted as the "effective" interest rate. For example, lowering $F_{t+1}$ below $F^*$, or borrowing when $F^*=0$, increases the effective interest rate. Note that this mechanism generates a spread between borrowing and saving rates.

consumption and the stock of durables by the same percentage. However, adjusting the stock of durables requires a much higher percentage change of durable purchases than nondurable purchases.

In contrast with the household's demand side, labor supply is exogenous, and is determined by the working age population per household, $Pop_t$, the participation rate, $Part_t$, hours per worker, $Hours_t$ and the natural unemployment rate $u_t^*$.

$$L_t^s = Pop_t Part_t Hours_t (1 - u_t^*). \tag{3.8}$$

# 4. Government

The modeling of the public sector is aimed at capturing its behavior since 2003. In that year, the government announced a multi-year tax-cut program together with a commitment to reduce the debt-to-GDP ratio. Between the years 2003 and 2008, the tax-cut program was implemented fully, while the debt-to-GDP ratio decreased from 100 to 76 percent. Accordingly, government consumption dropped from 27.8 percent of GDP in 2003 to 23.8 percent in 2008. In December 2009, in spite of the global economic crisis, the government renewed its committed to cut tax rates and further reduce the public debt/GDP ratio to 60 percent—the Maastricht Accord benchmark—in approximately a decade. Accordingly, we model government expenditure as endogenous to the path for the debt, exogenous tax rates, and expected tax revenue.

The government has an exogenous target for the public debt/GDP ratio and a rate of convergence towards that target. Defining $\eta$ as the target ratio of government debt to GDP, the target debt is

$$B_{t+1}^{**} = \eta E_t \left( Y_{t+1} \right), \tag{4.1}$$

and the government plans to achieve this target gradually. The intermediate target for the debt is

$$B_{t+1}^{*} = B_t^* \left( B_{t+1}^{**} / B_t^* \right)^{\lambda^b}, \ 0 < \lambda^b < 1. \tag{4.2}$$

The total revenue from taxing households and firms is

$$R_t = \tau_t^l W_t L_t + \tau_t^c \left( Q_t - W_t L_t - P_t^m M_t \right) + \tau_t^n C_t^n + \tau_t^d C_t^d. \tag{4.3}$$

The government spends $G_t$ in goods and services, $T_t$ in transfers to the public, and $(1+r_t)B_t$ in debt servicing and repayment. "Normal" government spending, $G_t^*$, is planned $\zeta$ quarters earlier given the expected values of the tax revenue, transfers, and the deficit complying with the intermediate target for the debt:

$$G_t^* = E_{t-\zeta}\left[R_t + B_{t+1}^* - T_t - (1+r_t)B_t\right]. \tag{4.4}$$

Actual spending, however, includes a stochastic term, representing unexpected events.

$$G_t = G_t^* \exp(\upsilon_t^x), \quad 0 < \zeta < 1. \tag{4.5}$$

This error term is assumed to be serially uncorrelated. Although the unexpected event may continue, the total expenditure in the next budget complies again with the fiscal rule.

Actual debt at the end of period $t$ is

$$B_{t+1} = G_t + T_t - R_t + (1+r_t)B_t. \tag{4.6}$$

If there are no unexpected events in period $t$ relative to expectations in period $t-\zeta$, then $B_{t+1} = B_{t+1}^*$. However, if, for example, economic activity is lower than expected, government revenue is correspondingly lower, and then $B_{t+1}$ increases. Then, in the following periods $G$ is adjusted downwards.

## 5. Rest of the World

The rest of the world demands domestic output according to the function

$$X_t = X^0 WT_t \left(P_t^x\right)^{\chi} \exp(\upsilon_t^x), \quad \chi > 0, \tag{5.1}$$

where $X^0$ is a scale parameter, $WT$ is the volume of world trade, treated as an exogenous stochastic variable. The relative price $P_t^x$, which is the relative price of the foreign substitute in terms of the domestic goods, has a positive effect.

The relative price of the foreign export substitutes in terms of imports is

$$P_t^{xm} = \frac{P_t^x}{P_t^m}, \tag{5.2}$$

which is exogenously given from the world markets.

The interest rate is exogenous, reflecting the behavior of the world financial markets. The time-series formalization of the interest rate process has an error-correction format. The interest rate level is determined by

$$r_t = \frac{1}{\beta} + \upsilon_t^r, \tag{5.3}$$

where $\upsilon_t^r$ is a stationary deviation from the long-run level $1/\beta$.

The dynamic, or "error-correction", behavior is described by

$$\Delta r_t = \rho^r \Delta r_{t-1} - \lambda^r \upsilon_{t-1}^r + \varepsilon_t^r, \quad \lambda^r > 0, \ \ 0 < \rho^r < 1, \tag{5.4}$$

which includes the innovation $\varepsilon_t^r$. One force affecting the interest rate change is inertia, represented by the first term. The other is mean reversion, modelled as "error correction". For example, if initially $r_t = 1/\beta$ and then $\varepsilon_t^r > 0$, the interest rate increases not only currently but, if $\rho^r - \lambda^r > 0$, it increases further in the following period. Eventually, the mean reversion should dominate because $0 < \rho^r < 1$, which weakens the persistence of the effect over time.

### 5.1. Approximation of Long Maturities

In the present model we use the standard modeling assumption that the debt and assets are one-period instruments. Hence, interest rate changes affect the entire stock of government debt and households' assets. In reality, debt and assets have long maturities, and thus, next-period payment and receipts are only marginally affected by the current interest change. To incorporate this consideration in the quantitative calculations, we define an "average" interest rate as

$$\hat{r}_t = \xi r_t + (1 - \xi)(1/\beta), \quad 0 < \xi < 1, \tag{5.5}$$

which will be the relevant interest rate for current interest income and payments. Hence, this interest rate will replace $r_t$ in the household's budget constraint (3.4), and for the government, in the planned expenditure equation (4.4) and in the current budget constraint (4.6). Modeling households and government as deciding on the maturity structure of assets or debt given the term structure of the interest rate would complicate the model considerably.

# 6. Equilibrium

## 6.1. The Labor Market

Using the labor demand and supply equations, (2.8) and (3.8), the equilibrium wage rate $W_t^*$ is given by

$$L_t^s = Pop_t Part_t Hours_t(1 - u_t^*) = \vartheta^l A_t^{\frac{(1-\alpha)}{\alpha}} K_t \left(W_t^*\right)^{-\frac{1}{\alpha}} \left(P_t^m\right)^{-\frac{1-\gamma}{\alpha\gamma}},$$

where $\vartheta^l = \left[(1-\alpha)\beta\right]^{\frac{1}{\alpha\beta}} \left(\frac{1-\beta}{(1-\alpha)\beta}\right)^{\frac{1-\beta}{\alpha\beta}}$, or

$$W_t^* = \left(\vartheta^l\right)^{\alpha} A_t^{1-\alpha} K_t^{\alpha} \left(P_t^m\right)^{-\frac{1-\gamma}{\gamma}} \left(Pop_t Part_t Hours_t\right)^{-\alpha} (1 - u_t^*)^{-\alpha}. \tag{6.1}$$

The equilibrium wage rate prevails in the long run. In the short run, we assume real wage rigidity: it adjusts gradually to its long-run level according to

$$W_t = W_{t-1} \left(\frac{W_t^*}{W_{t-1}}\right)^{\lambda^w} \exp\left(\upsilon_t^w\right), \quad 0 < \lambda^w < 1. \tag{6.2}$$

The speed of adjustment is governed by $\lambda^w$, and $\upsilon_t^w$ is a stochastic term.

Actual employment is determined by substituting (6.2) in the labor demand equation (2.8). Because the wage can be lower or higher than $W_t^*$, the unemployment rate can accordingly be higher or lower than the natural rate $u_t^*$. Thus,although labor supply is inelastic in the long run, it can fluctuate in the short run due to real wage rigidity. Note that in the case of a fully flexible wage, that is $\lambda^w = 1$,then $W_t = W_t^*$ and $L_t = L_t^s$ at all times.

## 6.2. The Output Market

The present model, as a version of the Bruno and Sachs' (1985) framework, is based on an aggregate supply and an aggregate demand for output—which in equilibrium solve for the relative price of imports $P_t^m$. Aggregate supply is obtained by substituting labor demand from (2.8) and imports demand (2.9) into the production function (2.3), and using the current wage as set by (6.1) and (6.2). The current relative price of imports has a negative effect on aggregate supply because it reduces the demand for imports and labor.

The aggregate demand for goods is positively affected by the relative price of imports via exports from (5.1), given $P_t^{xm}$—the price of export substitutes relative to imports. The other components, consumption, government expenditure and investment, are not directly affected by $P_t^m$.

The equilibrium condition is

$$Q_t = C_t^n + C_t^d + I_t + G_t + X_t + J_t^k + J_t^d + J_t^f. \tag{6.3}$$

### 6.3. The Current Account

The current account surplus ($CA_t$), and the corresponding capital flows, follow from (6.3) by adding $r_{t-1}(F_t - B_t) - P_t^m M_t$ on both sides:

$$Q_t + r_{t-1}(F_t - B_t) - P_t^m M_t = C_t^n + C_t^d + I_t + G_t + X_t + J_t^k + J_t^d + J_t^f + r_{t-1}(F_t - B_t) - P_t^m M_t,$$

or, rearranging,

$$CA_t \equiv X_t + r_{t-1}(F_t - B_t) - P_t^m M_t = Q_t + r_{t-1}(F_t - B_t) - P_t^m M_t - C_t^n - C_t^d - I_t - G_t - J_t^k - J_t^d - J_t^f. \tag{6.4}$$

The current account balance can be measured in two equivalent ways: one is the difference between receipts from abroad from exports and assets less payments abroad from imports and debt. The other is total income less total expenditures.

### 6.4. Steady State

In this version of the model there is no long-run growth. If growth is introduced, the long run can be defined by translating the growing variables into stationary "efficient per-capita" terms, i.e., dividing these variables by $A \cdot Pop$. Note that at the steady state $J_t^k = J_t^d = J_t^f = 0$. The equations for the long-run values are the following.

The capital stock from (2.7):

$$K = \alpha\gamma \frac{(1-\tau^c)\,Q}{r + \delta^k}.$$

Labor input from (3.8):

$$L^* = Pop \cdot Part \cdot Hours \cdot (1 - u^*).$$

Output from (2.3):

$$Q = A^{(1-\alpha)\gamma} K^{\alpha\gamma} \left(L^*\right)^{(1-\alpha)\gamma} (M)^{1-\gamma}.$$

Imports from (2.9):

$$M = \vartheta^m (A^{\frac{1-\alpha}{\alpha}} K \left(W^*\right)^{-\frac{1-\alpha}{\alpha}} (P^m)^{-\frac{1-\gamma(1-\alpha)}{\alpha\gamma}},$$

where $\vartheta^m = [(1-\alpha)\beta]^{\frac{1}{\alpha\beta}} \left(\frac{1-\beta}{(1-\alpha)\beta}\right)^{\frac{1-\beta(1-\alpha)}{\alpha\beta}}$.

Dividends from (2.6):

$$\Pi = (1-\tau^f)(Q - W^* L - P^m M) - \delta^k K.$$

The wage from (6.1):

$$W^* = \left(\vartheta^l\right)^\alpha A^{1-\alpha} K^\alpha (P^m)^{-\frac{1-\gamma}{\gamma}} (L^*)^{-\alpha}.$$

Nondurable and durable consumption from (3.4) and (3.5):

$$C^n = \frac{1}{1+\tau^n}\left[(1-\tau^l) W^* L^* + \Pi + T + rF^* - \left(1+\tau^d\right)\delta^d D\right],$$

$$D = \left(\frac{\beta\theta}{1-\theta}\right)\left(\frac{1+\tau^n}{1+\tau^d}\right)\left(\frac{1}{1-\beta\left(1-\delta^d\right)}\right) C^n.$$

Exports from (5.1) and (5.2):

$$X = X^0 \cdot WT \cdot (P^m \cdot P^{xm})^\chi.$$

Equilibrium in the output market from (6.3):

$$Q = C^n + \delta^d D + \delta^k K^* + G + X.$$

Government revenue, debt and budget constraint from (4.3)-(4.6):

$$R = \tau^l W^* L^* + \tau^c (Q - W^* L^* - P_m M) + \tau^n C^n + \tau^d \delta^d D,$$

$$B^* = \eta A K^\alpha (L^*)^{(1-\alpha)},$$

$$G = R - T - rB^{*}.$$

This is a system of 13 equations in the 13 unknowns $K$, $L^{*}$, $Q$, $M$, $\Pi$, $W^{*}$, $D$, $C^{n}$, $X, R$, $B^{*}$, $G$, $P^{m}$, given the exogenous values of $r$, $Pop$, $Part$, $Hours$, $u^{*}$, $F^{*}$, $WT$, $P^{xm}$, $U$, $\tau^{l}$, $\tau^{c}$, $\tau^{n}$, $\tau^{d}$ and $T$. The values of $A$, $Pop$, $Part$, $Hours$, $WT$, and $P^{xm}$ can be normalized to 1.

Note that the current account balance should be zero in the long run. We can see this using the steady-state version of (6.4):

$$CA = Q + r\left(F^{*} - B^{*}\right) - P^{m}M - C^{n} - \delta^{d}D - \delta^{k}K^{*} - G. \tag{6.5}$$

Using now the expression for dividends, the expenditure on nondurable consumption is

$$\left(1+\tau^{n}\right)C^{n} = (1-\tau^{l})W^{*}L^{*} + (1-\tau^{c})\left(Q - W^{*}L - P^{m}M\right) - \delta^{k}K + T + rF^{*} - \left(1+\tau^{d}\right)\delta^{d}D.$$

Substituting the government's budget constraint yields:[5]

$$C^{n} = Q + r\left(F^{*} - B^{*}\right) - P^{m}M - \delta^{d}D - \delta^{k}K - G.$$

This implies in (6.5) that $CA = 0$.

## 7. The System of Dynamic Equations

For econometric purposes, also the equations for $L_t$, $M_t$, and $P_t^{m}$ include error terms, thought of as reflecting measurement errors.

[5] These are steps in this substitution:

$$\begin{aligned}
C_n &= Q + rF^{*} - P^{m}M - \delta_k K - \delta_d D - \tau^{l}W^{*}L^{*} - \tau^{c}\left(Q - W^{*}L - P^{m}M\right) + T - \tau^{n}C_n - \tau^{d}\delta_d D,\\
C_n &= Q + rF^{*} - P^{m}M - \delta_k K - \delta_d D - \left(\tau^{l}W^{*}L^{*} + \tau^{c}\left(Q - W^{*}L - P^{m}M\right) + \tau^{n}C_n + \tau^{d}\delta_d D\right) + T,\\
C_n &= Q + rF^{*} - P^{m}M - \delta_k K - \delta_d D - \left(R - T\right),\\
C_n &= Q + rF^{*} - rB^{*} - P^{m}M - \delta_k K - \delta_d D - G.
\end{aligned}$$

### 7.1. Aggregate Supply

Output and domestic product from (2.3) and (2.2):

$$Q_t = A_t^{(1-\alpha)\gamma} K_t^{\alpha\gamma} L_t^{(1-\alpha)\gamma} M_t^{1-\gamma}, \tag{7.1}$$

$$Y_t = A_t K_t^{\alpha} L_t^{(1-\alpha)}. \tag{7.2}$$

Here the "errors" come only from $A_t$, which is a residual when $Y_t$, $K_t$ and $L_t$ are observed.

The demand for labor and imports from (2.8) and (2.9):

$$L_t = \vartheta^l A_t^{\frac{(1-\alpha)}{\alpha}} K_t W_t^{-\frac{1}{\alpha}} \left(P_t^m\right)^{-\frac{1-\gamma}{\alpha\gamma}} \exp(\upsilon_t^l), \tag{7.3}$$

$$M_t = \vartheta^m A_t^{\frac{(1-\alpha)}{\alpha}} K_t W_t^{-\frac{1-\alpha}{\alpha}} \left(P_t^m\right)^{-\frac{1-\gamma(1-\alpha)}{\alpha\gamma}} \exp(\upsilon_t^m). \tag{7.4}$$

Labor supply from (3.8):

$$L_t^* = \left(Pop_t Part_t Hours_t\right)\left(1 - u_t^*\right). \tag{7.5}$$

The wage from (6.1) and (6.2):

$$W_t^* = \left(\vartheta^l\right)^{\alpha} A_t^{1-\alpha} K_t^{\alpha} \left(P_t^m\right)^{-\frac{1-\gamma}{\gamma}} \left(L_t^*\right)^{-\alpha}, \tag{7.6}$$

$$W_t = W_{t-1} \left(\frac{W_t^*}{W_{t-1}}\right)^{\lambda^w} \exp\left(\upsilon_t^w\right), \quad 0 < \lambda^w < 1. \tag{7.7}$$

### 7.2. Aggregate Demand

#### 7.2.1. Households

The equations for $F_{t+1}$, $C_t^n$, $\Pi_t$, $D_{t+1}^*$, $D_{t+1}$ and $C_t^d$ are the Euler equation (3.6), the budget constraint (3.4), the dividends in (2.6), the condition for optimal durable goods (3.5), the average interest rate in (5.5), and the definition of durable goods purchases:

$$\frac{1}{C_t^n}\left[1+\omega^f\left(F_{t+1}-F^*\right)\right]=\beta(1+r_t)\frac{(1+\tau_t^n)}{\left(1+\tau_{t+1}^n\right)}\frac{1}{C_{t+1}^n}, \tag{7.8}$$

$$C_t^n=\frac{1}{(1+\tau_t^n)}\left[\begin{array}{c}(1-\tau_t^l)W_tL_t+\Pi_t+T_t-\left(1+\tau_t^d\right)C^d+(1+\hat{r}_{t-1})F_t-F_{t+1}\\ -J_t^d-J_t^f\end{array}\right], \tag{7.9}$$

$$\Pi_t=(1-\tau_t^c)\left[Q_t-W_tL_t-P_t^mM_t\right]-J_t^k-I_t, \tag{7.10}$$

$$\frac{(1-\theta)}{C_t^n\left(1+\tau_t^n\right)}\left[1+\tau_t^d+\left(J_t^d\right)'\right]=\beta\frac{\theta}{D_{t+1}}+\beta E_t\left[\frac{(1-\theta)}{C_{t+1}^n\left(1+\tau_{t+1}^n\right)}\left(\left(1-\delta^d\right)\left(1+\tau_{t+1}^d\right)+\left(J_{t+1}^d\right)'\right)\right], \tag{7.11}$$

$$C_t^d=D_{t+1}-(1-\delta^d)D_t. \tag{7.12}$$

### 7.2.2. Firms and Exports

Investment is obtained from the optimal capital stock in (2.7) and the investment equation (2.4):

$$1+\left(J_t^k\right)'=\frac{1}{1+r_t}E_t\left[\left(1-\tau_{t+1}^c\right)\alpha\gamma E_tQ_{t+1}/K_{t+1}+1-\delta^k+\left(J_{t+1}^k\right)'\right]. \tag{7.13}$$

$$I_t=K_{t+1}-(1-\delta^k)K_t. \tag{7.14}$$

Exports from (5.1):

$$X_t=X^0WT_t\left(P_t^m\cdot P_t^{xm}\right)^{\chi}\exp(\upsilon_t^x). \tag{7.15}$$

### 7.2.3. Government

Revenue from (4.3):

$$R_t=\tau_t^lW_tL_t+\tau_t^c\left(Q_t-W_tL_t-P_t^mM_t\right)+\tau_t^nC_t^n+\tau_t^dC_t^d. \tag{7.16}$$

In this version of the model, we set $\zeta = 1$, i.e., budgeting is carried out one period in advance.

Government spending from (4.4), (4.5) and (5.5):

$$G_{t+1}^{*} = E_t \left[ R_{t+1} - T_{t+1} + B_{t+2}^{*} - (1 + \hat{r}_t) B_{t+1} \right], \tag{7.17}$$

$$G_t = G_t^{*} \exp(\upsilon_t^{g}). \tag{7.18}$$

Path for the debt from (4.1) and (4.2):

$$B_{t+1}^{**} = \eta E_t Y_{t+1}, \tag{7.19}$$

$$B_{t+1}^{*} = B_t^{*} \left( B_{t+1}^{**} / B_t^{*} \right)^{\lambda^b}. \tag{7.20}$$

Actual debt from (4.6) and (5.5):

$$B_{t+1} = G_t + T_t - R_t + (1 + \hat{r}_t) B_t. \tag{7.21}$$

### 7.3. Adjustment Cost Functions

Levels:

$$J_t^k = \frac{\omega^k}{2} \left( K_{t+1} \exp(\upsilon_t^k) - K_t \right)^2 \exp(\upsilon_t^k), \tag{7.22}$$

$$J_t^d = \frac{\omega^d}{2} \left( D_{t+1} \exp(\upsilon_t^d) - D_t \right)^2 \exp(\upsilon_t^d), \tag{7.23}$$

$$J_t^f = \frac{\omega^f}{2} \left( F_{t+1} - F^{*} \right)^2. \tag{7.24}$$

Derivatives:

$$\left( J_t^k \right)' = \omega^k \left( K_{t+1} \exp(\upsilon_t^k) - K_t \right) \exp(\upsilon_t^k), \tag{7.25}$$

$$\left( J_t^d \right)' = \omega^d \left( D_{t+1} \exp(\upsilon_t^d) - D_t \right) \exp(\upsilon_t^d), \tag{7.26}$$

$$\left( J_t^f \right)' = \omega^f \left( F_{t+1} - F^{*} \right). \tag{7.27}$$

### 7.4. Interest Rate Determination

From (5.3) and (5.4):

$$r_t = r_{t-1} + \rho^r (r_{t-1} - r_{t-2}) - \lambda^r (r_{t-1} - 1/\beta) + \varepsilon_t^r. \tag{7.28}$$

$$\hat{r}_t = \xi r_t + (1-\xi)(1/\beta), \quad 0 < \xi < 1, \tag{7.29}$$

### 7.5. Equilibrium

Equilibrium in the goods market from (6.3), (5.1) and (5.2):

$$P_t^m = \left( \frac{Q_t - C_t^n - C_t^d - I_t - G_t - J_t^k - J_t^d - J_t^f}{X^0 W T_t} \right)^{\frac{1}{\chi}} \frac{1}{P_t^{xm}} \exp(\upsilon_t^p). \tag{7.30}$$

### 7.6. Solution of the System

The system (7.1)-(7.30) has 30 equations in the 30 unknowns $Q_t$, $Y_t$, $L_t$, $M_t$, $L_t^*$, $W_t^*$, $W_t$, $F_{t+1}$, $C_t^n$, $\Pi_t$, $D_{t+1}$, $C_t^d$, $K_{t+1}$, $I_t$, $X_t$, $R_t$, $G_t^*$, $G_t$, $B_{t+1}^{**}$, $B_{t+1}^*$, $B_{t+1}$, $J_t^k$, $J_t^d$, $J_t^f$, $\left(J_t^k\right)'$, $\left(J_t^d\right)'$, $\left(J_t^f\right)'$, $r_t$, $\hat{r}_t$ and $P_t^m$. Given the forward looking nature of the problem, the solution involves the future variables $Q_{t+1}$, $Y_{t+1}$, $C_{t+1}^n$, $K_{t+2}$, $D_{t+2}$ and $B_{t+2}^*$. The predetermined endogenous variables are $K_t$, $D_t$, $W_{t-1}$, $G_{t-1}^*$, $r_{t-1}$, $r_{t-2}$, $B_t^{**}$, $B_t^*$ and $B_t$. The stochastic exogenous variables are $A_t$, $WT_t$, $P_t^{xm}$ and the shocks, and the deterministic exogenous variables are $Pop_t$, $Part_t$, $Hours_t$, $u_t^*$, $B^*, T_t$, and the tax rates.

### 7.7. Stochastic Structure

$$\log A_t = \rho^a \log A_{t-1} + \varepsilon_t^a, \tag{7.31}$$

$$\log WT_t = \rho^{wt} \log WT_{t-1} + \varepsilon_t^{wt}, \tag{7.32}$$

$$\log P_t^{xm} = \rho^{xm} \log P_{t-1}^{xm} + \varepsilon_t^{xm}, \tag{7.33}$$

with standard deviations $\sigma^a, \sigma^{wt}, \sigma^{xm}$, and $\sigma^r$ for the interest rate innovation.

The shocks follow the processes

$$\upsilon_t^i = \rho^i \upsilon_{t-1}^i + \varepsilon_t^i, \tag{7.34}$$

with standard deviations $\sigma^i$, for $i = l, m, w, d, k, x, g, p$. Restrictions to these parameters are $\rho^g = 0$, assumed above in Section 4, and $\rho^p = 0$, i.e., we wish to assign all the dynamics of the key market clearing price to the model itself.

With the addition of these 11 equations and 11 variables in (7.31)-(**??**), the system has a total of 41 equations and 41 unknowns.

### 7.8. Conversion of Variables from Output Units to GDP Units

National income accounts emphasize GDP, or domestic product, rather than domestic output. In particular, relative prices of imports and exports are computed using GDP price indices, and not output indices—as the model naturally involves. Hence, prior to reporting the empirical part of the paper, we derive the theoretical counterparts of variables as they are usually measured.

From equation (2.1), efficient production implies that the relative price of value added in terms of output equals

$$P_t^y = \gamma \frac{Q_t}{Y_t}.$$

Substituting $Y$ in terms of $Q$ from (2.1) we get

$$P_t^y = \gamma \frac{Q_t M_t^{\frac{1-\gamma}{\gamma}}}{Q_t^{\frac{1}{\gamma}}} = \gamma \left( \frac{M_t}{Q_t} \right)^{\frac{1-\gamma}{\gamma}}. \tag{7.35}$$

Then, to convert variables in terms of output to GDP terms we divide by $P_t^y$.

In particular, the relative price of imports in terms of GDP, which we define as $Rer_t$ (real exchange rate) is

$$Rer_t = \frac{P_t^m}{P_t^y} = P_t^m \frac{1}{\gamma} \left( \frac{Q_t}{M_t} \right)^{\frac{1-\gamma}{\gamma}}, \tag{7.36}$$

i.e., the relative price of imports in terms of output divided by the relative price or GDP in terms of output.

## 8. Calibration and Estimation

The parameters determining the steady state of the model are calibrated using long-run averages of relevant ratios in the data, available information, and standard values in the literature, as for $\beta$. The other parameters, which can be identified only from second moments, are estimated using a maximum likelihood procedure, taking the calibrated parameters and exogenous variables as given. Exceptions are the fiscal parameters $\lambda^b$, governing the speed of convergence of public debt to its target and $\eta$, the target public debt/GDP ratio. These parameters cannot be estimated with the available sample because they reflect government behavior since 2003, and the new fiscal rule put forward in December 2009. Hence, we set these values based on the details of this fiscal rule. The tax rates, depreciation rates, and the other parameters in Table 1 are also calibrated.

### 8.1. Calibration

The calibrated values are listed in Table 1. Below, we discuss and justify the calibrated values we use.

**Technology.** The technology parameters $\gamma$ and $\alpha$ were calibrated using the relevant shares during the 2000s. The parameter $\gamma$ was set equal to the average ratio of imports to output, which corresponds to $1-\gamma$ , and the value of $\alpha$ equals the average nonlabor income share in GDP.

The depreciation rate of productive capital $\delta^k$ is 2 percent, the average of the quarterly depreciation rates across capital goods.[6] The depreciation rate of durable goods, $\delta^d$, was set at 2.5 percent. Note that durable goods do not include housing; thus, the depreciation of these goods should be higher than that of productive capital which includes structures—the productive counterpart of housing, that depreciate slowly.

[6] This is the average depreciation rate from the detailed perpetual capital stock system at the Bank of Israel. This system is composed of highly detailed investment items—construction, machinery, vehicles, non-tangibles etc. Every item is matched with the appropriate depreciation rate.

| Table 1: Calibrated Parameters | | |
|---|---|---|
| Definition | Symbol | Value |
| Basic Parameters | | |
| GDP share in output | $\gamma$ | 0.7 |
| Capital share in GDP | $\alpha$ | 0.3 |
| Depreciation rate of capital | $\delta^k$ | 0.02 |
| Utility function parameter | $\theta$ | 0.165 |
| Depreciation rate of durables | $\delta^d$ | 0.025 |
| Discount rate | $\beta$ | 0.99 |
| Real interest rate | $r$ | 0.99 |
| Net portfolio position | $F^*$ | 0 |
| "Natural" unemployment rate | $u^*$ | 0.05 |
| Fiscal Parameters | | |
| Debt to GDP target | $\eta$ | 0.6 |
| Public debt convergence | $\lambda^b$ | 0.025 |
| Fiscal plan horizon (periods) | $\zeta$ | 1 |
| Effective corporate tax | $\tau_t^c$ | 15 |
| Average tax on labor | $\tau_t^l$ | 15 |
| Tax on nondurables (VAT) | $\tau_t^n$ | 16.5 |
| Tax on durables (VAT + purchase) | $\tau_t^d$ | 50 |
| Unilateral transfers (% of GDP) | $T$ | 5 |

**Preferences**. For the calibration of $\theta$ in the utility function we use the steady-state version of equation (3.7), which can be expressed as

$$\frac{\theta}{1-\theta} = \left(\frac{\delta^d + r}{1+r}\right)\frac{1}{\beta\delta^d}\frac{C^d}{C^n} \equiv \Delta,$$
$$\theta = \frac{\Delta}{1+\Delta}.$$

Given the average ratio of $C^d/C^n = 0.141$ during the 2000s (excluding housing services in $C^n$), and the other parameter values, the resulting estimate is $\theta = 0.165$.

The discount rate $\beta$ was set such that the steady-state level of the real interest rate $(1/\beta - 1)$ equals one percent, or 4 percent annualized.

**Fiscal Policy.** The target ratio of fiscal debt to GDP, $\eta$, is set equal to the Maastricht Treaty required ratio, 0.6, adopted by the Israeli government as well. The rate of convergence of $B$ to $B^{**}$ is determined by $\lambda^b$. According to the rule adopted by the government in December 2009, this target should be met by the end of the decade (2020). The decline in $B^*$ when starting from a debt of 80% of GDP implies that, approximately, $\lambda^b = 0.025$ (40 quarters).[7] Note this applies to any tax schedule, because we assume that $G$ adjusts one-to-one with tax revenue given the path for the debt.

The tax rates were calibrated by the average effective rates during the 2000s. Although theses rates were changed several times, we treat them as fixed parameters when studying the dynamics around the steady state. The effects of changes in the tax rates are analyzed in Section 10.

The net portfolio position $F^*$ is calibrated to zero, in line with the observed Net International Investment Position of the Israeli economy as of 2010.

### 8.2. Estimation

While the parameters which determine the steady state of the model are calibrated, almost all the parameters governing out-of-steady-state dynamics are estimated.

The model is estimated by maximum likelihood using the sample period 1999Q1 - 2010Q1. Although data from 1995Q1 are available, we do not include the first four years because they reflect the large adjustment process following the mass-immigration of the 1990s. The shorter sample looks more as "balanced-growth"—for example, the average current account is close to zero—with fluctuations around it.

Taking the model to the data requires adjustments as there are discrepancies between the definitions of the variables in the model and their actual measurement. First, we need to transform the data to match deviations from a steady state. For this we compute the deviations of the log of each variable from its trend. The basic detrending procedure is the Hodrick-Prescott (HP) filter, but we check robustness by applying a linear trend as well. Because we do not have an explicit treatment of inventories, we exclude the change in inventories from the data. About 70 percent of inventories consist of imported goods. We adopt the extreme assumption that all inventories are imported. Hence, we subtract the change in inventories from both investment and imports. Diamonds are excluded from both import and exports because theses series have large fluctuations while having little effect on GDP, as they are usually match each other. We also exclude exports of start-up

[7] The approximation follows from looking at achieving the middle range ratio 0.7 in half the time, 20 quarters.

companies.

The raw data series for estimation are described in Table 2.

| **Table 2: Data for Estimation*** | | |
|---|---|---|
| Name | Symbol | Details |
| Gross Domestic Product | $Y$ | – |
| Exports | $X$ | – |
| Imports | $M$ | Civilian |
| Investment | $INV$ | Fixed Capital Formation |
| Gross Capital Stock | $K$ | – |
| Nondurable Consumption | $CN$ | – |
| Purchases of Durable Goods | $CD$ | – |
| Output | $Q$ | Civilian |
| Government Consumption | $G$ | General government excl. defense |
| Imports/Output relative price | $PM$ | Import price index/output price deflator |
| Real Exchange Rate | $RER$ | Import price index/GDP deflator |
| Wage per hour | $W$ | Real wage per employee post/ average weekly hours |
| Labor Input | $L$ | Total hours worked by Israelis, Palestinians and Foreigners |
| Foreign relative price | $Pxm$ | Export substitutes price index/ import price index (excl. defense) |
| World Trade ** | $WT$ | Trade volume: OECD countries |
| Real Interest Rate | $R$ | Short term |
| Stock of Durables | $D$ | – |
| * National accounts are seasonally adjusted, chained to 2005 prices | | |

Table 3 below presents the estimated parameters and, in parentheses, $t$-statistics. With 12 shocks in the model, it is possible to use up to 12 observed series for estimation. Column (1) includes output and the real exchange rate based on output prices, and column (2) includes instead GDP and the real exchange rate based on the GDP deflator. The rest of the variables are the same, and are listed in the table.

| Table 3: Estimation Results | | |
|---|---|---|
| Parameter | (1) | (2) |
| $\omega^k$ | 1.54 (2.0) | 1.28 (2.2) |
| $\omega^d$ | 2.75 (5.4) | 2.69 (5.2) |
| $\lambda^w$ | 0.13 (2.2) | 0.13 (2.0) |
| $\lambda^f$ | 0.021 (3.1) | 0.016 (2.7) |
| $\rho^{tt}$ | 0.47 (3.7) | 0.48 (3.8) |
| $\rho^{wt}$ | 0.83 (23.3) | 0.84 (21.9) |
| $\rho^a$ | 0.50 (4.3) | 0.82 (8.7) |
| $\rho^r$ | 0.43 (3.2) | 0.43 (3.0) |
| $\lambda^r$ | −0.47 (5.7) | −0.48 (5.5) |
| $\rho^l$ | 0.52 (6.9) | 0.59 (7.9) |
| $\rho^w$ | −0.11 (0.8) | −0.10 (0.7) |
| $\rho^m$ | 0.19 (1.9) | 0.30 (3.4) |
| $\rho^d$ | 0.62 (5.4) | 0.63 (5.5) |
| $\rho^k$ | 0.69 (8.6) | 0.66 (8.0) |
| $\rho^x$ | 0.73 (7.3) | 0.74 (7.5) |
| Variables used | Set 1* | Set 2** |
| Likelihood | −1342 | −1354 |
| Sample: $1999q1 - 2010q1$ | | |
| Detrending: HP filter. Estimation Method: MLE | | |
| * $Q, X, M1, CN, INV1, CD, WT, L, Pxm, W, R1, PM$ | | |
| ** $Y, X, M1, CN, INV1, CD, WT, L, Pxm, W, R1, RER$ | | |

The parameters determining adjustment costs and liquidity effects—$\omega^k, \omega^d$, $\lambda^w$ and $\lambda^f$—are all statistically significant and sizable. However, the fact that $\rho^k$ and $\rho^d$ are also large and significant suggests that those mechanisms are not sufficient for explaining the behavior of investment and purchase of durables; other forces, captured by the error terms, seem to play a role as well. The adjustment of real wage to labor productivity—governed by $\omega^k$—takes about two years.[8] In this case, however, $\omega^k$ fully explains wage inertia; $\rho^w$ is insignificant. The low estimated value of $\lambda^w$ justifies our treatment of the labor market as real wage rigidity seems to be high.

Shocks to world trade have high persistence ($\rho^{wt}$~0.85). The coefficients $\lambda^r$ and $\rho^r$ have similar magnitude in absolute value, which means that the stochastic

[8] This adjustment is slower than in the empirical findings of Lavi and Sussman (2005), who find a one-year adjustment.

process of the real interest rate from equation (5.4) is approximately a simple mean reversion process.

Another interesting result is the difference between the estimates of $\rho^a$ identified using output (0.5, column (1)), and GDP (0.8, column (2)). The lower persistence of productivity in the first case suggests that output data may capture, at least partially, changes in capacity utilization: Energy share in imports is high, so there could be a positive correlation between imports and unobserved capacity utilization. Hence, part of the productivity persistence may be captured by capital utilization.

Some identities in the model do not hold in the data. Hence, there are variables that cannot be used simultaneously. For example, capital stock and investment cannot be included simultaneously: The identity of the capital evolution equation (2.4) does not hold in the data. We preferred to use investment, which is measured more reliably than capital stock.

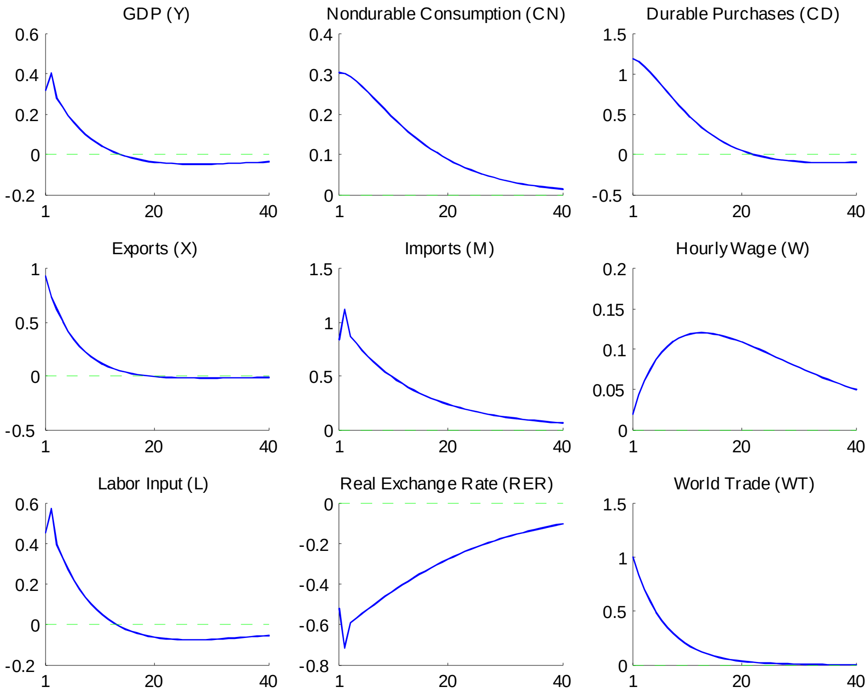
The results in Table 3 are robust to the extension of the sample period backwards to 1995. Detrending the data with linear trends instead of the HP filter generates higher persistence in most of the shocks of the model. However, the HP-filter is likely to be preferable because it also filters out low frequency movements that we do not focus on here.

## 9. Impulse Response Analysis

We present here the results of the impulse responses to the main shocks of the model: world trade, productivity, public consumption, and the interest rate. The first three are of one percent, and the shock to the interest rate is of one percentage annualized point. Although public consumption is endogenous in the model, the shock to it reflects unexpected events at the time of last budget preparation, such as a recent deterioration of security conditions. All the results in this section are based on the parameters of Model A in Table 2.

**Shock to World Trade (**Figure 9.1)**.** A one-percent shock to world trade impacts directly on exports. This increases output demand and thus causes an appreciation—as shown by the decline of the real exchange rate. The main result is an increase of GDP by about 0.4 percent. Note that the increase in output exceeds that of GDP due to the increase in imports. The demand shock is boosted by the positive wealth effect which increases consumption of nondurables, as well as durable purchases and public consumption (not shown). The sizable income effect on private consumption is also a result of the high persistence of the world

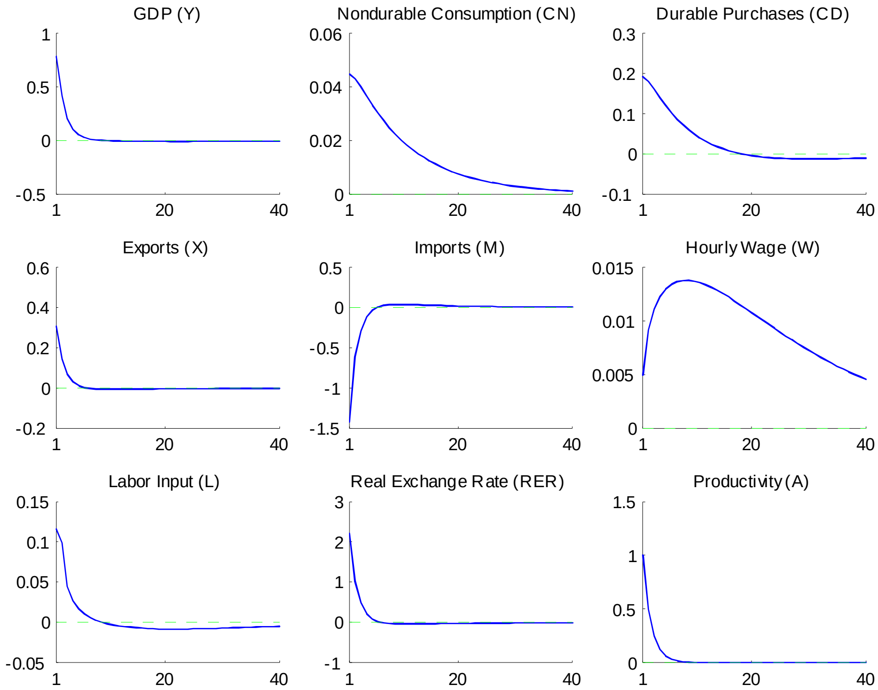
Figure 9.1: Shock to World Trade




trade shock. The spike in the period after the shock follows from the mechanism of public consumption assumed, which requires one period budgeting in advance. Comparing the responses of GDP and exports, we can compute here a GDP multiplier of exports. Given the average ratio of exports to GDP of about 0.43 percent, and the exports response of close to one percent, the exports multiplier is about 0.9.

**Shock to Productivity** (Figure 9.2). Affecting directly the supply side, this shock increases the relative price of imports—as shown by the real exchange rate. Recall that this shock is labor augmenting, so its direct effect on GDP is the labor share. A one percent shock increases GDP by 0.8 percent, slightly higher than the share of labor (0.7). This is due to the increase in hours worked, which rise temporarily above their long-run level given that wages do not adjust instantaneously. The effect on final output is relatively small due to the depreciation, which lowers imports. The model has the Keynesian characteristic that shocks to aggregate demand have a strong effect on production. A demand shock lowers

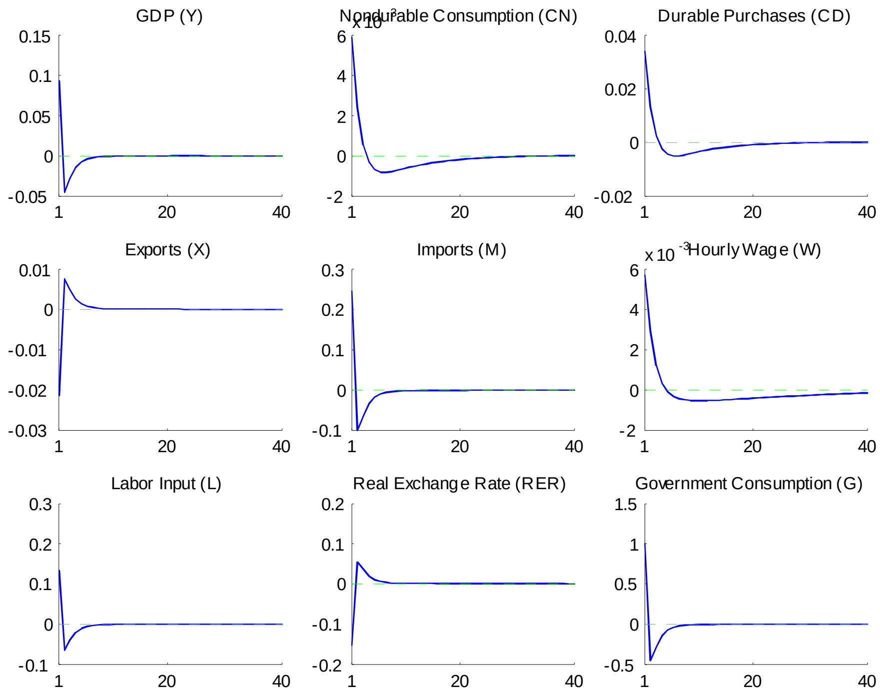
Figure 9.2: Shock to Productivity




the relative price of imports, which are a complementary factor of production, and hence generates higher demand for imports as well as for labor and capital. Therefore, the model includes a mechanism linking positively output demand and the firms' demand for factors. This link is absent in the neoclassical framework.

**Shock to Public Consumption** (Figure 9.3)**.** A one percent shock to government consumption increases GDP by 0.1 percent on impact. The shock is followed by a contraction of public consumption—which in the real world can be interpreted as a cut in other spending items. This spending response is fully expected, so the transitory increase in $G$ does not crowd out private consumption. Moreover, private consumption increases slightly because of higher income flows. However, exports are crowded out due to the real appreciation. The contemporaneous response of GDP is 0.1 percent, and thus, given the average share of government consumption in GDP of about 0.25, the fiscal multiplier is 0.4. This is relatively low compared to the empirical findings of Mazar (2010) for Israel, who estimated a fiscal multiplier of 0.7. The model's multiplier is much lower than the

Figure 9.3: Shock to Government Consumption



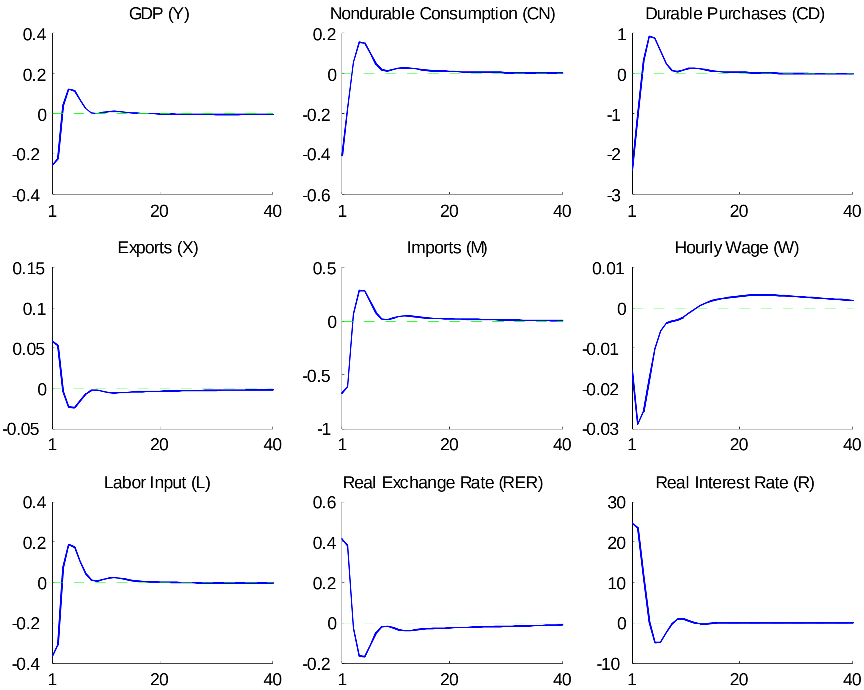

export multiplier of 0.9 computed above for the world trade shock. The difference is due to the strong wealth effect on consumption that world trade generates, which boosts output demand further.

The comparison of the model's fiscal multiplier with the standard one-good open economy model is also of interest. In such a model, a transitory increase in government spending has a very small multiplier, while excess exports go down almost one-to-one with the government spending shock. Here, in comparison, imports are a different good so the positive effect on imports works via the real appreciation. Then, because imports are intermediate goods complementing domestic inputs, the latter increase—thereby causing higher domestic GDP.

**Shock to the Real Interest Rate** (Figure 9.4). A one percentage point shock to the real interest rate in annual terms decreases GDP by 0.2 percent.[9] The

[9]The steady-state level of the real interest rate is one percent per quarter. Hence, a 25 *percent* increase in the real interest rate is equivalent to a one *percentage point* increase in annual terms.

Figure 9.4: One Percentage Point Annualized Shock to the Real Interest Rate




main channel of effect is a 0.4 percent drop in consumption. Note that the interest rate in the model comes from the world capital markets. Thus, the resulting depreciation follows from the reduced demand for consumption and investment.[10]

## 9.1. Impulse Response Functions: A Summary

We conclude this discussion of impulse responses by stressing the stronger effect of demand shocks than supply shocks on quantities—while for relative price of output the effects are the opposite. This follows from the high elasticity of aggregate supply and the low elasticity of aggregate demand. A one percent shock in productivity increases output by 0.15 percent and the real exchange rate depreciates by 2 percent. In contrast, a one percent shock to world trade increases

[10]In contrast, the Keynesian models generate an appreciation in response to an interest rate shock. In that case, the shock affects directly the domestic interest rate rather than the world interest rate, and thus generates a capital inflow.

output by 0.5 percent and the real exchange rate appreciates by 0.5 percent. This feature gives the model a kind of Keynesian flavor, as least quantitatively.

## 10. Policy Analysis

In this section we present results for the analysis of Israeli consolidation policies in the 2000s: pre-announced tax cuts and adoption of lower public debt/GDP target. The tax cuts are permanent and announced Ten quarters in advance. The public debt target is reduced simultaneously with the announcement. Income tax changes are likely to have different effects before and after their implementation.

### 10.1. Expected Tax Changes

We discuss three tax changes: reductions in the labor income tax rate $\tau^l$, the corporate tax rate $\tau^c$, and the tax rate on durable goods $\tau^d$. All these changes are pre-announced ten quarters in advance. Hence, we can follow the effects from the time of the announcement to the time of implementation, and from then onwards.

Figure 10.1 shows the effects of a one percentage point reduction of $\tau_l$. The response of $Y$ summarizes the effects of this policy: output expands prior to the actual tax decline and declines at that time. From then onwards, output converges gradually to the original level. This cycle in economic activity is started by the wealth effect on $C^n$ and $C^d$ of the forthcoming tax cuts, and continued later by the downward adjustment of $G$ required by the tax cut. This fluctuation in the demand for output brings about first an appreciation—a decline of the real exchange rate—and then a devaluation. These changes in the relative price of imports triggers a corresponding fluctuation of $M$, as reflected in the panel for the trade balance ($TB$). Given that imports are intermediate inputs which interact positively with $K$ and $L$ in production, both these factors increase first and then decline. This is the essence of the generated cycle. Note that wage rigidity contributes to the magnitude of this fluctuation. If wages are flexible, labor input remains constant and thus the appreciation/depreciation cycle affects only $K$.

Figure 10.2 addresses a one percentage point reduction in the corporate income tax. Unlike $\tau_l$, the change in $\tau_c$ leads to a new steady state: The lower tax rate increases the optimal capital stock, and hence long-run output is higher. For $L$ and $Y$, however, there are similarities with the cycle shown in Figure 10.1. Labor input follows a similar expansion/contraction cycle. In the present case, the initial

Figure 10.1: Future Labor Tax Cut

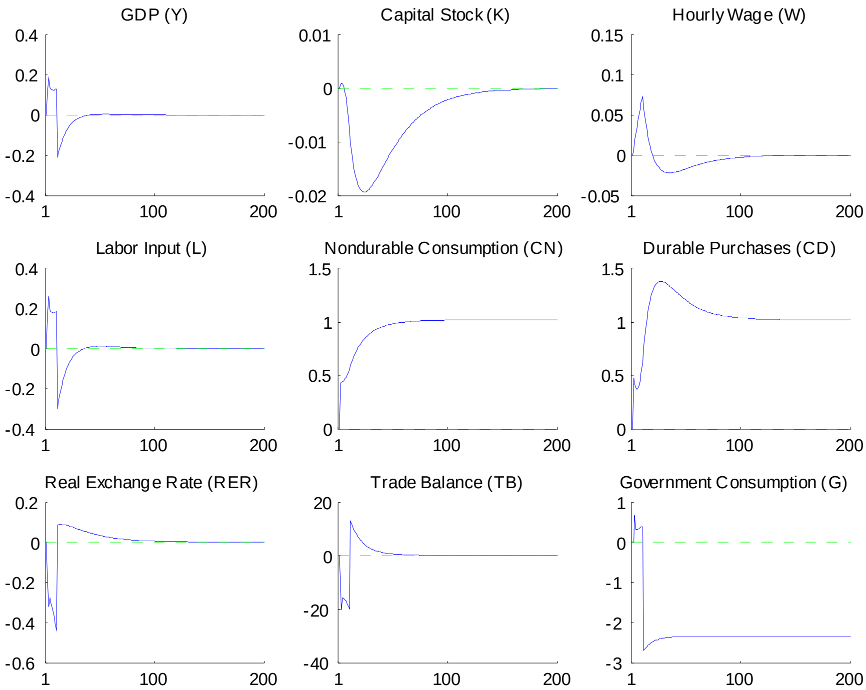
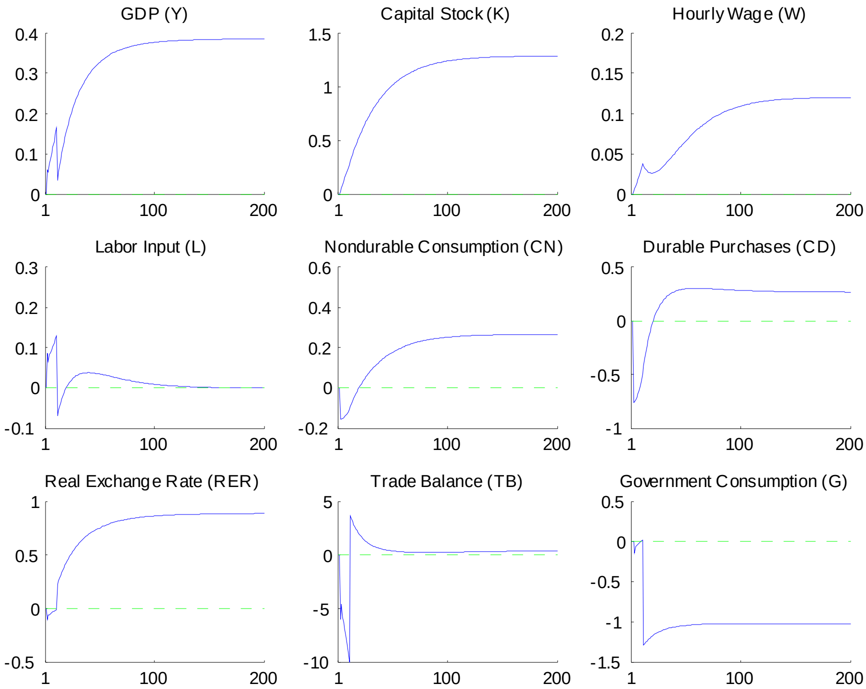


Figure 10.2: Future Corporate Tax Cut




demand expansion does not come from consumption, but from investment by the firms. High investment reduces dividends during the process, and thus households wish to borrow to smooth consumption. However, reducing $F$ below $F^*$ is costly. As mentioned in Section 3, this works like an increase in the effective interest rate, which reduces consumption of the two types, and $C^d$ in particular.

Figure 10.3 shows the responses to a five percentage points reduction of the tax rate on durable goods.[11] This generates a cycle associated with the demand for durable goods. As expected, durable purchases decline sharply at the time of the announcement. Demand remains low until the implementation, and then demand soars. This is reflected in corresponding fluctuations of the real exchange rate, and thus in production and employment. In the long run, lower taxes lead to higher durable purchases, while public consumption is lower due to less tax revenue.

[11]The tax rate on durable goods is calibrated to fifty percent. Therefore, in order to get a non-negligible effect, we present a five percentage point tax-cut.

Figure 10.3: Future Tax Cut on Durable Goods of Five Percentage Points

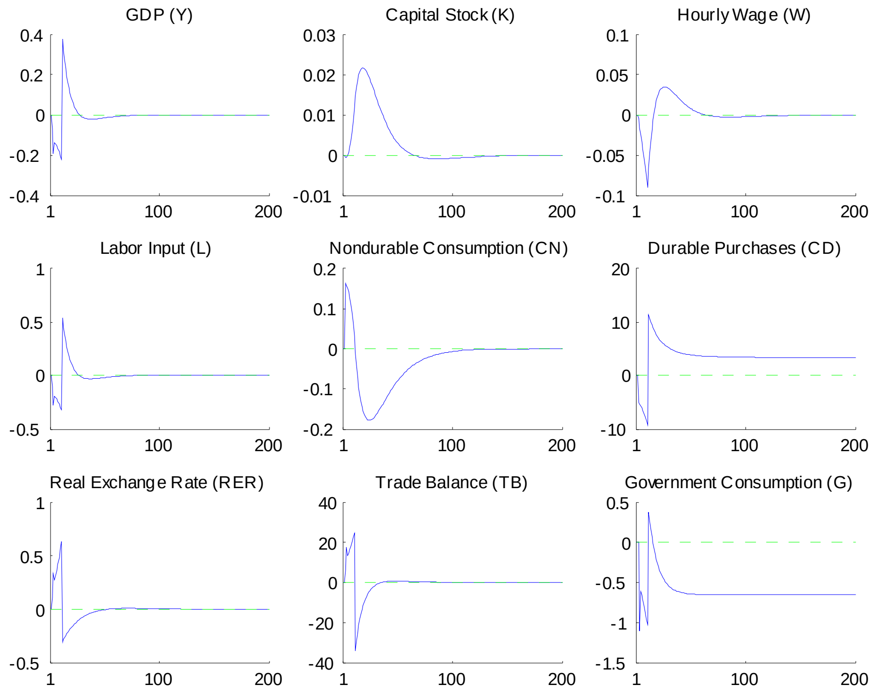
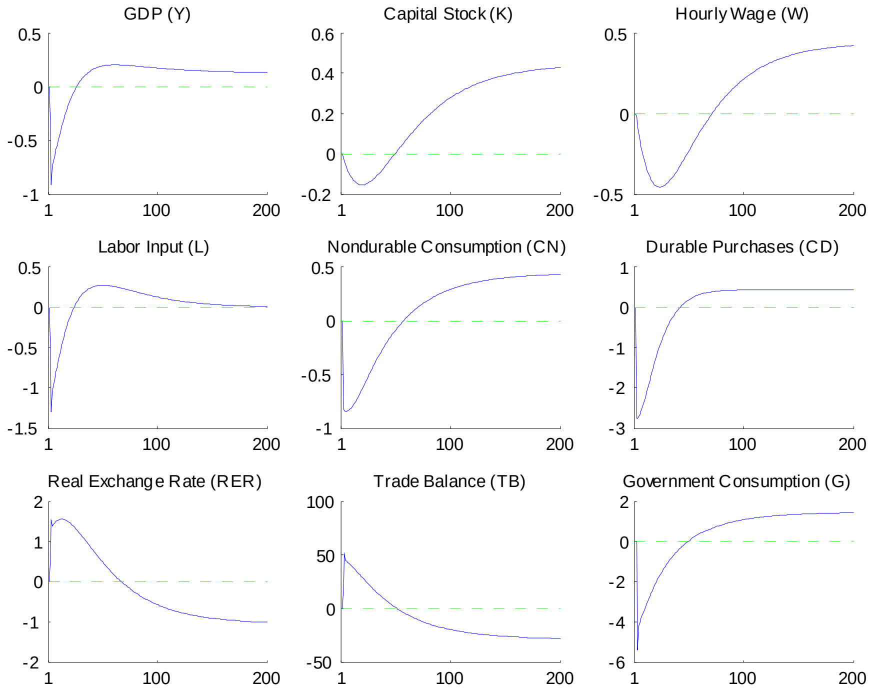


Figure 10.4: Reduction of the Debt to GDP Target




## 10.2. Adopting a Lower Public Debt Target

Figure 10.4 shows the responses to adopting a public debt/GDP target of 0.5 when starting from 0.8. We use a large horizon to show the slow convergence to a new steady state. The initial effect is mildly contractionary, due to the government spending cut—GDP declines by about 0.2 percent.

Interestingly, the long-run effects are expansionary due to the decline in the government interest payments. This induces a shift of public expenditure towards a higher $G$, which in turn causes a long-run appreciation. The resulting larger imports increase the optimal size of capital, due to the complementarity in production, while long-run labor input is constant given wage adjustment. Hence, the capital share links the long-run increase in capital and GDP.

### 10.3. An Alternative Fiscal Behavior

In the previous setup, tax rates are exogenous. However, there is evidence of a cyclical behavior of the Value Added Tax. In the 2009 slowdown, for example, VAT was increased following the drop in tax revenue, while in 2010, following the rebound, VAT was reduced. In addition, unlike other tax rates, the government does not commit to a long-run VAT trajectory. These changes, however, take the form of discrete jumps. To model discrete changes requires a different and much more challenging solution methodology.

Here, we report briefly the results from the following continuous approximation of that behavior, concentrating on the tax on nondurables:[12]

$$\tau_t^n = (\tau_t^n)^* + \varsigma \left( \frac{E_{t-\varsigma} R_t - R_t}{E_{t-\varsigma} R_t} \right), \quad \varsigma > 0,$$

where $(\tau_t^n)^*$ is the exogenous target tax rate. In this specification, an unexpected decrease in government revenue following a downturn leads to a higher tax rate. This, in turn, slows down economic activity further by reducing consumption demand.

Surprisingly, the simulation results show that the inclusion of this mechanism reduces output volatility, rather than increasing it. The reason is the cross effect on government consumption: The pro-cyclical tax rate changes offset, at least partially, the pro-cylicality of government consumption: Given the trajectory of the debt, higher tax revenue in a recession imply a smaller reduction of government spending. With a larger expenditure multiplier than tax multiplier, aggregate demand declines by less.

## 11. Concluding Remarks

This paper presents a stochastic open-economy model of the Israeli economy. The model has two non-standard features. One is a fiscal rule appropriate for Israel in the second decade of the millennium: tax rates, a public debt/GDP target, and a speed of adjustment to that target are exogenous, and government spending is endogenous to these factors.

The other feature is a liquidity effect on households spending decisions. This is modeled with a cost of holding assets away from an exogenous target level. A

[12] In reality, this tax rate corresponds to the value added tax. Also durable goods are subject to the value added tax, but for these goods, tax rate changes are small relative to their level.

temporary income increase leads to a desire to save in order to smooth consumption, but the cost of increasing assets leads to spending more than permanent income theory predicts. This captures a positive impact of a higher income flow on available liquidity. A persistent income increase creates the desire to upgrade the durable goods stock—which consequently causes the assets level to decline. The costs involved generate a gradual adjustment of consumption and durable purchases, capturing a liquidity shortage generated by the increased demand for durable goods.

Another important feature of the model is the treatment of all imports as intermediate goods in the production of the domestic good. This reflects the real world characteristic that final prices of imported goods contain a large value added domestic component.

Overall, the model is an updated version of the Bruno-Sachs framework, where the relative price of imports in terms of the domestic good clears the output market. The relative price of exports substitutes abroad in terms of domestic output is exogenously linked to the relative price of imports. The demand for goods depends positively on the relative price of foreign goods through exports. As foreign exports substitutes become more expensive, the demand for the economy's exports increase. The supply of goods depends negatively on the relative price of foreign goods via the cost of imported intermediate products.

The model has the Keynesian feature that aggregate demand shocks have a strong positive effect on production. The mechanism here involves the specification of imports as factors of production complementing capital and labor. A positive demand shock reduces the relative price of imports, and thus motivates higher imports, labor and capital. This mechanism, linking production to demand shocks is absent in the neoclassical framework.

There are a number of possible extensions to the model. In the current version, government budgeting is carried out one period (quarter) ahead, while the realistic budgeting horizon in Israel has been two years since 2009. Another direction to explore is a link between the tax rates, consumption taxes in particular, and current government revenue. In practice, Israel witnessed several procyclical adjustments of the value added tax to economic conditions. Endogenous capital utilization is another possible extension, although the current version already reflects this consideration indirectly: imports of intermediate products in the production function of output may capture capital utilization, as they include energy.

## References


[1] Barro, Robert J. (1979) "On the Determination of the Public Debt," *Journal of Political Economy*, 87, pp. 940-971.

[2] Bruno, Michael and Jeffrey Sachs (1985) "Economics of Worldwide Stagflation", Harvard University Press, Cambridge, MA.

[3] Caballero, Ricardo J. (1990) "A Case for Slow Adjustment", *The Quarterly Journal of Economics*, 105(3), pp. 727-743.

[4] Campbell, Jeffrey R. and Zvi Hercowitz (2009), "Liquidity Constraints of the Middle Class".

[5] Deaton, Angus (1987) "Life-Cycle Models of Consumption: Is the Evidence Consistent with the Theory?", in T. Bewley (ed.) *Advances in Econometrics*, Fifth Wold Congress, Vol 2, Cambridge University Press.

[6] Flavin, Marjorie (1985) "Excess Sensitivity of Consumption to Current Income: Liquidity Constraints or Myopia?", *Canadian Journal of Economics* 18, pp. 117–136.

[7] Friedman, Amit and Yaacov Lavi (2006) "The Real Exchange Rate and Foreign Trade in Israel", *Bank of Israel Review* 79, pp.37-86 (Hebrew).

[8] Hercowitz, Zvi, Yaacov Lavi and Rafi Melnick (1997) "A Macroeconomic Model of the Israeli Economy" (Hebrew).

[9] Lavi, Yaakov and Nathan Sussman (2005), "The Determinants of the Real Wage in Israel in the Long Run and in the Short Run", *Bank of Israel Review* 77, pp. 43-75 (Hebrew).

[10] Lavi, Yaacov (2004) "Do Changes in Current Income Help to Explain Changes in Consumption in Israel?", Israel Economic Review 2, pp. 113-135.

[11] Lavi, Yaacov and Michel Strawczynski (2005), "The Impact of Fiscal Policy on Private Consumption in Israel With Emphasis on the Fiscal Expectations Approach", *Israel Economic Review* 3 (1), pp. 53-86.

[12] Ljungqvist, Lars and Thomas J. Sargent (2004), *Recursive Macroeconomic Theory*, MIT Press.

[13] Mazar, Yuval (2010) "The Impact of Fiscal Policy and its Components on Output in Israel", *Bank of Israel*, Research Department Discussion Paper No 2010.07.