Optimal Unemployment Insurance with Monitoring

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March 4, 2012

Abstract

Monitoring the job-search activities of unemployed workers is a common government intervention. I model monitoring in the optimal unemployment insurance framework of Hopenhayn and Nicolini (1997), where job-search effort is private information for the unemployed worker. In the model, monitoring provides costly imperfect information upon which the government conditions the unemployment benefits. In the optimal monitoring scheme, random monitoring, together with endogenous sanctions and rewards, create effective job-search incentives for the unemployed worker. For CRRA utility, the monitoring frequency increases and the spreads decrease with promised utility, if and only if the coefficient of risk aversion is greater than $\frac{1}{2}$. Compared to optimal unemployment insurance, monitoring saves, at the balanced budget point, about eighty percent of the cost associated with moral hazard. The gain is achieved by a decrease of more than half in the standard deviation of consumption.

JEL Classification: D82; H21; J64; J65

Keywords: Recursive Contracts; Unemployment Insurance; Job Search Monitoring

*I am grateful to Gianluca Violante for his valuable guidance on the original version of this paper. I thank Gian Luca Clementi, Chris Flinn, Ady Pauzner, Nicola Pavoni, and Roine Vestman for their useful comments. Yair Antler provided outstanding research assistance. All remaining errors are mine. Correspondence: Ofer Setty, Department of Economics, Tel Aviv University. E-mail: ofer.setty@gmail.com.
1 Introduction

Most unemployment insurance programs in the United States include monitoring of the job-search effort of the worker (Grubb, 2000). A typical monitoring policy requires the unemployed worker to record his job-search activities by listing the employers he contacted in a given period. At the employment office, a caseworker evaluates occasionally whether the job-search requirements are met by verifying that the contacts are authentic. If the caseworker finds the report unsatisfactory, then she may impose sanctions, usually in the form of benefits' reduction for a limited period.

In the last three decades active labor market policies such as job-search monitoring have gained a higher share of the total spending on labor policies\(^1\). Such policies are receiving increasing attention as governments seek to insure unemployed workers without damaging their incentives for becoming employed.

Given that job-search monitoring is available and is implemented by governments, it is important to model this policy, and to examine to what extent these instruments increase the efficiency of unemployment insurance programs. This is a non-trivial task since such instruments, as valuable as they may be, are also costly.

In this paper I model monitoring in the framework of optimal unemployment insurance developed by Hopenhayn and Nicolini (1997) and characterize the optimal contract in the presence of monitoring. In optimal unemployment insurance, a risk neutral planner insures a risk averse worker against unemployment by setting transfers during unemployment and a wage tax or a subsidy during employment. During unemployment, the worker searches for a job by exerting an effort level which is his private information. The first best, had the information been observable to the planner, is to deliver constant consumption to the worker regardless of the employment status. Since, however, the planner cannot observe

\(^1\) Between 1985 and 2001 the share of active labor market policies in OECD countries increased from 35% to 52% (OECD 2005).
the job-search effort, constant benefits would flaw the worker’s incentives to search for a job. Therefore, to solve the incentive-insurance trade-off, benefits should continuously decrease during unemployment and the wage tax upon re-employment should continuously increase.

I incorporate monitoring into the optimal unemployment insurance framework as follows. The planner monitors the unemployed worker with some history-dependent probability. When a worker is monitored, the planner pays a cost and receives a signal that is correlated with the job-search effort of the worker. The planner uses that signal to improve the efficiency of the contract by conditioning future payments and the wage tax, not only on the employment outcome, but also on the signal. These future values create endogenous sanctions and rewards, that together with the random monitoring, create effective job-search incentives: the worker exerts a high job-search effort in order to increase the probability of a good signal, and consequently to increase the probability of higher payments.

Using a two period model I characterize the optimal contract for a worker with logarithmic utility from consumption. The monitoring frequency and the dispersion of future utilities complement each other in creating the incentives for the worker to search actively for a job. The specific combination of those two components depends on the generosity of the welfare system. As the generosity of the welfare system increases, the planner monitors the unemployed worker more frequently but imposes lower sanctions. The driving force of this result is that while the cost of monitoring is independent of the generosity of the welfare system, the cost of spreading future utilities increases with generosity.

I then use an infinite horizon model to quantify the optimal contract. In this extended environment I show that the characteristics of the optimal contract can be generalized to any CRRA preferences with a coefficient of risk aversion $\sigma$ of at least 0.5. When $\sigma < 0.5$ the exact opposite happens: as the generosity increases, monitoring frequency decreases
and the spread between utilities increases. This happens because $\sigma = 0.5$ is the cutoff point between spreading costs that increase with generosity ($\sigma > 0.5$) and spreading costs that decrease with generosity.

The infinite time model is then used for estimating the value of the additional instrument of monitoring by comparing the results of the model to the results of a model where monitoring technology is unavailable. I calibrate the model to the US economy at the level of utility that balances the government budget for the model with no monitoring. Keeping utility fixed I compare the gain from monitoring to the gain from shifting to the first best allocation. I find that the gain from monitoring equals to roughly two thirds of the gain from shifting to the first best allocation. These savings stem from the ability of the planner to smooth the worker’s consumption across states. Indeed, monitoring decreases about half of the standard deviation of consumption.

Empirically, the effect of job-search monitoring on labor market outcomes such as unemployment duration is usually significant and positive\(^2\). Johnson and Klepinger (1994) used random assignment of unemployed workers to treatment groups that differed in the job-search requirements. They find that waving the weekly requirement to record three contacts increased the average unemployment spell by 3.3 weeks. Benus and Johnson (1997) find that increasing the number of required contacts from two to four decreased the average unemployment spell by 5.9%, and that informing the unemployed workers that the contacts will be verified decreased the average unemployment spell by 7.5%.

The evidence on the effects of sanctions is limited yet encouraging. In two empirical studies that were conducted in the Netherlands, Van den Berg et al. (2004) and Abbring et al. (2005) find that the unemployment exit rate doubles following a sanction. Lalive et al. (2005) use Swiss data on benefit sanctions and find that both warning about not

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\(^2\)Van den Berg et al. (2006) consider a model where the efficiency of search is damaged because unemployed workers substitute formal channels for informal channels. For adverse effects of job search assistance see Van den Berg (1994) and Fougere et al. (2009).
complying with eligibility requirements and enforcement have a positive effect on the exit rate out of unemployment, and that increasing the monitoring intensity reduces the duration of unemployment of the non-sanctioned.

Monitoring has been previously examined by several authors. A common assumption in this literature is that monitoring perfectly reveals the agent’s hidden information (or action) to the principal. This simplifying assumption, which goes back to the seminal paper by Becker (1968) on Crime and Punishment, has important implications. In a standard environment, using the signal allows the planner to get arbitrarily close to the first best allocation by using a combination of very low monitoring frequency (that costs very little) with an extremely severe punishment that will never be applied. This potential threat ensures that the worker exerts the high effort.

In practice, extracting the exact information on the worker’s job-search effort may not be possible. Furthermore, even if extracting the exact information was possible, it might be very costly, and it may be more effective for the planner to extract imperfect information on the job-search effort for a significantly lower cost.

In contrast with existing literature, my model allows for an imperfect signal. This is a key generalization that has three essential implications. First, the monitoring probability becomes a decision variable. Second, limited sanctions and rewards arise endogenously in the optimal contract. Third, sanctions are applied in equilibrium. These results are realistic for many applications of monitoring, including that of unemployment insurance. Specifically, maximal sanctions are usually not practiced and monitoring is not applied with certainty. The consistency between the model's results and the actual policy is a clear advantage for the welfare analysis.

One of the exceptions of modeling imperfect signal is Boone, Fredriksson, Holmlund, and van Ours (2007). They restrict the set of policies among which the optimal one is chosen. First, the planner does not condition the benefits on the worker’s history, and
second, the planner can only punish the worker by applying a fixed decrease in benefits for the remaining unemployment spell. Their model, however, has the advantage of general equilibrium which my model lacks.

Pavoni and Violante (2007) consider monitoring as part of an optimal Welfare-to-Work program. In their model the planner can perfectly observe the worker’s job-search effort by paying some cost. As a result, the planner monitors the job-search effort of the worker with certainty and therefore sanctions or rewards are never needed.

For a more complete review on models of job-search monitoring see Fredriksson and Holmlund (2006).

Although the focus of the paper is unemployment insurance, monitoring is a general mechanism. Hence, I review models of monitoring in other contexts.

Aiyagari and Alvarez (1995) consider the optimal contract including perfect signal monitoring given hidden information. In their model the planner may deprive the agent’s leisure and determine his consumption. They characterize the optimal monitoring frequency over compact consumption sets to avoid making the monitoring technology so powerful that the problem becomes uninteresting. The contract characteristics near the bounds of consumption lead monitoring frequency to be non monotone.

Popov (2009) models verification of hidden information reported by a worker. He keeps the problem nontrivial by assuming that the utility function is bounded from below and that the continuation utility is bounded. With this assumption the contract delivers bounded sanction and reward according to the verification result. He finds that monitoring never occurs with certainty and that for a certain class of utility functions the principal would use verification regardless of this cost.

Newman (2007) studies entrepreneurial risk and occupational self-selection. He uses a static principal agent problem to study the optimal match between exogenous monitoring technologies and workers who differ by their outside option. The available monitoring
technologies differ by their efficiency in a way that can also be interpreted as providing an imperfect signal. Newman shows that when workers have logarithmic utility, the optimal contract leads to positive assortative matching between workers with a higher level of promised utility and tasks that produce more observable output.

The rest of the paper is organized as follows. In Section 2, I describe the infinite horizon model. In Section 3, I use a two period model to derive the theoretical characterization of the optimal contract. In Section 4, I calibrate the model to the US economy. In Section 5, I characterize the optimal monitoring policy, and estimate the value of monitoring. In Section 6, I conclude and discuss further research.

2 The model

2.1 The economy

Preferences: Workers have a period utility $u(c) - a$ where $c$ is consumption, $a$ is disutility from job-search effort or work, and $u$ is assumed to be strictly increasing and strictly concave. Workers discount the future at the discount factor $\beta$.

Employment and Unemployment: The worker is either employed or unemployed. During employment, which is assumed to be an absorbing state, the worker exerts a constant effort level $e_w$, and receives a fixed periodic wage $w^3$.

During unemployment, the worker searches for a job with an effort level $a \in \{e_l, e_h\}$ that is either low or high and is private information of the worker. The job-finding probability increases with the job-search effort level $j \in \{l, h\}$ and is denoted by $\pi_j$. The low job-search effort is interpreted as not actively looking for a job, and therefore I set $e_l = 0$ and $\pi_l = 0$. For brevity of notation, denote henceforth $e_h$ as $e$, and $\pi_h$ as $\pi$.

\footnote{The assumption that employment is an absorbing state is widely used in the literature (e.g. Hopenhayn and Nicolini 1997, Pavoni 2009, and Pavoni and Violante 2007). This assumption allows us to analyze one unemployment spell at a time, and does not affect the qualitative characteristics of the optimal policy.}
Monitoring technology: The monitoring probability \( \mu \in [0, 1] \) is a decision variable of the planner. When the worker is monitored, the planner receives a signal on the worker’s job-search effort that is either good or bad, denoted by \( \{g, b\} \) respectively. The probability of a good signal given job-search effort \( j \in \{l, h\} \) is \( \theta_j \). The signal is only informative if \( \theta_h \neq \theta_l \), and I assume, without loss of generality, that \( \theta_h > \theta_l \). This means that following a high job-search effort, a monitored worker is more likely to receive the good signal, then following the low job-search effort. Note that this technology does not restrict \( \theta_h \) to be higher than 0.5. Indeed, there might be some strict monitoring tests generating a useful signal for which \( \theta_h \) might be very small.

Allowing \( \theta_h \) to be smaller than 1 indicate that the planner receives imperfect information regarding the worker’s effort. This false negative option is a realistic feature of the unemployment insurance system, representing a verification that fails unjustifiably. Allowing \( \theta_l \) to be \( (\theta_h) \) to be greater than 0 is another source of imperfection, representing a false positive result. This imperfection occurs, for example, due to an administrative failure or due to over-generosity of the caseworker.

The cost of monitoring is quasi-concave in the monitoring frequency and equals to \( \kappa \mu^\alpha \) per period, with \( \alpha \leq 1 \). This cost discourages the planner from setting the monitoring frequency to 1 under all circumstances. This cost structure covers both constant and increasing returns to scale. Decreasing returns to scale seem unreasonable since the monitoring application can be split between caseworkers\(^4\). Nevertheless, I study convex costs in the quantitative analysis and show that qualitatively the characteristics of the contract are identical to those of the quasi concave cost.

The assumption that only one monitoring technology is available to the planner can be relaxed by allowing the planner to choose a monitoring technology \( m \) from the set \( \mathcal{M} = \{ \kappa^i, \theta^h_i, \theta^l_i, \alpha^i \} \}_{i=1}^N \), which includes \( N \) monitoring technologies.

\(^4\)Given that the administrative institutions for unemployed workers already exist, I assume that monitoring has no additional fixed cost.
Information structure: Both the worker and the planner observe the employment state, the monitoring signal and the on-the-job effort level\(^5\). The job-search effort level of the worker is his private information. This leads to the moral hazard problem.

Timing: Figure 1 shows the timing of the model and the four possible outcomes at the end of the period. At the beginning of each period, the planner delivers consumption \(c\) to the worker. Then, the worker looks for a job with an effort level \(e_j\) and finds a job with probability \(\pi_j\). If the worker becomes employed then the planner does not apply monitoring\(^6\). If, on the other hand, the worker remains unemployed then he is monitored with probability \(\mu\). When monitoring takes place, the planner pays the cost \(\kappa\), and receives the signal \(s \in \{g, b\}\).

Figure 1 Approximately Here

Given the realizations of the employment state, monitoring, and the signal, the four possible outcomes at the end of the period are: employment \((e)\), unmonitored unemployment \((n)\), monitored unemployment with a good signal \((g)\), and monitored unemployment with a bad signal \((b)\).

2.2 The planner’s problem

The optimal contract between the planner and the worker requires, in general, conditioning the benefits and the wage tax on the entire history of the worker. Spear and Srivastava (1987), Thomas and Worrall (1988), Abreu, Pearce, and Stacchetti (1990), and Phelan and Townsend (1991) found that all the relevant information for the recursive contract is contained in a one-dimensional object. In the monitoring recursive contract, as in the

\(^5\)This assumption is standard in the optimal unemployment insurance literature, and goes back to Hopenhayn and Nicolini (1997). Wang and Williamson (2002) consider the case where the worker’s effort level affects the probability of transitions both from unemployment to employment and from employment to unemployment.

\(^6\)When a worker becomes employed, the effort level is perfectly revealed to the planner and therefore monitoring such a worker is never optimal.
unemployment insurance contract, this one-dimensional state is the expected discounted utility $U$ promised to the worker at the beginning of each period. This value is updated at the end of each period, according to the outcomes. Hence, the state is governed by all the relevant information in the worker’s history. Although this state is not a primitive of the model, using it makes the problem tractable. Once the model is solved, the state is used to back out the allocation for each type of worker.

In what follows, I present the planner problems during employment and during unemployment.

### 2.2.1 The planner’s problem during employment

Let $W(U)$ be the value for the planner from an employed worker who has promised utility $U$. The planner’s problem during employment is:

$$ W(U) = \max_{c,U^e} -c + w + \beta W(U^e) $$

subject to:

$$ U = u(c) - \epsilon_w + \beta U^e, $$

where $U^e$ is the future promised utility contingent on employment. If $c > w$ then the planner delivers the difference to the worker as a wage subsidy and if $c < w$ then the planner extracts the difference as a wage tax. The constraint in the problem, commonly known as the promise keeping constraint, states that the expected utility for the worker given current consumption, disutility from work, and discounted future promised utility, has to deliver, in expected terms, the utility $U$ that was promised to the worker at the beginning of the period.

Given the absence of moral hazard during employment, the solution to the employment problem is full insurance and constant benefits, which implies a constant wage tax or subsidy.
2.2.2 The planner’s problem during unemployment

I assume that the government announces the optimal policy at time zero and commits itself to it. This assumption eliminates policies in which the planner deviates from the announced policy (e.g., the government does not monitor ex-post) and workers update their beliefs according to the observed government policy. Given the commitment assumption, the question of whether the planner should monitor or not, need only be examined ex-ante: if the addition of monitoring improves the effectiveness of the contract then the government should use monitoring and follow the monitoring scheme.

For an unemployed worker, the planner chooses for each possible state six variables: consumption $c$, monitoring probability $\mu$, and four continuation values, one for each possible outcome: employment $U^e$, unmonitored unemployment $U^u$, monitored unemployment with a good signal $U^g$, and monitored unemployment with a bad signal $U^b$. In addition to these six decisions, the planner recommends a job-search effort level. When the planner recommends a high job-search effort level, he needs to support this recommendation by making it worthwhile for the worker to follow the recommendation. This is achieved by the incentive compatibility constraint that guarantees that the expected utility for a worker who exerts the high job-search effort is at least as high as that of a worker who exerts the low job-search effort. Let $V(U)$ be the value for the planner, who recom-

7The commitment assumption is typical in the unemployment insurance literature, e.g., Pavoni (2007) and Hopenhayn and Nicolini (1997). There, too, the planner never deviates from the declared scheme.

8If the planner recommends the low effort level then there is no need to set incentives and the solution is constant benefits and a constant wage tax. This solution can be achieved because while $\pi > 0$, the probability of finding a job associated with zero effort is zero. Therefore the planner knows that a worker who received a job offer must have searched for a job with a high effort level. The planner can use this observation to apply a punishment severe enough to discourage workers from not following the low job-search effort recommendation.

9For high enough values of promised utility, creating incentives by spreading future promised utilities is too costly and the planner recommends low job-search effort and implements full insurance (Pavoni and Violante (2007) refer to this state as Social Assistance). In the current calibration social assistance is optimal only for values of promised utility associated with extremely high consumption levels, around 10 times the wage. Thus, in the simulations the promised utility values of the workers at the balanced budget point are much lower than these values. To fully characterize the optimal monitoring policy, I describe the monitoring policy as if creating incentives for the worker to extract the high job-search effort is always desirable.
mends the high job-search effort, from an unemployed worker who has promised utility $U$. The problem of that planner during unemployment is:

$$V(U) = \max_{c, U^e, U^g, U^b, U^n, \mu} -c + \beta \left\{ \pi W(U^e) + (1 - \pi) \left\{ (1 - \mu) V(U^n) + \mu \left[ \theta_h V(U^g) + (1 - \theta_h) V(U^b) \right] - \kappa \mu \right\} \right\}$$

s.t.:

$$U = u(c) - e + \beta \pi U^e + \beta (1 - \pi) \left\{ (1 - \mu) U^n + \mu \left[ \theta_h U^g + (1 - \theta_h) U^b \right] \right\}$$

$$U \geq u(c) + \beta \left[ (1 - \mu) U^n + \mu \left[ \theta_i U^g + (1 - \theta_i) U^b \right] \right],$$

where the objective function includes: the cost of consumption payments to the worker; the discounted value of employment at future promised utility $U^e$; the discounted value of unmonitored unemployment; the discounted value of monitored unemployment; and the monitoring cost. The constraints are the promise keeping constraint and the incentive compatibility constraint. Since the incentive compatibility constraint is clearly binding at the optimum, I will use equality below for this constraint.

By applying fairly standard results in dynamic programming, one obtains that $W(U)$ and $V(U)$ are continuous functions, which are decreasing, concave and continuously differentiable in $U^{10}$.

### 3 The optimal contract

While the model presented above is suited for quantitative study, it is too cumbersome for theoretical analysis. To this end I analyze in this section a two period model that caries the same economic forces as the full blown model. Specifically, the possible outcomes, their probabilities, and the choice variables of the planner are all identical to the full blown problem. The following adjustments take place in the two period model: $W(U)$ becomes

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10Since the monitoring cost is concave in $\mu$, $V(U)$ can be improved by using lotteries over the monitoring probability. This is a linear programming problem that can be solved numerically as shown by Phelan and Townsend (1991). I abstract from such lotteries in this paper. In the quantitative analysis I use a linear monitoring cost that eliminates the use of such lotteries.
\( w - c^e; U^i \) becomes \( u(c^i) \) for \( i \in \{e, g, n, b\} \); and \( V(U^i) \) becomes \( c^i \) for \( i \in \{g, n, b\} \).

I impose \( u(c) \) to be logarithmic and discuss the generalization of the results to CRRA preferences in section 5. Finally, I assume for simplicity that \( \beta = 1 \).

The problem of the planner for an unemployed worker is then:

\[
\begin{align*}
V(U) &= \max_{c, c^e, c^g, c^b, c^n} \quad -c - \pi (c^e - w) - (1 - \pi) \left\{ (1 - \mu) c^n + \mu [\theta_h c^g + (1 - \theta_h) c^b] - \kappa \mu^a \right\} \\
\text{s.t.} \quad U &= u(c) - e + \pi u(c^e) + (1 - \pi) \left[ (1 - \mu) u(c^n) + \mu (\theta_h u(c^g) + (1 - \theta_h) u(c^b)) \right] \\
U &\geq u(c) + [(1 - \mu) u(c^n) + \mu (\theta_h u(c^g) + (1 - \theta_h) u(c^b))] \\
\end{align*}
\]

For the proof exposition it is useful to set the problem in two steps. In the first step we solve for \( M(U, \mu) \), which is identical to (3) above except that the monitoring frequency is given exogenously. In the second step we solve for \( V(U) = \max_\mu M(U, \mu) \).

The Lagrangian of \( M(U, \mu) \) is:

\[
\begin{align*}
&-c - \pi (c^e - w) - (1 - \pi) \left\{ (1 - \mu) c^n + \mu [\theta_h c^g + (1 - \theta_h) c^b] - \kappa \mu^a \right\} \\
&+ \lambda_1 \left\{ U - u(c) + e - \pi u(c^e) - (1 - \pi) \left[ (1 - \mu) u(c^n) + \mu (\theta_h u(c^g) + (1 - \theta_h) u(c^b)) \right] \right\} \\
&+ \lambda_2 \left\{ U - u(c) - (1 - \mu) u(c^n) - \mu (\theta_h u(c^g) + (1 - \theta_h) u(c^b)) \right\},
\end{align*}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multipliers of the promise keeping and incentive compatibility constraints, respectively. Note that both \( \lambda_1 \) and \( \lambda_2 \) are negative.

I start the characterization of the optimal contract with two results regarding the relative values of consumption levels. The first result refers to the ranking of the future consumption levels.

**Lemma 1** In the optimal solution \( c^e > c^g > c^n > c^b \).

**Proof.** This follows directly from the likelihood ratios\(^{11}\), with \( l^e > l^g > l^n > l^b \), where:

\(^{11}\)The likelihood ratio is defined as follows. For each outcome \( i \in \{e, g, b, n\} \) define \( p^i_1, p^i_2 \) as the probability of the outcome given high and low effort levels, respectively. Then \( l^i = \frac{p^i_1 - p^i_2}{p^i_1} \).
According to Lemma 1 the monitoring technology creates an endogenous prize \( c^g - c^n > 0 \) when the good signal is realized, and an endogenous sanction \( c^n - c^b > 0 \) when the bad signal is realized. The next result refers to the relationship between future consumption levels and the shadow prices.

**Lemma 2** In the optimal solution current consumption equals to an average of future consumption levels, weighted by the probabilities given high effort for outcomes \( i \in \{ e, g, n, b \} \).

**Proof.** The first order conditions of (4) with respect to the choices of future consumption levels \( c^i, i \in \{ e, g, n, b \} \) are\(^{12}\):

\[
c^i = -\lambda_1 - \lambda_2 l^i
\]

Averaging (5) over outcomes \( i \) with the probabilities of each outcome given the high effort yields that \( \lambda_1 = -\pi c^e - (1 - \pi) \left\{ (1 - \mu) c^n + \mu [\theta_h c^g + (1 - \theta_h) c^b] \right\} \). Using the first order condition of (4) w.r.t. the choice of current consumption yields \( \lambda_1 = -c \), implying that \( \pi c^e + (1 - \pi) \left\{ (1 - \mu) c^n + \mu [\theta_h c^g + (1 - \theta_h) c^b] \right\} = c = -\lambda_1 \).

Lemma 2 simply states that the shadow prices of the promise keeping constraint is equal to the weighted average of the reciprocals of marginal utility, which for log utility equals to consumption itself.

Before moving to the main propositions regarding the optimal contract, we can gain insights on the optimal contract from the two Lemmas above. First, using (5) for \( c^g, c^n \) and \( c^b \) implies that \( c^n = \theta_h c^g + (1 - \theta_h) c^b \). Thus, in the optimal contract the prize and the sanction balance each other. Also observe that as the precision of the monitoring signal increases, the ratio of sanction to prize, \( \frac{\theta}{1-\theta} \), increases. Therefore, for high precision

\(^{12}\)Note that although \( \{ c^i, \lambda_1, \lambda_2 \} \) are functions of the state \( \{ U, \mu \} \) I omit the state from these functions for brevity of notation.
signals monitoring is applied with a modest prize with a high probability and a severe sanction with a low probability.

Second, by the likelihood ratios, as the monitoring signal becomes more precise ($\theta_h$ increases, $\theta_l$ decreases, or both), $c^g$ and $c^b$ move further away from $c$ relative to the other two levels of consumption $c^e$ and $c^n$. In the extreme case of a non-informative signal, when $\theta_h = \theta_l$, $c^g$ and $c^b$ are equal to $c^n$. In the other extreme case, when $\theta_h = 1$, the sanction explodes relative to any other spread. Note that as long as $\theta_h > \theta_l$, $\theta_h = 1$ provides a perfect signal regardless of the value of $\theta_l$. This is the case because upon a bad signal, the planner knows with certainty that the worker deviated from the recommended level of effort. This event, regardless of its probability, can be leveraged as much as needed to provide the incentives for the worker to exert the high level of effort.

I now move to the effect of the worker’s state on the monitoring frequency and the spread of future consumption. The proof is inspired by Newman (2007).

**Proposition 1** The optimal monitoring frequency increases with $U$.

**Proof.** I first show that $M(U, \mu)$ is super modular in $U, \mu$. For a twice differentiable function this is equivalent to showing that $\frac{\partial^2 M(U, \mu)}{\partial U \partial \mu} \geq 0$.

Since $\lambda_1$ is the shadow price of the promised utility constraint $\frac{\partial M(U, \mu)}{\partial U} = \lambda_1$, and therefore the supermodularity condition becomes:

$$
\frac{\partial^2 M(U, \mu)}{\partial U \partial \mu} = \frac{\partial \left\{ \frac{\partial M(U, \mu)}{\partial U} \right\}}{\partial \mu} = \frac{\partial \lambda_1}{\partial \mu}
$$

From Lemma 2 $\lambda_1 = -\frac{\pi c^e - (1-\pi) \left\{ (1-\mu)c^n + \mu \left[ \theta_h c^g + (1-\theta_h) c^b \right] \right\} - c}{2}$ and therefore:

$$
\frac{\partial^2 M(U, \mu)}{\partial U \partial \mu} = \frac{\partial \left\{ -\frac{\pi c^e - (1-\pi) \left\{ (1-\mu)c^n + \mu \left[ \theta_h c^g + (1-\theta_h) c^b \right] \right\} - c}{2} \right\}}{\partial \mu}
$$

Note that $-\frac{\pi c^e - (1-\pi) \left\{ (1-\mu)c^n + \mu \left[ \theta_h c^g + (1-\theta_h) c^b \right] \right\} - c}{2}$ is the total cost for the planner provid-
ing that level of utility excluding the monitoring cost. Since the monitoring cost is excluded the cost of providing utility cannot increase when monitoring increases\textsuperscript{13}. Therefore:

$$f(1) f(1) cn + \left[h+c + (1-h)b \right] g_c \atop 0 \), and supermodularity between $U$ and $V$ holds. Supermodularity implies that increasing one variable increases the returns to increasing the other variables (Athey, 2002). In the monitoring context, supermodularity implies that increasing the promised utility improves the return to monitoring. Therefore, as long as the first order condition with respect to the monitoring frequency holds for $V (U)$, the monitoring frequency increases with promised utility. \Box

This result is consistent with the result of positive assortative matching in a model where high promised-utility workers are matched with tasks that produce more observable output (Newman, 2007).

The monitoring frequency is one of the two instruments of the monitoring policy. The second instrument of monitoring is the spread between future utilities (henceforth, spreads), defined as $u(c^i) - u(c^j) i \in \{e,g,n,b\}$, such that $c^i \geq c^j$. The next main result, proposition 2, complements the first proposition by showing that as promised utility increases the spreads decrease. The next Lemma is an important building block for this result:

**Lemma 3** When either $\mu$ or $U$, or both change, all the spreads move in the same direction.

**Proof.** See Appendix C. \Box

**Proposition 2** The spread between future utilities decreases with promised utility.

\textsuperscript{13}Denote the initial monitoring probability with $\mu_1$ and the new higher probability by $\mu_2 > \mu_1$. The planner can ignore the additional information by using a lottery on the monitoring application with weight $\delta = \mu_2 / \mu_1$. When monitoring is applied (with probability $\mu_2$) the planner will use the same allocation given $\mu_1$ with probability $\delta$ and the allocation given that monitoring was not applied with probability $1 - \delta$. 

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Proof. Rewrite the incentive compatibility constraint as a linear combination of spreads.

\[ \pi[u(c^e) - u(c^g)] + \pi(1 - \mu)[u(c^g) - u(c^\alpha)] + \mu(\pi(1 - \theta_h) + \theta_h - \theta_l)[u(c^g) - u(c^b)] = e. \]  

(6)

Taken together with Lemma 3, (6) implies that if \( \mu \) is constant then all spreads are constant as well. This is the case since the coefficients of all spreads in (6) are all positive.

Using (6) the effect of an increase in the monitoring probability on the left hand side of the constraint can be written as:

\[ \pi[u(c^\alpha) - u(c^b) - \theta_h(u(c^g) - u(c^b))] + (\theta_h - \theta_l)[u(c^g) - u(c^b)] \]

The right term is strictly positive because \( \theta_h > \theta_l \) and \( c^g > c^b \). Therefore, the total effect of the increase in the monitoring frequency on the left hand side is strictly positive if the first term is non-negative. For log utility this is equivalent to showing that:

\[ \log(c^\alpha) > \theta_h \log(c^g) + (1 - \theta_h) \log(c^b) \]

Since at the optimum \( c^\alpha = \theta_h c^g + (1 - \theta_h)c^b \) (see Lemma 2 above), the inequality above holds as the logarithmic function is a concave transformation of \( f(c) = c \).

Therefore, following an increase in \( \mu \), at least one spread must drop to keep the constraint tight, and by Lemma 3 all the spreads drop together.

Since we know from Proposition 1 that an increase in \( U \) leads to an increase in the monitoring frequency, and since an increase in the monitoring frequency leads to a decrease in the spreads, it follows that an increase in \( U \) also leads to a decrease in the spreads\(^{14}\).

\(^{14}\)Equation (6) also shows that \( U \) may only affect the spreads by a change in \( \mu \). This rules out a combined effect such as a decrease in the spreads because of the increase in the monitoring probability and an independent increase in the spread following a change in \( U \).
Taken together, the two propositions above suggest that as the promised utility of the worker increases the planner shifts the composition of the two components of monitoring: increasing the monitoring frequency and decreasing the spreads. I now discuss the driving force behind this result.

The gain of either of the two components is satisfying the IC constraint. The cost for the planner of increasing the spreads is an increase in the average cost of providing consumption due to the worker’s risk aversion. The cost of increasing the frequency is the marginal cost of monitoring.

The dynamics of the monitoring frequency and the spreads are rooted in the risk aversion of the worker’s utility from consumption. Log utility implies that as the promised utility increases, the cost of spreading out the future promised values (i.e. the cost of providing utility in spread values above the cost of providing the certainty equivalent) increases as well.

Since the monitoring cost is independent of the promised utility level, the cost of the sanction relative to the cost of applying monitoring increases with promised utility and the planner substitutes sanctions with more frequent monitoring.

I show in section 5 that: this characterization can be extended to CRRA utility with $\sigma > \frac{1}{2}$; for $\sigma = \frac{1}{2}$ the monitoring frequency and the spreads are invariant to $U$; for $\sigma < \frac{1}{2}$ the monitoring frequency decreases and the spreads decrease with $U$.

4 Calibration

The next task is to quantify the optimal contract for the US economy. To this end I calibrate the infinite horizon model presented in section 2.

Table 1 lists the parameters of the model. The unit of time is set to one month, and preferences are log utility in consumption. The monthly discount factor $\beta$ is set to 0.9959 to match an annual interest rate of 5% (Cooley, 1995). Monthly earnings, $w$, are set
to $2,800, which is the median monthly earnings of all workers (DOL, 2006a). The job finding probability \( \pi \), is set to 0.17, based on the CPS derived data constructed by Shimer (2005). The disutility of work effort, \( e_w \), which equals the disutility of job-search effort, \( e \), is equal to 0.67 (Pavoni, Setty and Violante, 2010).

The monitoring technology is characterized by four parameters: the probabilities of a good signal given high and low job-search effort, \( \theta_h \) and \( \theta_l \), respectively, the monitoring cost per unit of monitoring \( \kappa \), and the curvature of the monitoring cost \( \alpha \).

The calibration of a good signal given high and low job-search effort levels \((\theta_h, \theta_l)\) of the current US system is quite challenging for two reasons. First, while in the model all workers have the incentives to search for a job with a high effort, it is unclear what fraction of workers indeed have those incentives in the US. I therefore assume that in the current system only a fraction \( \xi \) search for a job with a high effort.

Second, \( \theta_h \) and \( \theta_l \) stem from the imperfection of the system, which is unobservable to either the case worker or the economist. Fortunately, the US department of labor is engaged in systematic and detailed analysis of the adequacy of payments in the current UI system (Woodbury (2002), DOL (2006b) and Vroman and Woodbury (2001)). Specifically, these projects reveal the fraction of overpayment and underpayment (denial errors) paid to workers specifically for non-separation errors, thus excluding reasons such as ineligibility due to insufficient previous earnings and quits.

Additional useful piece of information is the fraction of monitored workers who were sanctioned, which is equal to the monthly probability of sanctions \( \kappa \), over the monthly monitoring frequency.

I proceed by writing down explicit equations that connect \( \theta_h \) and \( \theta_l \) to the observed data, taking into account that only a fraction \( \xi \) exerts the high effort level. First, the fraction of overpayment, denoted by \( z_1 \) is equal to those who did not exert the high effort and received payments relative to all those who received payments: \( \frac{(1-\xi)\theta_l}{(1-\xi)\theta_l + \xi \theta_h} \). Similarly,
the fraction of overpayment, denoted by $z_2$, is equal to: \( \frac{\xi(1-\theta_h)}{(1-\xi)(\theta_l+\xi\theta_h)} \). Finally, the fraction of monitored workers who were sanctioned, denoted by $z_3$ is equal to: $\xi \ast (1 - \theta_h) + (1 - \xi) \ast (1 - \theta_l)$.

Those three equations can be rewritten as an explicit unique solution of \( \{z_1, z_2, z_3\} \) as follows:

\[
\begin{align*}
\theta_h &= \frac{1 - z_1}{z_2 + 1 - z_1} \\
\theta_l &= \frac{z_1 \ast (1 - z_3)}{z_3 + (1 - z_3) \ast (z_1 - z_2)} \\
\varepsilon &= (1 - z_3) \ast [z_2 + 1 - z_1].
\end{align*}
\]

Based on the sources above the values for \( \{z_1, z_2, z_3\} \) are \{1.4\%, 1.9\%, 16\%\}, respectively\(^{15}\). The implied values for the monitoring technology are: $\theta_h = 0.98$, $\theta_l = 0.08$ and $\xi = 0.84^{16}$.

The calibration of the signal probabilities implies a rather precise monitoring technology. The high value of $\theta_h$ is influenced by actions taken after the sanctions such as appeals, and redetermination (see Table ES-1 in DOL (1999) and DOL (2006b)). The low level of $\theta_l$ on the other hand can be affected by measures taken by workers to manipulate the system. If this is the case then the value of $\theta_l$ above is a lower bound of the correct value. The sensitivity analysis below, however, shows that the results are robust even to large increases of that value.

Before moving to the calibration of the rest of the monitoring parameters it should be emphasized that the analysis in this paper is based on the actual monitoring technology in the US. Another direction for the analysis is the optimal monitoring precision given a

\(^{15}\)The basis for $z_1$ and $z_2$ is Table 1 in Woodbury (2002) that gives the percentage of overpayment of 7.2\% and underpayment of 3.4\% in the five states pilot. The fraction of overpayment due to non-separation errors is 19.8\% (DOL (2006b)) and for wrongful denials it is 57\% (Vroman and Woodbury (2001)). $z_3$ is equal to the monthly probability of sanctions (\( \phi \)) of 3.3\% (Grubb 2000), over the monthly monitoring frequency (\( \mu^{ACT} \)) of 0.20 (see Appendix B).

\(^{16}\)The high measure of workers who exert the high effort of 0.84 is consistent with the observation of Pavoni and Violante (2007) that the current system in the US the exceeds in providing incentives.
monitoring cost that increases with precision as in Boone, Fredriksson, Holmlund, and van Ours (2007). Such cost-effectiveness analysis may lead to using a lower precision-lower cost monitoring technology.

The monitoring cost $\kappa$ is based on data from The Minnesota Family Investment Program (2000), where each caseworker was responsible for 100 clients, and among other tasks, applied sanctions, assisted with housing, and documented client activities. Based on monthly gross earnings of $3,000 per caseworker and the caseload described above, the value of $\kappa$ is $30 per month per monitoring of an unemployed worker\textsuperscript{17}. This value is an approximation because on one hand the caseworkers were also engaged in activities other than monitoring, and on the other hand they may have not monitored every month. Interestingly, although Boone, Fredriksson, Holmlund, and van Ours (2007) use a completely different data sources, their equivalent value of $\kappa = $27 is surprisingly close to the calibration here.

The interpretation of the monitoring action as a verification of employment contacts is consistent with a linear monitoring cost, and therefore I assume that $\alpha = 1$.

Table 1 Approximately Here

5 Results

This section includes two parts. First, I discuss the characteristics of the optimal monitoring policy for the infinite horizon model. Second, I estimate the value of monitoring by comparing the current model to a model without monitoring technology. Appendix A describes the solution method.

\textsuperscript{17}The median of annual earnings for Community and Social Services Occupations in the US is $36,390 (Department of Labor, 2006).
5.1 Optimal monitoring policy

The optimal contract is described recursively by six functions of the state variable $U$. These are $\{c, U^e, U^g, U^b, U^n, \mu\}$. I start with the mapping of current promised utility to next period’s promised utility, conditional on outcomes. In the optimal contract, the four future values, corresponding to the four possible outcomes, endogenously create implicit rewards and sanctions.

Figure 2 shows the mapping of promised utility across periods in utility units by outcome. The horizontal axis is the promised utility at the beginning of the period and the vertical axis is the next period’s promised utility by outcome. As in the two period model, the four future promised utilities are ordered by the likelihood ratios: $U^e, U^g, U^n, U^b$. Upon employment, an outcome that can only happen in the model if the worker exerts a high job-search effort, promised utility increases; upon monitoring with a good signal, the worker receives a reward that is only slightly lower than that of employment; upon unmonitored unemployment, the promised utility changes only slightly; finally, upon monitoring with a bad signal the worker experiences a severe decrease in promised utility, implying that the planner finds the bad signal informative and helpful in creating the necessary incentives\(^{18}\).

The values of $U^n, U^g, and U^b$, are determined jointly by the following condition, based on the three first order conditions: $V'(U^n) = \theta_h V'(U^g) + (1 - \theta_h) V'(U^b)$. The calibration of $\theta_h$ implies that the sanction level is significantly higher than the reward\(^{19}\). This result is consistent with the absence of prizes in the actual monitoring scheme, as they are relatively small.

I now move to discuss the monitoring frequency and the utility spreads decisions.

\(^{18}\)By the likelihood ratios the good signal state can be more or less informative than the no monitoring state depending on the parameters. In contrast, the bad signal state is always more informative than the no monitoring state.

\(^{19}\)In the two period model the ratio $\frac{\theta_h}{1 - \theta_h}$ is equal to the ratio of the sanction over prize. This ratio, whose value in the calibration is 49, plays a similar role here.
According to the theoretical analysis in section 3 we expect that as promised utility increases, the monitoring frequency would increase and the spreads would decrease.

Figure 3 shows the monitoring frequency by promised utility\textsuperscript{20}. The monitoring frequency varies across its complete support: for low enough values, the cost of spreading out future utilities is lower than the cost of monitoring and no monitoring takes place; for high enough values of promised utility the opposite is the case and the monitoring frequency is at its maximum value.

As for the spreads, observe that given the calibration of $\theta_h$, $U^b$ is by far lower than the other three future utilities. To demonstrate the dynamics of the spreads, I therefore concentrate on the level of $U^b$ relative to $U^n$. The dynamics of the rest of the spreads are identical.

Define the relative consumption sanction as the fraction by which the next period’s current consumption decreases upon a bad signal relative to that of unmonitored unemployment\textsuperscript{21}. Figure 4 shows the relative consumption sanction by promised utility. Note that the sanction is plotted only for levels of promised utility for which $\mu > 0$. Taken together, figures 3 and 4 demonstrate additional features of the optimal contract that were proved in section 3. First, the sanction and the spread complement each other and as one increases the other decreases. Second, when the monitoring frequency is constant, the sanction remains constant as well. This can be seen for when $\mu = 1$, but also for low levels of promised utility when the resolution of $\mu$, creates steps in the monitoring frequency.

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\textsuperscript{20}The high span of promised utility in this figure, ranging from equivalent consumption levels of $\$20$ to $\$200,000$ a month, is used to demonstrate the qualitative characteristics of the contract. As I discuss below the span of promised utility that includes the complete support of $\mu$ can be much smaller (Figure 7). The subsequent quantitative results are based on a fine grid over a much smaller span.

\textsuperscript{21}The relative consumption sanction is different from the spread defined earlier, which refers to the absolute decrease in the lifetime expected discounted utility.
The result that as the generosity of the welfare system increases, the planner monitors the unemployed more frequently but imposes more moderate sanctions is supported by some empirical evidence. Boone, Fredriksson, Holmlund, and van Ours (2007) find that when comparing the monitoring policies of the US and Sweden, the number of sanctions is inversely related to the severity of the penalty.

Running the model with different levels of risk aversion in the CRRA class reveals an interesting pattern: the dynamics of the monitoring probability and the spreads flip exactly when the coefficient of relative risk aversion ($\sigma$) falls below $\frac{1}{2}$. In other words $\mu$ increases in $U$ (and the spreads decrease) for any $\sigma > \frac{1}{2}$, $\mu$ and the spreads are invariant in $U$ if $\sigma = \frac{1}{2}$, and $\mu$ decreases in $U$ (and the spreads decrease) for $\sigma < \frac{1}{2}$.

I demonstrate the importance of $\sigma = \frac{1}{2}$ in a simple two period model with no monitoring and no current compensation. The problem becomes:

$$ V(U) = \max_{c^e,c^n} \{ w - \pi c^e - (1-\pi)c^n \} $$

s.t.:

$$ U = -e + \pi u (c^e) + (1 - \pi) u (c^n) $$

$$ U \geq u (c^n). $$

The solution to this problem is $c^n = u^{-1}(U) \, , \, c^e = u^{-1} \left( U + \frac{e}{\pi} \right)$. The difference between the first best (constant consumption across both future states) and the second best is then equal to the cost of spreading the future values:

$$ \pi u^{-1} \left( U + \frac{e}{\pi} \right) + (1-\pi)u^{-1}(U) - u^{-1}(U + e). \tag{8} $$

Using CRRA utility and differentiating this cost with respect to promised utility gives:

\[^{22}\text{Newman (2007) finds that } \sigma = 0.5 \text{ is the critical value for the distribution of wealth across entrepreneurs and workers.}\]
Note that the first two terms compose a lottery, whose expected prize value is \( U \). The value of the derivative is positive if and only if \( f(x) = x^{\frac{\sigma}{1-\sigma}} \) is a strictly convex function.

The second derivative of (9) is:

\[
\sigma (2\sigma - 1) (1 - \sigma)^{\frac{\sigma}{1-\sigma}} \left\{ \frac{\pi}{\pi} \left[ \left( U + e \right)^{\frac{\sigma}{1-\sigma}} + (1-\pi)U^{\frac{\sigma}{1-\sigma}} - (U + e)^{\frac{\sigma}{1-\sigma}} \right] \right\},
\]

which is positive if and only if \( \sigma > \frac{1}{2} \). Since the cost of spreading out utilities is increasing in promised utility if and only if the derivative is positive, it follows that the cost is increasing if and only if \( \sigma > \frac{1}{2} \).

An interesting implication of this result is that as the level of risk aversion increases, the change in the monitoring probability will be faster, i.e. the sensitivity of \( \mu \) to promised utility will be higher because the increase in the cost of spreading consumption increases as well.

### 5.2 The value of monitoring

The planner’s value from the optimal monitoring policy lies between the value of optimal unemployment insurance, which is a special case of monitoring with \( \mu = 0 \), and the value of the first best. Thus, to study the effectiveness of monitoring relative to unemployment insurance, I define the following metric: \( \frac{V_{MON} - V_{OUI}}{V_{FB} - V_{OUI}} \), where \( V_{MON} \), \( V_{OUI} \) and \( V_{FB} \) are the planner values for optimal unemployment insurance with monitoring, optimal unemployment insurance, and the first best, respectively\(^{23} \). The difference \( V_{FB} - V_{OUI} \) can be considered as the moral hazard cost if no monitoring was available. Therefore, the metric is the percentage of the moral hazard cost saved by including monitoring.

\(^{23}\) The model with no monitoring is closely related to the model used in Hopenhayn and Nicolini (1997). The main difference is that in Hopenhayn and Nicolini the job-search effort level is continuous and not discrete. I use a discrete level of effort in both models for consistency.
Figure 5 shows the value of monitoring over the support of promised utility. Monitoring is relatively more effective at high levels of promised utility because for log utility the cost of spreads increases with promised utility. At low enough levels of promised utility, the optimal monitoring policy coincides with the optimal unemployment insurance policy. At the other extreme of promised utility, savings strictly increase even though \( \mu \) is constant. This happens because the cost of spreads continue to increase.

Since the effectiveness of monitoring varies significantly across the state, it is of interest to measure the savings at the level of promised utility that balances the government’s budget. The balanced budget point is \( U_0^* \) such that \( V(U_0^*) = 0 \). This is the level of promised utility for which the costs of benefits, wage subsidies and monitoring are exactly covered by the tax revenues\(^2\). At \( U_0^* \) for the model with no monitoring the addition of monitoring saves 82% of the moral hazard cost.

At \( U_0^* \) the monitoring frequency is 6% and the relative consumption sanction, which is approximately a permanent decrease in consumption, is 4.8%. The other three states lead to deviations on a scale much smaller than that of the relative consumption sanction.

In absolute value the savings at the balanced budget point amounts to $87, out of $106 possible. The potential savings of $106 is limited since the optimal unemployment insurance contract a la Hopenhayn and Nicolini (1997) already gets quite close to the first best by conditioning on the complete history of the worker and allowing the tax to depend on the history as well.

5.2.1 Who should be monitored?

As noted above, monitoring is especially effective for workers for whom the cost of spreading utilities is high. So far I discussed the combined effect of promised utility and risk

\(^2\)This point is unique because \( V(U) \) is strictly monotone in \( U \).
aversion on this cost. There are, however, other parameters that affect the cost of spreading utilities.

As can be seen in (8), the cost of spreading utilities increases with \( \xi \). Both an increase in disutility and a decrease in the job-finding probability (given high effort) decrease the left hand side of the incentive compatibility constraint without changing the right hand side, making it harder to satisfy this constraint.

Thus, monitoring will be relatively more effective, not only for more generous welfare systems, but also for two types of workers. These are discouraged unemployed workers who have a high disutility from work (or a high utility from leisure) and workers with a low job-finding probability. This finding fits well the application of monitoring to those types of workers.

Quantitatively, changes in disutility from work or in the job-finding probability can have a substantial effect on the contract. In the case of log utility, for example, changes in \( e \) or in \( \pi \) affect the cost of spreads exponentially. Note that unlike the effect of promised utility on the spreading costs that is sensitive to risk aversion, the effects of \( e \) and \( \pi \) on the cost of spreads are more general as they are independent of risk aversion.

What about the dynamics of monitoring over the unemployment spell? Conditional on unemployment, promised utility decreases along the unemployment spell and therefore qualitatively monitoring should decrease and sanctions should increase (for \( \sigma > \frac{1}{2} \)). Quantitatively, however, for log utility the optimal monitoring scheme is fairly insensitive to such changes in promised utility. The planner can therefore use a fixed monitoring frequency together with a fixed sanction. This relatively simple monitoring scheme would deliver almost the same gains as the optimal one.
5.2.2 What makes monitoring an effective policy?

The reduction in the planner's cost due to monitoring is achieved by consumption smoothing. To demonstrate this and to assess the smoothing intensity, I simulate the consumption paths for the optimal unemployment insurance model and for the monitoring model. Figure 6 shows three examples of consumption paths according to the two policies. In each example, the worker starts off as unemployed with a promised utility level of $U_0^*$, stays unemployed for 4 periods and then finds a job. In the top panel, there is no monitoring. In the middle panel, monitoring is applied in periods 1, 2 and 3 and in all three cases the signal is good. In the bottom panel, monitoring is applied once in period 1, and results with a bad signal.

Figure 6 Approximately Here

Consumption in the unemployment insurance model, where the planner has only two outcomes to condition on, decreases monotonically, and then increases significantly when the worker finds a job. These shifts in consumption are required for creating the necessary incentives for the unemployed worker to look for a job with a high effort.

In contrast, consumption in the monitoring model varies very little, except for the third panel where the worker was sanctioned. Note, however, that sanctions are a rare event as they only happen when both monitoring and a bad signal happen. At the balanced budget point the unconditional probability of a sanction, $\mu \ast (1 - \theta_h)$, is around 0.12%. This is equivalent, on average, to sanctioning one of about 800 unemployed workers or sanctioning an unemployed worker once every 70 years!

The simulation shows that due to the additional information regarding the job-search effort, monitoring allows the planner to smooth the unemployed worker's consumption. Simulating the model over 60 periods and 5,000 workers shows that the standard deviation of consumption in the monitoring model is less than half the standard deviation of consumption in the model without monitoring.
5.2.3 Sensitivity Analysis

The comparison between the models with and without monitoring relies on the effectiveness of monitoring, which in turn relies on the four parameters of the monitoring technology: the probabilities of a good signal given high and low job-search effort $\theta_h, \theta_l$, respectively, the cost per unit of monitoring $\kappa$, and the concavity of the monitoring cost $\alpha$. In order to examine the robustness of the savings to these parameters I analyze the response of savings at the balanced budget point to various values of these parameters.

The probability of a good signal given the high job-search effort $\theta_h$ determines the precision of the information extracted by applying monitoring. As $\theta_h$ increases, the planner receives more accurate information on the worker’s job-search effort level and is therefore encouraged to monitor more frequently. Furthermore, as $\theta_h$ increases, the probability of a sanction decreases and the planner can use more severe sanctions. In the extreme case when $\theta_h = 1$ it is possible to get arbitrarily close to the first best allocation by using a combination of a very low monitoring frequency (that costs very little) with an extremely severe punishment that will never be applied.

Table 2 presents the savings at the balanced budget point for various levels of $\theta_h$. Holding $\theta_l$ and $\kappa$ fixed, as $\theta_h$ increases beyond the benchmark value, the efficiency of monitoring increases as expected. As $\theta_h$ decreases, the savings level decreases sharply and at a value of $\theta_h = 0.90$ (close to the unlikely lower bound) the savings is $61\%$.\footnote{Note that according to the monitoring frequency and the proportion of sanctions in the US the lower bound for $\theta_h$ is 0.84. For $\theta_h = 0.84$, the monitoring technology is useless and the savings would be 0, as $\theta_l = 0.84$ as well. This description of the monitoring technology is inconsistent with the calibration equations shown above.}

| Table 2 Approximately Here |
| Table 3 Approximately Here |

The sensitivity analysis of $\theta_l$ in Table 3 shows that monitoring’s efficiency depends on the difference between the precision of the two signals $(\theta_h, \theta_l)$. As $\theta_l$ gets closer to $\theta_h$ the
savings decrease significantly.

Table 4 shows the savings for various values of the monitoring cost \( \kappa \). First, note that when \( \kappa = 0 \), the first best is not achieved because the free monitoring provides imperfect information\(^{26}\). Second, as the cost of monitoring increases the planner uses monitoring less frequently and the level of savings decreases. Nevertheless, even when \( \kappa = 100 \), a monitoring cost that is higher by more than three times than the benchmark calibration, monitoring savings stands at about 70%.

Table 4 Approximately Here

Table 5 shows the savings for various values of the cost curvature \( \alpha \). Since \( \kappa \) is kept fixed, increasing the curvature parameter \( \alpha \) implies lower costs and increasing savings. More interesting is the dynamics of the monitoring frequency and the spreads over \( U \) given convex costs. While \( V \) is concave in \( \mu \) for any quasi-concave cost, it is also concave for some convex costs (see proof of Claim 1 in Appendix C). Since supermodularity in proposition 1 holds for any cost curvature, the dynamics of the monitoring probability and the spreads are expected to hold for convex costs as well.

With the caveat that \( V \) may not be concave in \( \mu \), figure 7 shows the monitoring frequency for \( \alpha \in \{0.5, 1, 5\} \). Figure 8 complements figure 7 by showing the relative consumption sanction for the same three cases. The two extreme cases show that while qualitatively the characteristics of the contract are the same, the sensitivity to changes in promised utility depends on the cost curvature. This demonstrates the wide spectrum of results that the model can generate. Whereas for some parameters choosing \( \mu \{0, 1\} \) may be a decent rule of thumb, for other parameters a fixed monitoring frequency and a fixed sanction may be close to optimal.

\(^{26}\)When \( \kappa = 0 \), the planner monitors with probability 1.0 but since the signal is imperfect, the planner cannot know for sure what the job-search effort level was. Therefore, the planner still needs to condition the promised utility on outcomes that will happen in equilibrium, which is costly.
Note that the monitoring frequency for any two levels of curvature \( \alpha_1, \alpha_2 \) cross when the marginal cost of monitoring is equalized at \( \mu = \exp \left( \frac{\log(\alpha_1/\alpha_2)}{\alpha_2 - \alpha_1} \right) \).

Table 5 Approximately Here

6 Concluding remarks

Governments monitor the job-search activities of unemployed workers in order to increase the effectiveness of unemployment insurance. They randomly monitor job-search effort and receive, at a cost, a signal that is related to the effort level. This additional information plays an important role in the design of unemployment insurance schemes.

This paper uses the recursive contracts framework to model monitoring that may result in an imperfect signal. This framework allows characterizing the optimal contract given this realistic technology, and evaluating the gain from using the monitoring technology.

I show how the two components of monitoring, the monitoring frequency and the spread of continuation values complement each other depending on the state of the worker. For CRRA utility, the monitoring frequency increases and the spreads decrease with promised utility if and only if \( \sigma > \frac{1}{2} \). The driving force behind the result is that the cost of spreading out future utilities increases with the continuation value if and only if \( \sigma > \frac{1}{2} \), while the cost of monitoring is invariant to the continuation value.

In the quantitative analysis at the balanced budget point I show that compared to optimal unemployment insurance, monitoring saves about 80% of the cost associated with moral hazard. The gain is achieved by a decrease of more than half in the standard deviation of consumption.

One limitation of the framework used in this paper is that the tractability of the paper depends on the assumption that the planner controls the consumption of the worker, i.e. no savings on the worker’s side allowed. As pointed out by Abdulkadiroglu, Kuruscu, and
Sahin (2002) and Shimer and Werning (2008), allowing the workers to hold unobservable savings may significantly affect the results. Nevertheless, the recursive contract framework demonstrates the main trade-offs when a costly imperfect signal is available. It seems that as long as differentiating future levels of payments is necessary not only that monitoring can be effective, but also the trade-offs presented in this framework should hold. Also note, that as the environment presented here can be applied to other contexts such as crime and punishment, the no-savings assumption might be perfectly reasonable.

Another limitation of that framework is that all sanctions in equilibrium are unjustified. This happens because the incentive compatibility constraint holds. These sanctions are necessary in the contract to keep the worker’s incentives in place. The same concept of unjustified punishments holds in optimal unemployment insurance as well. There, conditional on unemployment, the worker experiences benefit cuts even though the planner is aware that the effort recommendation is followed. A more realistic model would include unobserved heterogeneity in disutility from job-search and from work. Then, the sanctions in equilibrium would be partially justified.

Alternatively, heterogeneity can be introduced through wages. This would allow conditioning the initial level of promised utility on the wage of each type of worker as applied in most OECD countries\textsuperscript{27}.

According to Grubb (2000), there are significant differences across countries in all the main characteristics of the policy. As an example, consider Australia and the US. In Australia a moderate sanction of 18\% of the benefits level is applied for a duration of 26 weeks, equivalent to roughly 4.5 weeks of benefits. This is considerably higher than the one week denial of benefits in the US. At the same time the annual sanction rate in Australia is relatively low, standing at 1.2\%, compared with 33\% in the US. An extended

\textsuperscript{27}This heterogeneity in wages would reduce and possibly eliminate the result that for high levels of promised utilities the planner recommends a low job-search effort: once high levels of promised utilities will be associated with high wages, the gain from employment to the planner will dominate the cost of setting the incentives for the high job-search effort.
model could reveal whether the variation in policies follows labor market characteristics or some inefficiencies.

Although the focus of the paper is monitoring of unemployed workers, the modeling environment presented in this paper is rather general. The model can be used for a wide array of problems, where a planner uses a costly imperfect signal to learn about the agent’s hidden information or action.

The analysis of monitoring can also be applied to systems in which the probability of a good signal given the high effort is very low. In this case the prize relative to the sanction will be very high and a reasonable implementation of the contract could include a prize only. This characterization is consistent with competitions.
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*Unemployment Fraud and Abuse*, Hearing Before the Subcommittee on Human Resources of the Committee on Ways and Means, House of Representatives. Available at: http://research.upjohn.org/testimonies/10.
APPENDIX A: COMPUTATIONAL METHOD

This appendix describes the solution method for the problem of a planner who recommends the high job-search effort during unemployment\textsuperscript{28}.

Transform the maximization problem with six decision variables and two constraints into a maximization problem with four decision variables and no constraints.

Write the incentive compatibility constraint as follows\textsuperscript{29}:

\[
\begin{align*}
    u(c) - e + \beta \pi U^e + \beta (1 - \pi) \left[ (1 - \mu) U^n + \mu (\theta_h U^g + (1 - \theta_h) U^b) \right] \\
    = u(c) + \beta \left[ (1 - \mu) U^n + \mu (\theta_l U^g + (1 - \theta_l) U^b) \right]
\end{align*}
\]

and express \( U^e \) in terms of \( U^n, U^g, U^b \) and \( \mu \):

\[
U^e = \frac{e}{\beta \pi} + (1 - \mu) U^n - \frac{\mu}{\pi} \left\{ [ (1 - \pi) \theta_h - \theta_l ] U^g + [ (1 - \pi) (1 - \theta_h) - (1 - \theta_l) ] U^b \right\}
\]

(10)

Use the promise keeping constraint to express \( c \) in terms of \( U^e, U^n, U^g, U^b \) and \( \mu \):

\[
c = u^{-1} \left\{ U + e - \beta \left[ \pi U^e + (1 - \pi) \left( (1 - \mu) U^n + \mu (\theta_h U^g + (1 - \theta_h) U^b) \right) \right] \right\}
\]

(11)

Use (10) in the right hand side of (11) to express the consumption level \( (c) \) in terms of \( U^n, U^g, U^b \) and \( \mu \). Substitute this value of \( c \) and the value for \( U^e \) from (10) into (2) to receive the maximization problem with four decision variables: \( U^n, U^g, U^b \) and \( \mu \), with no constraints.

Those four remaining decision variables consist of three continuation values \( (U^n, U^g, U^b) \) and the monitoring frequency \( \mu \). While the support for the continuation values is, in general, the real line, the support for the monitoring frequency is \([0, 1]\). This closed support presents a computational challenge, which I overcome by discretizing the support of the

\textsuperscript{28} In absence of asymmetric information, the solution to the employment problem consists of constant benefits for the complete duration of employment.

\textsuperscript{29} In the optimal solution, the incentive compatibility constraint always holds with equality. This is the case simply because delivering an expected discounted utility that is higher than the required one, costs more.
monitoring frequency into 151 values and then solve the maximization problem for each of those 151 values\(^\text{30}\).

Thus, the maximization problem is reduced to three continuous variables: \(U^n, U^g, U^b\). The solution to this problem is based on the three first order conditions with respect to \(U^n, U^g, \) and \(U^b\) respectively:

\[
(u^{-1})' (c_{\text{arg}}) + \pi W' (U^e) + (1 - \pi)V'(U^n) = 0
\]

\[
(u^{-1})' (c_{\text{arg}}) (1 - \theta) - W' (U^e) ((1 - \pi) \theta_h - \theta_l) + (1 - \pi)\theta_h V'(U^g) = 0
\]

\[
(u^{-1})' (c_{\text{arg}}) \theta_l - W' (U^e) ((1 - \pi) (1 - \theta_h) - (1 - \theta_l)) + (1 - \pi) (1 - \theta_h) V'(U^b) = 0
\]

where I have defined for brevity of notation:

\[
c_{\text{arg}} = U + e - \beta \left[ \pi U^e + (1 - \pi) ((1 - \mu) U^n + \mu (\theta_h U^g + (1 - \theta_h) U^b)) \right]
\]

\(^\text{30}\)The sensitivity of the solution is, therefore, 0.0033 of monitoring frequency.
The calibration of the actual monthly monitoring probability in the US, $\mu^{ACT}$, is based on the frequency of required reports of employment contacts that the unemployed workers fill in and on the probability that these contacts are verified. While the weekly frequency of reports is fairly consistent across states (O’Leary 2004), the probability or verifying these contacts varies vastly across states: some states (e.g. Pennsylvania) do not monitor at all; some states (e.g. Washington) have a target monitoring frequency of 10%; and some states (e.g. South Dakota) consistently review contacts every 4-6 weeks (DOL 2003).

In addition to the vast variance in the probability of verifying contacts across states, the information is also usually vague, possibly because it is of the interest of states to conceal the actual probability of verifying contacts. As a benchmark for the probability of verifying employment contacts in the US I use a conservative value of 5% (the lower this probability the lower is $\theta_h$), which determines, together with a weekly frequency of reports, a monthly monitoring probability ($\mu^{ACT}$) of 20%\textsuperscript{31}.

\textsuperscript{31}The unemployed worker submits $\frac{52}{12} = 4\frac{1}{3}$ reports a year. The probability of being monitored at least once in a month is: $1 - 0.95^{4.33} = 0.20$, where 0.95 is the probability of not being monitored in a given week.
APPENDIX C: PROOFS

Lemma 3 When either \( \mu \) or \( U \), or both change, all the spreads move in the same direction.

Proof. Recall that the spread between any two future utilities is defined as \( u(c^i) - u(c^j) \) \( i \in \{e, g, n, b\} \), such that \( c^i \geq c^j \). The ordering of consumption levels is used only for convenience, as it guarantees that the spreads are non-negative.

Let \( \{c^1_1, c^1_2, c^2_1, c^2_2\}, \{c^3_1, c^3_2, c^4_1, c^4_2\} \) be the optimal consumption levels for \( \{U_1, \mu_1\}, \{U_2, \mu_2\} \), respectively, and assume without loss of generality that \( \frac{c^1_1}{c^1_2} \geq \frac{c^3_1}{c^3_2} \), i.e., that the spread is larger when \( \{U, \mu\} = \{U_1, \mu_1\} \). Using the first order conditions \( \frac{c^i}{c^j} = \frac{\lambda^i+\lambda^j\mu}{\lambda^i+\lambda^j\theta} \). Therefore:

\[
\frac{\lambda^1+\lambda^2\mu}{\lambda^1+\lambda^2\theta} \geq \frac{\lambda^3+\lambda^4\mu}{\lambda^3+\lambda^4\theta}.
\]

After multiplying by both denominators (note that \( \lambda^i+\lambda^j\mu \geq \lambda^3+\lambda^4\mu \)), we get that: \( \lambda^1+\lambda^2\theta \geq \lambda^3+\lambda^4\theta \). Therefore:

\[
\frac{\lambda^1}{\lambda^3} \geq \frac{\lambda^2}{\lambda^4}.
\]

Using the first order conditions:

\[
\lambda^1 \geq \lambda^3,
\]

and rearranging we get that: \( \lambda^1 \leq \lambda^3 \). By repeating the same steps backwards for any \( j, k \), we get that: \( \frac{c^k}{c^j} \geq \frac{c^i}{c^j} \), with strict inequality if \( \frac{c^k}{c^j} > \frac{c^i}{c^j} \).

Since \( u(c^i) - u(c^j) = \log \left( \frac{c^i}{c^j} \right) \), if one spread decreases, then the rest of the spreads must decrease as well.■

Claim 1 \( \frac{\partial^2 V(U)}{\partial \mu^2} < 0 \)

Proof. Use the promise keeping and incentive compatibility constraints to derive \( c \), and \( U^e \) as a function of \( U^g, U^n, U^b \) and \( \mu \).

\[
U^e = \frac{c}{\pi^2} + (1 - \mu) U^n + \frac{\mu}{\pi} \left\{ \left[ (\theta_t U^g + (1 - \theta_t) U^b) \right] - (1 - \pi) \left[ (\theta_h U^g + (1 - \theta_h) U^b) \right] \right\}
\]

\[
c = u^{-1} \left[ U - \beta \left[ (1 - \mu) U^n + \mu \left( \theta_t U^g + (1 - \theta_t) U^b \right) \right] \right]
\]

Substitute the constraints into the maximization problem:

\[
V(U) = \max_{\mu, U^n, U^b} -\kappa U^n - u^{-1} \left[ U - \beta \left[ (1 - \mu) U^n + \mu \theta_t U^g + (1 - \theta_t) U^b \right] \right] + \beta \left\{ \pi W \left( \frac{c}{\pi^2} + (1 - \mu) U^n + \frac{\mu}{\pi} \left[ (\theta_t U^g + (1 - \theta_t) U^b) \right] - (1 - \pi) \left[ (\theta_h U^g + (1 - \theta_h) U^b) \right] \right) \right\} (1 - \pi) \left( \left( 1 - \mu \right) V(U^n) + \mu \theta_t V(U^g) + (1 - \theta_t) V(U^b) \right)
\]

41
Differentiate twice with respect to $\mu$:

\[
\frac{\partial^2 V(U)}{\partial \mu^2} = -c \left[ -\beta \left( -U^n + (\theta_i U^g + (1 - \theta_i) U^b) \right) \right]^2 + \\
\beta \pi W^m \{ U^e \} \left[ -U^n + \frac{1}{\pi} \left\{ \left( \theta_i U^g + (1 - \theta_i) U^b \right) - (1 - \pi) \left( \theta_h U^g + (1 - \theta_h) U^b \right) \right\} \right]^2 - \\
\alpha (a - 1) \kappa \mu^{a - 2},
\]

which is strictly negative for every $\alpha \leq 1$ since $u^{-1}$ is convex and $W$ is concave. Note that since the first two terms are strictly negative for any $\alpha$, the claim also holds for some convex cost of monitoring. \[\blacksquare\]
### TABLE 1

*Calibration parameters*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good signal probability given high effort</td>
<td>$\theta_h$</td>
<td>0.98</td>
<td>See text</td>
</tr>
<tr>
<td>Good signal probability given bad effort</td>
<td>$\theta_l$</td>
<td>0.08</td>
<td>See text</td>
</tr>
<tr>
<td>Monitoring cost</td>
<td>$\kappa$</td>
<td>$30$</td>
<td>See text</td>
</tr>
<tr>
<td>Monitoring cost curvature</td>
<td>$\alpha$</td>
<td>1.0</td>
<td>See text</td>
</tr>
<tr>
<td>The actual monitoring frequency in the US</td>
<td>$\mu^{ACT}$</td>
<td>0.20</td>
<td>See Appendix B</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.9959</td>
<td>Cooley (1995)</td>
</tr>
<tr>
<td>Wage</td>
<td>$w$</td>
<td>$2,800$</td>
<td>National compensation survey (2006)</td>
</tr>
<tr>
<td>Unemployment exit rate</td>
<td>$\pi$</td>
<td>0.17</td>
<td>Shimer (2007)</td>
</tr>
<tr>
<td>Disutility from effort</td>
<td>$e$</td>
<td>0.67</td>
<td>Pavoni and Violante (2007)</td>
</tr>
</tbody>
</table>

### TABLE 2

*Sensitivity analysis for the value of $\theta_h$*

<table>
<thead>
<tr>
<th>$\theta_h$</th>
<th>0.90</th>
<th>0.95</th>
<th>0.98</th>
<th>0.99</th>
<th>1.00</th>
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<tbody>
<tr>
<td>$\nu$</td>
<td>0.61</td>
<td>0.72</td>
<td>0.82</td>
<td>0.87</td>
<td>1.00</td>
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</table>

### TABLE 3

*Sensitivity analysis for the value of $\theta_l$*

<table>
<thead>
<tr>
<th>$\theta_l$</th>
<th>0.0</th>
<th>0.08</th>
<th>0.30</th>
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<th>0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings*</td>
<td>0.84</td>
<td>0.82</td>
<td>0.77</td>
<td>0.70</td>
<td>0.35</td>
</tr>
</tbody>
</table>

### TABLE 4

*Sensitivity analysis for the value of $\kappa$* ($\)$

<table>
<thead>
<tr>
<th>$\kappa$ ($)</th>
<th>0</th>
<th>10</th>
<th>30</th>
<th>50</th>
<th>100</th>
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</thead>
<tbody>
<tr>
<td>Savings*</td>
<td>0.99</td>
<td>0.89</td>
<td>0.82</td>
<td>0.77</td>
<td>0.70</td>
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</tbody>
</table>

### TABLE 5

*Sensitivity analysis for the value of $\alpha$*

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings*</td>
<td>0.37</td>
<td>0.6</td>
<td>0.82</td>
<td>0.93</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Fig. 1. The timing of the model and the four possible end-of-period outcomes: employment, unmonitored unemployment, monitored unemployment with a good signal, and monitored unemployment with a bad signal.

Fig. 2. The mapping of promised utility from the current period to the next period, conditioned on the four possible outcomes: employment, unmonitored unemployment, monitored unemployment with a good signal, and monitored unemployment with a bad signal. The values for employment and monitoring with a good signal are above the diagonal (the diagonal itself is not illustrated) and include a reward. The value for unmonitored unemployment is only slightly below the diagonal. Finally, the value for monitored unemployment with a bad signal is low due to the sanction.
Fig. 3. The monitoring frequency by promised utility. As the generosity of the welfare system increases, the monitoring frequency increases and the relative consumption sanction (Fig. 3) decreases.

Fig. 4. The relative consumption sanction by promised utility. The sanction responds to changes in the monitoring frequency (Figure 3).
Fig. 5. The value of monitoring as the fraction of the moral hazard cost of optimal unemployment insurance that is saved when the monitoring technology is available.

Fig. 6. Simulated consumption paths according to optimal monitoring and optimal unemployment insurance policies. The consumption paths for the unemployment insurance policy are identical. The consumption paths for the monitoring policy depends on whether monitoring was applied and the signal’s result.
Fig. 7. Monitoring frequency for various monitoring cost types. As the cost becomes more convex (alpha increases) the marginal cost of monitoring increases and the increase in monitoring becomes more moderate.

Fig. 8. The relative consumption sanction for various monitoring cost types. The spreads change according to the monitoring frequency (Figure 7).