Trade and Labor Market Outcomes*

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Abstract

This paper reviews a new framework for analyzing the interrelationship between inequality, unemployment, labor market frictions, and foreign trade. This framework emphasizes firm heterogeneity and search and matching frictions in labor markets. It implies that the opening of trade may raise inequality and unemployment, but always raises welfare. Unilateral reductions in labor market frictions increase a country’s welfare, can raise or reduce its unemployment rate, yet always hurt the country’s trade partner. Unemployment benefits can alleviate the distortions in a country’s labor market in some cases but not in others, but they can never implement the constrained Pareto optimal allocation. We characterize the set of optimal policies, which require interventions in product and labor markets.

Keywords: inequality, unemployment, trade, labor market policy

JEL Classification: F12, F16, J64

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1 Introduction

For understanding the causes and consequences of international trade, recent research has increasingly focused on individual firms. While this research emphasizes reallocations of resources across heterogeneous firms, it typically assumes frictionless labor markets in which all workers are fully employed for a common wage. In reality, labor markets feature both unemployment and wage inequality, and labor market institutions are thought to play a prominent role in propagating the impact of external shocks. In this paper, we draw on recent research in Helpman and Itskhoki (2010) and Helpman, Itskhoki and Redding (2010), to discuss interdependence across countries.

This framework incorporates a number of features of product and labor markets. Firms are heterogeneous in productivity, which generates differences in revenue across firms. There are search and matching frictions in the labor market, which generate equilibrium unemployment, and give rise to multilateral bargaining between the firms and their workers. While workers are \textit{ex ante} homogeneous, they draw a match-specific ability when matched with a firm, which is not directly observed by either the firm or the worker. Firms, however, can invest resources in screening their workers to obtain information about ability. Larger, more-productive firms, screen workers more intensively to exclude those with low-ability. As a result, they have workforces of higher average ability and they pay higher wages. These differences in firm characteristics are systematically related to export participation. Exporters are larger and more productive than nonexporters; they screen workers more intensively; and they pay higher wages in comparison to firms with similar productivity that do not export. The resulting framework highlights a new mechanism through which trade affects inequality, based on variation in wages across firms and the participation of only the most-productive firms in exporting.

We use a simplified version of this framework to examine interdependence across countries through labor market frictions. Cross-country differences in labor market characteristics shape patterns of comparative advantage. A reduction in a country’s labor market frictions in the differentiated sector reduces unemployment within that sector and expands the share of workers searching for employment there, which affects aggregate unemployment through a change in sectoral composition. Depending on the relative values of unemployment rates across sectors, aggregate unemployment may rise or decline. The expansion in a home country’s differentiated sector increases
its welfare, but enhances the degree of product market competition faced by foreign firms, which leads to a contraction in the foreign country’s differentiated sector and a reduction in its welfare. Unilateral labor market reforms, therefore, can have negative externalities across countries, whereas coordinated reductions in labor market frictions raise welfare in every country.

As well as providing a platform for analyzing the positive economic effects of trade and labor market characteristics, our framework can be used to address normative issues. We first examine the impact of unemployment benefits on resource allocation and welfare, and show that they raise welfare in some circumstances and reduce welfare in other. We also present new results on policies that implement a constrained Pareto optimum. When the Hosios (1990) condition is satisfied, these policies do not require intervention in the labor market. Otherwise, a combination of subsidies to the cost of posting vacancies/hiring, subsidies to output/employment, and a common subsidy to all fixed costs (entry, production and exporting) implement the constrained Pareto optimal allocation. These product market policies apply equally to exporting and nonexporting firms. Unemployment benefits can be part of the optimal policy package under some circumstances, but even then more direct interventions in the labor market are preferable on informational grounds.

The remainder of the paper is structured as follows. In Section 2 we discuss the motivation for our approach and some of the related literature. In Section 3 we introduce our framework and examine the relationship between inequality, unemployment and trade. In Section 4 we use a simplified version of the model to explore how changes in labor market frictions in one country affect its trade partners and how the removal of trade impediments affects countries with different labor market frictions. Section 5 examines unemployment benefits and optimal policies. Section 6 concludes.

2 Background and Motivation

Traditional explanations of international trade have emphasized comparative advantage based on variation in technology across countries and industries (Ricardo 1817) or the interaction between cross-country differences in factor abundance and cross-industry differences in factor intensity (Heckscher 1919, Ohlin 1924, Jones 1965 and Samuelson 1948). In the 1980s, economies of scale and monopolistic competition were merged with factor proportions–based explanations for trade in Dixit
and Norman (1980), Helpman (1981), Krugman (1981) and Lancaster (1980). While economies of scale and love of variety preferences together generated two-way trade within industries, as observed empirically, the assumption of a representative firm implied that all firms exported.

More recently, firm heterogeneity has been introduced into general equilibrium trade theory following Melitz (2003) and Bernard, Eaton, Jensen and Kortum (2003). The resulting models of firm heterogeneity and trade provide a natural explanation for empirical findings from micro data that only some firms within industries export and these exporters are larger and more productive than nonexporting firms. Table 1 reports some representative evidence on export participation from the World Trade Organization (2008). In each of the countries considered, only a minority of firms export. Furthermore, even within exporters, there is tremendous heterogeneity in productivity and size. As reported in Table 2, the top 1 percent of firms account for 81 percent of U.S. exports and a substantial percentage of exports in all countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Exporting firms, in percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>2002</td>
<td>18.0</td>
</tr>
<tr>
<td>Norway</td>
<td>2003</td>
<td>39.2</td>
</tr>
<tr>
<td>France</td>
<td>1986</td>
<td>17.4</td>
</tr>
<tr>
<td>Japan</td>
<td>2000</td>
<td>20.0</td>
</tr>
<tr>
<td>Chile</td>
<td>1999</td>
<td>20.9</td>
</tr>
<tr>
<td>Colombia</td>
<td>1990</td>
<td>18.2</td>
</tr>
</tbody>
</table>

Table 1: Share of manufacturing firms that export, in percent (Source: WTO 2008, Table 5)

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Top 1% of firms</th>
<th>Top 10% of firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>2002</td>
<td>81</td>
<td>96</td>
</tr>
<tr>
<td>Belgium</td>
<td>2003</td>
<td>48</td>
<td>84</td>
</tr>
<tr>
<td>France</td>
<td>2003</td>
<td>44</td>
<td>84</td>
</tr>
<tr>
<td>Germany</td>
<td>2003</td>
<td>59</td>
<td>90</td>
</tr>
<tr>
<td>Norway</td>
<td>2003</td>
<td>53</td>
<td>91</td>
</tr>
<tr>
<td>U.K.</td>
<td>2003</td>
<td>42</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 2: Share of exports of manufactures, in percent (Source: WTO 2008, Table 6)

This new theoretical literature on firm heterogeneity and trade emphasizes the self-selection of more-productive firms into exporting and foreign direct investment (FDI). As a result of this
self-selection, reductions in trade costs have uneven effects across firms, as low-productivity firms exit and high-productivity firms expand to serve foreign markets. The resulting changes in industry composition raise aggregate productivity, consistent with empirical findings from trade liberalization episodes, as reported in Pavcnik (2002) and Trefler (2004). Firm heterogeneity and selection also influence cross-section patterns of trade and FDI. For example, the ratio of exports to foreign subsidiary sales depends not only on the trade-off between proximity and concentration, but also on the dispersion of firm productivity, as shown in Helpman, Melitz and Yeaple (2004) and Yeaple (2009). Similarly, the decision whether to offshore stages of production within or outside the boundaries of the firm is systematically related to firm productivity, as shown theoretically in Antràs and Helpman (2004) and empirically in Nunn and Trefler (2008) and Defever and Toubal (2010).

Although this theoretical literature emphasizes reallocations across firms, the modelling of the labor market has, until recently, been highly stylized. All workers are fully employed at a common wage and hence are affected symmetrically by the opening of trade. These model features sit uncomfortably with a large empirical literature that finds an employer-size wage premium (see the survey by Oi and Idson 1999) and with extensive evidence that exporters pay higher wages than nonexporters (see in particular Bernard and Jensen 1995, 1997). While this theoretical literature assumes no labor market frictions and costless reallocations across firms, search and matching frictions occupy a prominent position in macroeconomics (following Diamond 1982a,b, Mortensen 1970, Pissarides 1974, and Mortensen and Pissarides 1994). More generally, labor market institutions have been found to be influential in shaping the responses of European countries to external shocks (Blanchard and Wolfers 2000) and in understanding the evolution of unemployment rates in OECD countries over time (Nickell, Nunziata, Ochel and Quintini 2001).

Evidence on the magnitude of cross-country differences in labor market institutions is presented in Table 3. Even among countries at similar levels of economic development, such as OECD countries, there are substantial differences in the ease of hiring and firing workers and the rigidity of hours worked. In the European Union, member states have focused on labor market policies for more than a decade following the Luxembourg Extraordinary European Council Meeting on Employment in 1997. This meeting produced the European Employment Strategy, which was incorporated into the broader Lisbon Strategy, designed to turn Europe into a more competitive
and dynamic economy. To address such policy issues, we require theoretical models that pay more than usual attention to features of labor markets. And the high levels of international integration in the contemporary world economy suggest the need for frameworks within which it is possible to examine interdependence in labor market outcomes across nations.

<table>
<thead>
<tr>
<th>Country</th>
<th>Difficulty of Hiring</th>
<th>Rigidity of Hours</th>
<th>Difficulty of Redundancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Uganda</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rwanda</td>
<td>11</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>11</td>
<td>7</td>
<td>30</td>
</tr>
<tr>
<td>Japan</td>
<td>27</td>
<td>30</td>
<td>23</td>
</tr>
<tr>
<td>OECD</td>
<td>33</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Italy</td>
<td>67</td>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>France</td>
<td>78</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>Morocco</td>
<td>89</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3: Cross-country Differences in Labor Market Frictions (Source: Botero et al. 2004). Downloaded from the World Bank’s website http://www.doingbusiness.org/ExploreTopics/EmployingWorkers/ on September 25, 2009.

Our analysis builds on a long line of research on trade and labor market frictions. This literature has considered a number of different sources of labor market frictions, including minimum wages (Brecher 1974), implicit contracts (Matusz 1986), efficiency wages (Copeland 1989), fair wages (Agell and Lundborg 1995 and Kreickemeier and Nelson 2006), search and matching frictions (Davidson, Martin and Matusz 1988, 1999), and labor immobility and volatility (Cuñat and Melitz 2010). More recently, a surge of research has begun to incorporate labor market frictions into theories of firm heterogeneity and trade, including models of fair wages (Egger and Kreickemeier 2009, Amiti and Davis 2008), efficiency wages (Davis and Harrigan 2007), and search and matching frictions (Helpman and Itskhole 2010, Helpman, Itskhole and Redding 2010, Mitra and Ranjan 2010, and Felbermayr, Prat and Schmerer 2010).

Our analysis focuses on search frictions as the source of labor market imperfections and is based squarely in the new view of foreign trade that emphasizes firm heterogeneity in differentiated-product markets. The discussion of inequality, unemployment and trade in Section 3 draws on
Helpman, Itskhoki and Redding (2010), while the analysis of interdependence in labor market outcomes in Section 4 is based on Helpman and Itskhoki (2010). In Section 5, we present new results on the design of labor market policies in economies with firm heterogeneity and labor market frictions.¹

3 Inequality

The traditional framework for examining the distributional consequences of trade liberalization is the Stolper-Samuelson Theorem of the Heckscher-Ohlin model. Recent research, however, has identified a need to rethink the links between trade and wage inequality. While the Stolper-Samuelson Theorem predicts that trade raises wage inequality in skilled-labor–abundant countries and reduces wage inequality in unskilled-labor–abundant countries, empirical studies of recent trade liberalization episodes typically find rising wage inequality in both developed and developing countries (see for example the survey by Goldberg and Pavcnik 2007).² Furthermore, whereas the Stolper-Samuelson Theorem emphasizes changes in the relative wages of skilled and unskilled workers, there is evidence of changes in within-group inequality for workers with the same observed characteristics in the aftermath of trade reforms, as in Attanasio, Goldberg and Pavcnik (2004) and Menezes-Filho, Muendler and Ramey (2008).

In contrast to the Stolper-Samuelson Theorem’s reliance on reallocations of resources across industries, the key predictions of our framework relate to the distribution of wages and employment across firms and workers within a sector. We derive these distributions from comparisons across firms that hold in sectoral equilibrium for any value of a worker’s expected income outside the sector, i.e., his outside option. An important implication is that the predictions of our model for sectoral wage inequality hold regardless of general equilibrium effects. Throughout this section, all prices, revenues and costs are measured in terms of a numeraire, where the choice of this numeraire depends on how the sector is embedded in general equilibrium, as discussed further in Helpman, Itskhoki and Redding (2010).

¹See also Itskhoki (2010) for an analysis of the optimal design of a tax system in an open economy with heterogeneous firms.
²See, however, Feenstra and Hanson (1996), Zhu and Trefler (2005), and Sampson (2010) for trade mechanisms that can raise inequality in rich and poor countries alike.

6
3.1 Model Setup

We consider a differentiated-product sector. Consumer preferences take the constant elasticity of substitution (CES) form, and the real consumption index for the sector ($Q$) is

$$Q = \left[ \int_{j \in J} q(j)^{\beta} dj \right]^{1/\beta}, \quad 0 < \beta < 1,$$

where $j$ indexes varieties; $J$ is the set of varieties within the sector; $q(j)$ denotes consumption of variety $j$; and $\beta$ controls the elasticity of substitution between varieties.

There is a competitive fringe of potential firms who can choose to enter this sector by incurring a sunk entry cost of $f_e > 0$. Once the sunk entry cost is paid, a firm observes its productivity $\theta$, which is drawn from an independent Pareto distribution, $G_{\theta}(\theta) = 1 - (\theta_{\text{min}}/\theta)^z$ for $\theta \geq \theta_{\text{min}} > 0$ and $z > 1$. Once firms observe their productivity, they decide whether to exit, produce solely for the domestic market, or produce for both the domestic and export markets. Production involves a fixed cost of $f_d > 0$ units of the numeraire. Exporting involves an additional fixed cost of $f_x > 0$ units of the numeraire and an iceberg variable trade cost, such that $\tau > 1$ units of a variety must be exported in order for one unit to arrive in the foreign market.

There is a continuum of ex ante identical workers, who choose whether or not to search for employment in the sector. The labor market is subject to search and matching frictions. Workers draw a match-specific ability $a$ when matched with a firm in the differentiated sector. This match-specific ability, which is observed neither by the worker nor the firm, is drawn from an independent Pareto distribution, $G_a(a) = 1 - (a_{\text{min}}/a)^k$ for $a \geq a_{\text{min}} > 0$ and $k > 1$.

The output of each firm variety ($y$) depends on the productivity of the firm ($\theta$), the measure of workers hired ($h$), and the average ability of these workers ($\bar{a}$):

$$y = \theta h^\gamma \bar{a}, \quad 0 < \gamma < 1,$$

where this production technology can be interpreted as capturing either human capital complementarities (e.g., production in teams where the productivity of a worker depends on the average productivity of her team) or a managerial time constraint (e.g., a manager with a fixed amount of time who needs to allocate some time to each worker). A key feature of this production technol-
ogy is complementarities in worker ability, where the productivity of a worker is increasing in the abilities of other workers employed by the firm.

Search and matching frictions in the labor market are modelled following the standard Diamond-Mortensen-Pissarides approach. A firm that pays a search cost of $bn$ units of the numeraire can randomly match with a measure of $n$ workers, where the search cost $b$ is endogenously determined by the tightness of the labor market $x$:

$$b = \zeta x^\alpha.$$  \hspace{1cm} (3)

This search technology can be derived from a Cobb-Douglas matching function; $\zeta$ is a parameter that is increasing in the cost of posting vacancies and decreasing in the Hicks-neutral efficiency of the matching process; $\alpha$ is the ratio of the Cobb-Douglas coefficients on the number of workers searching for jobs and vacancies; and the tightness of the labor market, $x = N/L$, is the ratio of the measure of matched workers, $N$, to the measure of workers searching for employment in the differentiated sector, $L$.

Once matched with workers, firms can invest resources in screening them to obtain an imprecise signal of match-specific ability. By incurring a screening cost of $c\sigma \epsilon/\delta$, where $c > 0$ and $\delta > 1$, a firm can identify those workers with an ability below $a_c$, but cannot determine the abilities of the individual workers with any greater precision. We focus on interior equilibria in which $c$ is sufficiently small that all firms screen their workers.

The timing of decisions is as follows. Firms and workers decide whether or not to enter the differentiated sector. The outside option of firms is zero. The outside option of workers is expected income in other employment, $\omega$, where workers are assumed to be risk neutral and $\omega$ is determined in general equilibrium. After incurring the sunk entry cost for the differentiated sector, firms learn their productivity $\theta$ and choose whether to exit or produce. If firms choose to produce, they post a measure of vacancies and choose whether to serve only the domestic market or also export. Workers are next matched with firms. Unmatched workers become unemployed and receive unemployment benefits of zero. Firms screen their $n$ matched workers by choosing a screening threshold $a_c$. Only workers with abilities above the screening threshold are hired and those with abilities below the screening threshold become unemployed. The firm and its $h$ hired workers engage in multilateral bargaining over the division of the surplus from production as in Stole and Zwiebel (1996). Finally,
output is produced and markets clear.

### 3.2 Firm’s Problem

Given the specification of differentiated-sector demand, the equilibrium domestic-market revenue of a firm can be written as

$$ r(j) = p(j)q(j) = Aq(j)^\beta, $$

where $A$ is a demand-shifter, that is increasing in total expenditure on varieties within the sector, $E$, and in the sector’s ideal price index, $P$, which summarizes the prices of competing varieties.

If a firm exports, it allocates its output between the domestic and export markets to equate its marginal revenues in the two markets, so that total firm revenue can be expressed as

$$ r(\theta) = r_d(\theta) + r_x(\theta) = \Upsilon(\theta)^{1-\beta} Ay(\theta)^\beta, $$

(4)

where $r_d(\theta) \equiv Ay_d(\theta)^\beta$ is revenue from domestic sales; $r_x(\theta) \equiv Ay_x(\theta) / \tau^\beta$ is revenue from exporting; $y_d(\theta)$ is output for the domestic market; $y_x(\theta)$ is output for the export market; and $y(\theta) = y_d(\theta) + y_x(\theta)$. The variable $\Upsilon(\theta)$ captures a firm’s “market access,” which depends on whether it chooses to serve both the domestic and foreign markets or only the domestic market:

$$ \Upsilon(\theta) = 1 + I_x(\theta) \tau^{-\frac{\beta}{1-\beta}} \left(\frac{A^*}{A}\right)^{\frac{1}{r/\beta}}, $$

(5)

where $I_x(\theta)$ is an indicator variable that equals one if the firm exports and zero otherwise.

The solution to the bargaining game implies that the firm receives a share $1 / (1 + \beta \gamma)$ of revenue, while each worker receives a wage equal to a constant share of revenue per worker:

$$ w(\theta) = \frac{\beta \gamma}{1 + \beta \gamma} \frac{r(\theta)}{h(\theta)}. $$

Anticipating this outcome of the bargaining game, a firm chooses the measure of workers to match
with, \( n \), the screening threshold, \( a_c \), and whether or not to export to maximize its profits:

\[
\pi(\theta) \equiv \max_{n \geq 0, \ a_c \geq a_{\text{min}}, \ I_x \in \{0,1\}} \left\{ \frac{1}{1 + \beta \gamma} \left[ 1 + I_x \tau^{-1/\beta} \left( \frac{A^*}{A} \right)^{1/\beta} \right]^{1-\beta} A \left( \kappa_y \theta n^{\gamma} a_c^{1-\gamma k} \right)^{\beta} - bn - \frac{c}{\delta} a_c^\delta - f_d - I_x f_x \right\}
\]

(6)

where \( \kappa_y \) is a derived parameter and we have used the properties of the Pareto distribution of worker ability. The latter implies that a firm choosing a screening threshold \( a_c \) hires a measure

\[ h = n \left( \frac{a_{\text{min}}}{a_c} \right)^k \]

of workers with average ability \( \bar{a} = k a_c / (k - 1) \). Firms of all productivities have an incentive to screen for \( 0 < \gamma k < 1 \) and sufficiently small values of \( c \).

As a result of fixed costs of production and exporting, a firm’s decision whether or not to produce and export takes a standard form. Only the most-productive firms with productivities \( \theta \geq \theta_x \) export; firms with intermediate productivities \( \theta \in [\theta_d, \theta_x) \) serve only the domestic market; and the least-productive firms with productivities \( \theta < \theta_d \) exit. The firm’s market-access variable is therefore determined as follows:

\[
\Upsilon(\theta) = \begin{cases} 
1, & \theta < \theta_x, \\
\Upsilon_x, & \theta \geq \theta_x, 
\end{cases} \quad \Upsilon_x \equiv 1 + \tau^{-1/\beta} \left( \frac{A^*}{A} \right)^{1/\beta} > 1. 
\]

(7)

Using the first-order conditions to the firm’s problem (6), closed-form solutions for all firm-specific variables can be derived:

\[
\begin{align*}
\gamma(\theta) &= \Upsilon(\theta) \frac{1-\beta}{1-\gamma} \cdot r_d \cdot \left( \frac{\theta}{\bar{a}} \right)^{\beta} \gamma, \\
n(\theta) &= \Upsilon(\theta) \frac{1-\beta}{1-\gamma} \cdot n_d \cdot \left( \frac{\theta}{\bar{a}} \right)^{\beta} \gamma, \\
a_c(\theta) &= \Upsilon(\theta) \frac{1-\beta}{1-\gamma} \cdot a_d \cdot \left( \frac{\theta}{\bar{a}} \right)^{\beta} \gamma, \\
h(\theta) &= \Upsilon(\theta) \frac{1-\beta}{1-\gamma} \cdot h_d \cdot \left( \frac{\theta}{\bar{a}} \right)^{\beta} \gamma, \\
w(\theta) &= \Upsilon(\theta) \frac{1-\beta}{1-\gamma} \cdot w_d \cdot \left( \frac{\theta}{\bar{a}} \right)^{\beta} \gamma,
\end{align*}
\]

(8)

More-productive firms have larger revenues, match with more workers, and screen to higher ability thresholds. As a result they have workforces of higher average ability and pay higher wages. As long as screening costs are sufficiently convex and worker ability is sufficiently dispersed, \( \delta > k \).
more-productive firms also hire more workers, which implies that the model features the empirically-observed employer-size wage premium. The fixed costs of exporting imply that all firm variables, apart from profits, jump discretely at the productivity threshold for exporting, $\theta_x$, where $\Upsilon(\theta)$ jumps from one to $\Upsilon_x > 1$. Exporting firms are, therefore, more productive, larger, have workforces of higher average ability, and pay higher wages, as found empirically using micro data on firms and plants (e.g. Bernard and Jensen 1995, 1997) and matched employer-employee datasets (e.g. Frías and Kaplan 2009).

The wage schedule as a function of productivity is illustrated for particular parameter values in Figure 1. Although more-productive firms pay higher wages, they also screen more intensively, which implies that they hire a smaller fraction of their matched workers. Using the solution to the bargaining game and the firm’s first-order conditions, the higher wages of more-productive firms are exactly offset by the lower probability of being hired, since the Stole-Zwiebel bargaining solution implies that a firm’s equilibrium wage is equal to its replacement cost for each worker. As a result, the expected wage conditional on being matched is the same across all firms:

$$\frac{w(\theta)h(\theta)}{n(\theta)} = b,$$

which implies that workers have no incentive to direct their search across firms of differing productivities.
3.3 Labor Market Equilibrium

Worker indifference across sectors requires that expected income in the differentiated sector is equal to workers’ outside option, \( \omega \), where expected income in the differentiated sector equals the probability of being matched, \( x \), times the expected wage conditional on being matched, \( b \):

\[
\omega = xb. \tag{9}
\]

This indifference condition across sectors and the search technology (3) together determine the equilibrium tightness of the labor market and hiring costs as a function of workers’ outside option:

\[
b = \frac{x}{1 + x} \quad \text{and} \quad x = \left( \frac{\omega}{\zeta} \right)^{1 + 1}, \tag{10}
\]

where \( \omega \) is determined in general equilibrium, as considered in Helpman, Itskhoki and Redding (2010).

3.4 Implications for Wage Inequality

Since wages and employment in (8) are power functions of productivity, which is Pareto distributed, we can solve in closed form for the wage distribution. The distribution of wages across all workers is a weighted average of the distributions of wages for workers employed by domestic firms and for workers employed by exporters, with weights equal to the shares of employment in the two groups of firms: \( S_{h,d} \) representing the share of employment by nonexporters and \( S_{h,e} = 1 - S_{h,d} \) representing the share of employment by exporters. The distribution of wages across workers employed by domestic firms is a truncated Pareto distribution while the distribution of wages across workers employed by exporters is an untruncated Pareto distribution, but these two wage distributions have the same shape parameter, \( 1 + 1/\mu \), where \( \mu \) is defined as

\[
\mu \equiv \frac{\beta k/\delta}{2 \Gamma - \beta}, \quad \text{where} \quad \Gamma \equiv 1 - \beta \gamma - \frac{\beta}{\delta} (1 - \gamma k),
\]

and we require \( 0 < \mu < 1 \) and hence \( z \Gamma > 2 \beta \) for the wage distribution to have a finite mean and variance.
In both the closed economy ($S_{h,d} \to 1$) and the open economy when all firms export ($S_{h,d} \to 0$), the distribution of wages across all workers is an untruncated Pareto distribution. One feature of an untruncated Pareto distribution is that all scale-invariant measures of inequality, such as the Coefficient of Variation, the Gini Coefficient and the Theil Index, depend solely on the distribution’s shape parameter, which is a sufficient statistic for inequality. As this shape parameter is the same for workers employed by domestic firms and by exporters, it follows that the level of wage inequality in the open economy when all firms export is the same as in the closed economy. In contrast, when only some firms export, it can be shown that there is strictly greater wage inequality in the open economy than in the closed economy.

This result highlights a new mechanism for international trade to affect wage inequality: the participation of some but not all firms in exporting. This mechanism applies in any heterogeneous-firm model in which firm wages are related to firm revenue and there is selection into export markets. Our result holds whenever the following three conditions are satisfied: firm wages and employment are power functions of firm productivity, there is firm selection into export markets and exporting increases wages for a firm with a given productivity, and firm productivity is Pareto distributed. An important implication of this result, which applies for symmetric and asymmetric countries alike, is that the opening of trade can increase wage inequality in all countries. This result is therefore consistent with empirical findings of increased wage inequality in developing countries following trade liberalization. Similarly, our result is consistent with empirical evidence that much of the observed reallocation in the aftermath of trade liberalization occurs across firms within sectors and is accompanied by increases in within-group wage inequality.

Since sectoral wage inequality in an open economy in which all firms export is the same as in a closed economy, but sectoral wage inequality in an open economy in which only some firms export is higher than in a closed economy, it follows that the relationship between sectoral wage inequality and the fraction of exporters is at first increasing and later decreasing. The intuition for this result is that the increase in firm wages that occurs at the productivity threshold above which firms export is only present when some but not all firms export. When no firm exports, a small reduction in trade costs that induces some firms to start exporting raises sectoral wage inequality because of the higher wages paid by exporters. When all firms export, a small increase in trade costs that induces some firms to stop exporting raises sectoral wage inequality because of the lower
wages paid by domestic firms.

3.5 Implications for Unemployment

While we have so far focused on the distribution of wages across employed workers, income inequality in this framework also depends on the unemployment rate. Workers can be unemployed either because they are not matched with a firm or because their match-specific ability draw is below the screening threshold of the firm with which they are matched. The sectoral unemployment rate $u$ includes both of these components and can be written as one minus the product of the hiring rate $\sigma$ and the tightness of the labor market $x$:

$$u = \frac{L - H}{L} = 1 - \frac{H}{N} \frac{N}{L} = 1 - \sigma x,$$

(11)

where $\sigma \equiv H/N$, $H$ is the measure of hired workers, $N$ is the measure of matched workers, and $L$ is the measure of workers seeking employment in the sector.

As shown above, equilibrium labor market tightness, $x$, depends on worker’s outside option, $\omega$, which can either remain constant or rise following the opening of trade, depending on how the sector is embedded in general equilibrium (see Helpman, Itskhoki and Redding 2010). In contrast, the hiring rate, $\sigma$, is unambiguously lower in the open economy than in the closed economy, since the opening of trade reallocates employment within industries towards more-productive exporting firms, which screen more intensively and hire a smaller fraction of the workers with whom they are matched. Furthermore, this reduction in the hiring rate can dominate an increase in labor market tightness, so that the opening of trade not only increases wage inequality but also raises unemployment.

Although the opening of trade can increase both wage inequality and unemployment, it also reduces the CES ideal price index for the differentiated sector. Therefore, despite increasing social disparity, the opening of trade raises the expected welfare of risk-neutral workers.

3.6 Multiple Worker Types

Our main results on the impact of trade on wage inequality can be generalized to settings in which there are multiple types of workers with different observable characteristics. To illustrate, suppose
that there are two types of workers, indexed by $\ell = 1, 2$. There are separate labor markets for each type of worker, which are modelled as above, where the magnitude of search frictions can vary across worker types. Within each group of workers there is heterogeneity in the match-specific ability, $a_\ell$, which is not observable. As a result, workers of a given type $\ell$ are \textit{ex ante} homogeneous but \textit{ex post} heterogeneous, as for the case of a single type of worker discussed above.

Let the distribution of ability of type-$\ell$ workers be Pareto with shape parameter $k_\ell > 1$ for $\ell = 1, 2$, and let the production function be

$$y = \theta (\bar{a}_1 h_1^{\gamma_1})^{\kappa_1} (\bar{a}_2 h_2^{\gamma_2})^{\kappa_2}, \quad \kappa_1 + \kappa_2 = 1.$$  

Then Helpman, Itskhoki and Redding (2010) show that wage inequality is larger within each group of workers in an open economy in which only a fraction of firms export than in a closed economy. Moreover, for $k_1 < k_2$, more-productive firms employ relatively more workers of type-1—with the larger ability dispersion—and pay them relatively lower wages. The relatively larger number of type-1 workers in higher-productivity firms weakens these workers’ relative bargaining power, which translates into relatively lower wages. As a result, there is less wage dispersion among type-1 workers.

Importantly, while trade raises wage inequality within every group of workers, it may raise or reduce wage inequality between the two groups. Yet even if trade reduces wage inequality between the groups, overall wage inequality may still rise as a result of the increase in wage inequality within each group of workers with similar observable characteristics.

### 4 Interdependence

Having examined the impact of trade on sectoral inequality and unemployment, we now discuss interdependence between trading countries. Using the results from Helpman and Itskhoki (2010), we address the following questions: How do labor market frictions impact interdependence across countries? And, in particular, what are the impacts of a country’s labor market frictions on its trade partners?
4.1 Analytical Framework

For the purpose of addressing these questions, we consider a two-country world, say countries \textit{A} and \textit{B}, in which every country has the same technology in each one of two sectors. One sector produces varieties of a differentiated product while the other manufactures a homogeneous good. Preferences are quasi-linear, given by

\[ U = q_0 + \frac{1}{\eta} Q^\eta, \quad \eta < \beta < 1, \]  

(12)

where \( q_0 \) is consumption of the homogeneous good, \( Q \) is the real consumption index of the differentiated product, and we choose the homogeneous good as the numeraire. As before, \( \beta \) controls the elasticity of substitution across varieties, and the new parameter \( \eta \) controls the elasticity of substitution between the homogeneous good and the differentiated product. We think of \( U \) as the utility level of a family consisting of a continuum of workers of measure one. There exists a continuum of such families of measure \( \tilde{L} \). As a result, there are \( \tilde{L} \) workers in this economy. Each family chooses the allocation of family members across sectors to maximize family utility. Since the idiosyncratic risk faced by individual workers as a result of random search and matching is perfectly diversified across the continuum of workers within each family, each family behaves as if it is risk neutral.

The homogeneous good is produced according to a constant returns to scale technology, with one unit of labor required to produce one unit of output, and the homogeneous good is costlessly traded. The technology of the differentiated sector is a simplified version of the technology from the previous section, with no worker heterogeneity and no screening. In this case the production function of every variety is

\[ y = \theta h, \]

where, as before, \( \theta \) is the firm’s productivity and \( h \) is its employment. Varieties in the differentiated sector are again subject to iceberg trade costs, where \( \tau > 1 \) units must be shipped in order for one unit to arrive in the other country.

There are labor market frictions in each sector, similar to the labor market frictions described
in the previous section. In the homogeneous sector the cost of hiring is

\[ b_0 = \zeta_0 x_0^\alpha. \]

The derived parameter \( \zeta_0 \) is larger the higher the cost of vacancies is and the less efficient is the matching process in the homogeneous sector. Moreover, in equilibrium \( w_0 = 1/(1 + \lambda) \) and \( b_0 = \lambda/(1 + \lambda) \), where \( \lambda \) is the relative bargaining weight of the employer in the wage bargaining process (see Appendix).\(^3\) As a result,

\[ \zeta_0 x_0^\alpha = \frac{\lambda}{1 + \lambda}, \tag{13} \]

and equilibrium tightness in the homogeneous sector’s labor market, \( x_0 \), is decreasing in the level of labor market frictions in this sector, \( \zeta_0 \). The cost of hiring in the differentiated sector is given by (3). The two countries, A and B, differ only in labor market frictions \( (\zeta_0, \zeta) \). That is, they differ either in the sectoral levels of the efficiency of matching or in the costs of posting vacancies, which determine the equilibrium levels of the frictions \( (\zeta_0, \zeta) \).

In equilibrium, workers are indifferent between searching for jobs in the homogeneous or the differentiated sector, which implies that their expected income is the same in each sector, \( x_0 b_0 = x b \). Together with the search technology, this condition implies the following values of the wage rate, the cost of hiring, and labor market tightness in the differentiated sector in each country \( j \), independently of the trade regime:

\[ w_j = b_j = b_0 \left( \frac{\zeta_j}{\zeta_0_j} \right)^{\frac{1}{1+\alpha}}, \quad x_j = x_0 j \left( \frac{\zeta_j}{\zeta_0_j} \right)^{-\frac{1}{1+\alpha}}, \quad j = A, B. \tag{14} \]

For simplicity, and without loss of generality, we assume \( \zeta_A/\zeta_{0A} > \zeta_B/\zeta_{0B} \), which implies \( b_A > b_B \), i.e., labor market frictions in the differentiated sector are relatively larger in country A.

### 4.2 Trade and Welfare

Helpman and Itskhioki (2010) show that under these circumstances a larger fraction of differentiated-product firms export in country B, and that country B exports differentiated products on net and

\(^3\)In Helpman and Itskhioki (2010) the bargaining weights are equal, as a result of which \( \lambda = 1 \) and \( b_0 = 1/2 \). We generalize this result in order to better characterize optimal policies in the next section.
imports homogeneous goods. Since the only difference between the two countries is in their labor market frictions, it follows that this pattern of trade is determined by differences in labor market frictions across countries; the country that has the relatively lower level of labor market frictions in the differentiated sector exports differentiated goods on net. Moreover, in this world economy the share of intra-industry trade is smaller the larger the gap in relative hiring costs \( b_A/b_B \) is.

Another interesting result is that both countries gain from trade, in the sense that a representative family’s utility level \( U \) is higher in the trade equilibrium than in autarky. Since the idiosyncratic risk faced by individual workers is perfectly diversified within families, the expected utility of every worker is higher in the open economy than in autarky.

### 4.3 Interdependence in Labor Market Frictions

A reduction in labor market frictions in the differentiated sector of country \( j \), \( \zeta_j \), reduces the cost of hiring \( b_j \), raises country \( j \)’s welfare, and reduces its trade partner’s welfare. In this event a country loses from the lowering of labor market frictions in its trade partner. The intuition for this negative welfare effect is that indirect utility equals income plus consumer surplus in the differentiated sector. Lower labor market frictions in the differentiated sector in country \( j \) make this sector more competitive relative to that in its trade partner, which induces an expansion in the differentiated sector in country \( j \) and a contraction in this sector in its trade partner. These changes in the size of the differentiated sector raise consumer surplus and welfare in country \( j \) and reduce consumer surplus and welfare in its trade partner.

A simultaneous proportional reduction of \( \zeta_A \) and \( \zeta_B \) raises welfare in both countries, because it expands the size of the differentiated sector in each one of them. On the other hand, a reduction in \( \zeta_j \) and \( \zeta_{0j} \) at a common rate (which does not change the hiring cost \( b_j \)) raises country \( j \)’s welfare and does not affect the welfare level of its trade partner. This results from the fact that this type of reduction in labor market frictions does not impact competitiveness, yet it leads to higher aggregate utilization of resources in country \( j \) (see the discussion of unemployment below).

### 4.4 Trade Liberalization

Reductions of trade impediments, \( \tau \), raise welfare in both countries, because they also expand the size of the differentiated sector in each country. Unlike the welfare consequences of lower trade
frictions, however, the effects on unemployment can differ across countries. A country’s rate of unemployment equals a weighted average of its sectoral rates of unemployment—\((1 - x_{0j})\) in the homogeneous sector and \((1 - x_j)\) in the differentiated sector—with weights equal to the shares of workers seeking employment in these sectors. In other words, country \(j\)’s rate of unemployment is

\[
u_j = \frac{N_{0j}}{L_j} (1 - x_{0j}) + \frac{N_j}{L_j} (1 - x_j),
\]

where \(N_{0j}\) is the measure of workers seeking employment in the homogeneous sector and \(N_j\) is the measure of workers seeking employment in the differentiated sector, with \(N_{0j} + N_j = L_j\). Since trade impediments do not impact sectoral rates of unemployment, because tightness in labor markets does not depend on trade frictions, the only channel through which reductions in \(\tau\) can influence the rate of unemployment is through worker reallocation across industries. Therefore, if the rate of unemployment is higher in the differentiated sector than in the homogeneous sector, aggregate unemployment rises as a result of the expansion of the differentiated sector induced by lower trade frictions. And if unemployment is higher in the homogeneous sector than in the differentiated sector, aggregate unemployment declines as a result of the expansion of the differentiated sector induced by lower trade frictions. Moreover, (14) implies that the rate of unemployment is higher in the differentiated sector if and only if it has higher labor market frictions than the homogeneous sector, i.e., \(\zeta_j > \zeta_{0j}\).

Helpman and Itskhoki (2010) show that lower trade frictions may impact the rates of unemployment in the two countries in the same direction or in opposite directions. Moreover, the rate of unemployment can be higher in country \(A\) for some levels of trade frictions and higher in country \(B\) for other levels of trade frictions. As a result, differences in aggregate levels of unemployment do not necessarily reflect differences in labor market frictions; a country with more rigid labor markets may have a higher or lower rate of unemployment. Finally, since lower trade frictions raise welfare in both countries, but may raise the rate of unemployment in both or only in one of them, it is evident that the impact of lower trade frictions on unemployment provides no information on their impact on welfare; welfare goes up in both countries even when their rates of unemployment increase.
4.5 Unemployment and Labor Market Frictions

Of special interest is the relationship between labor market frictions and rates of unemployment. This relationship is sharpest in the case of symmetric countries, which have the same levels of labor market frictions ($\zeta_0, \zeta$). In this case, Helpman and Itskhoki (2010) show that raising the common level of labor market frictions in the differentiated sector raises the rate of unemployment in both countries if and only if $\zeta / \zeta_0$ is smaller than a threshold that exceeds one. It follows that whenever $\zeta < \zeta_0$, i.e., labor market frictions are lower in the differentiated sector, this condition is satisfied and raising $\zeta$ increases the rate of unemployment. This increase in the rate of unemployment occurs for two reasons: first, the sectoral rate of unemployment rises in the differentiated sector; second, workers move from the differentiated sector to the homogeneous sector and the latter has a higher sectoral rate of unemployment. Alternatively, when $\zeta > \zeta_0$ but $\zeta / \zeta_0$ is smaller than the threshold, higher frictions in the differentiated sector raise the sectoral rate of unemployment which raises in turn the aggregate rate of unemployment. But now the movement of workers from the differentiated to the homogeneous sector reduces aggregate unemployment, because the homogeneous sector has a lower rate of unemployment than the differentiated sector. The former effect dominates, however, as long as $\zeta / \zeta_0$ is below the threshold. Above the threshold higher frictions in the differentiated sector’s labor market reduce aggregate unemployment, because in this case the negative impact of worker reallocation across industries outweighs the positive impact of the rise in the rate of unemployment in the differentiated sector.4

When countries are not symmetric, the sectoral unemployment rate and labor force composition effects interact in complex ways. For example, starting with $\zeta > \zeta_0$ and raising labor market frictions in country A’s differentiated sector can initially raise the rate of unemployment in both countries but eventually reduce it in country A, whereas it continues to raise the rate of unemployment in country B. As a result, A may have a higher rate of unemployment for low values of $\zeta_A$ but a lower rate of unemployment for high values of $\zeta_A$, or it may have lower unemployment for all $\zeta_A > \zeta$. Again, we encounter a case in which knowledge of relative rates of unemployment across countries is not sufficient to draw inferences about their relative levels of labor market frictions.

4When $\zeta_0$ and $\zeta$ increase proportionately, aggregate unemployment rises.
5 Policy Implications

We now use the previous section’s analytical framework to study economic policies. One result of interest from that section is that a reduction in a country’s cost of hiring in the differentiated sector raises its competitiveness relative to its trade partner and thereby hurts the trade partner. In the previous section, the change in the cost of hiring was induced by a reduction in labor market frictions in the form of lower costs of vacancies or more efficient matching. In this section we examine instead how unemployment benefits—a prevalent labor market policy—influence the cost of hiring. The results from the previous section imply that if higher unemployment benefits raise a country’s cost of hiring then this policy benefits the trade partner, and if higher unemployment benefits reduce a country’s cost of hiring then this policy hurts the trade partner.

We have also seen that a country benefits from lower costs of hiring in its differentiated sector when this reduction is achieved through labor market frictions. If, alternatively, a similar reduction in the cost of hiring is attained with unemployment benefits, does this too raise welfare? One difference between this policy-induced reduction in the cost of hiring and a decline in labor market frictions is that unemployment benefits require financing in the form of taxes while the decline in labor market frictions does not. For this reason unemployment benefits that reduce the cost of hiring might be advantageous up to a point, while large unemployment benefits might be detrimental.

After discussing unemployment benefits in Sections 5.1 and 5.2, and the nature of the economy’s distortions in Section 5.3, we examine in Section 5.4 policies that implement a constrained Pareto optimum. The focus on a constrained rather than an unconstrained optimal allocation stems from our desire to treat search and matching in the labor market as a constraint on economic activity that a social planner cannot remove, and she therefore cannot costlessly allocate workers to firms. We show that there exists a simple set of policies in labor and product markets that support such a constrained Pareto optimal allocation. This set of policies is not unique, because there exist alternative combinations of labor market and product market policies that can achieve the same end. One conclusion from this analysis is that there are cases in which unemployment benefits can play a useful role in the optimal policy design, but that there are also cases in which unemployment benefits are not congruent with efficiency. Another conclusion is that optimal policies do not discriminate between firms by export status; the same policies should be applied to exporters and
nonexporters alike.

5.1 Unemployment Benefits

Unemployment benefits impact wages and the cost of hiring. Wages are affected directly when workers bargain with employers, because in the presence of unemployment benefits $b_u$—measured in units of the homogeneous numeraire good—the outside option of a worker in the bargaining game is $b_u$ instead of zero (we drop the country index in what follows). In addition, unemployment benefits affect tightness in labor markets and thereby the incentives of workers to search for jobs in the homogeneous versus differentiated sectors.

In the homogeneous sector the wage rate is now

$$w_0 = b_u + \frac{1}{1+\lambda} (1 - b_u),$$  \hspace{1cm} (15)

the cost of hiring is

$$b_0 = (1 - b_u) \frac{\lambda}{1+\lambda},$$

and tightness in the labor market (see Appendix for details) satisfies

$$\zeta_0 x_0^\alpha = (1 - b_u) \frac{\lambda}{1+\lambda},$$  \hspace{1cm} (16)

which is the same as (13) when the unemployment benefits are equal to zero. Evidently, in this case higher unemployment benefits reduce $x_0$ and raise the sectoral rate of unemployment. And, as before, higher frictions in the labor market reduce $x_0$. From (15) and (16) we obtain the expected income of a worker searching for employment in the homogeneous sector, $\omega = w_0 x_0 + b_u (1 - x_0)$, as a function of unemployment benefits. Moreover, $\omega$ is the outside option of workers searching for employment in the differentiated sector. Therefore, in an equilibrium with positive employment in both sectors, $\omega$ also equals the expected income of a worker searching for a job in the differentiated sector, and therefore $\omega = w x + b_u (1 - x)$.

In the differentiated sector bargaining over wages yields a wage rate equal to the fraction $\beta/ (\beta + \lambda)$ of revenue per worker plus $b_u \lambda / (\lambda + \lambda)$ (in the absence of unemployment benefits the second component equals zero). Accounting for the firm’s profit-maximizing choice of employment
and the requirement that the expected income of workers be the same in both sectors, we obtain

\[ x = x_0 \left( \frac{\zeta_0}{\zeta} \right)^{\frac{1}{1+\alpha}}. \]  

(17)

As a result, there is a proportional relationship between labor market tightness in the two sectors, and a change in unemployment benefits has the same proportional effect on labor market tightness in each sector. In particular, higher unemployment benefits reduce tightness in both labor markets. We also show in the Appendix that

\[ b = \zeta x^\alpha + \frac{\lambda}{1+\lambda} b_u. \]  

(18)

Therefore unemployment benefits, \( b_u \), directly affect the cost of hiring in the differentiated sector and also have indirect effects through labor market tightness, \( x \). Higher unemployment benefits raise \( b \) directly because they increase workers’ outside option in wage bargaining. But higher unemployment benefits reduce \( b \) indirectly because they reduce tightness in the labor market, \( x \). Equations (16)-(18) imply that higher unemployment benefits raise the cost of hiring, \( b \), on net if and only if labor market frictions are higher in the homogeneous sector; that is, if and only if \( \zeta_0 > \zeta \).

When labor market frictions are higher in the differentiated sector, the differentiated sector has a higher sectoral rate of unemployment than the homogeneous sector. Under these circumstances higher unemployment benefits reduce the hiring cost in the differentiated sector and lead to its expansion, as more workers choose to search for jobs in this industry. In other words, unemployment benefits have an uneven effect on sectoral employment, favoring the sector with higher unemployment. As a result, by raising unemployment benefits a country makes its differentiated sector more competitive on world markets if this sector has the higher sectoral rate of unemployment, in which case this policy hurts the country’s trade partner. Alternatively, by raising unemployment benefits a country benefits its trade partner when the country’s labor market frictions are higher in the homogeneous sector.

\footnote{Equations (16)-(18) can be used to derive a closed-form solution for the hiring rate:

\[ b = \frac{\lambda}{1+\lambda} \left[ b_u + (1-b_u) \left( \frac{\zeta}{\zeta_0} \right)^{\frac{1}{1+\alpha}} \right], \]

from which this result is transparent.}
5.2 Unemployment Benefits and Welfare

The next question is whether a country gains from raising its unemployment benefits. Figure 2 shows that the answer depends on structural features of the labor market. The figure depicts percentage changes in welfare, measured on the vertical axis, in response to changes in the level of unemployment benefits, measured as a replacement ratio of the homogeneous sector’s wage rate, $b_u/w_0$. It describes simulations of a closed economy in which labor market frictions are higher in the differentiated sector.\(^6\) In this case higher unemployment benefits always reduce the equilibrium cost of hiring in both sectors. Yet for high values of $\alpha \lambda$, welfare first rises in unemployment benefits and eventually declines, while for low values of $\alpha \lambda$, welfare always declines in unemployment benefits. It follows that when $\alpha \lambda$ is large welfare is maximized at a positive level of unemployment benefits, while the optimal level of unemployment benefits equals zero when $\alpha \lambda$ is small.

To gain further insight into these results, Figure 3 decomposes the changes in welfare that result from unemployment benefits for the case $\alpha \lambda = 1.2$. The North-Western panel describes the contribution of the differentiated sector to welfare, $Q^\beta$, and the contribution of income net of taxes, $E = \omega L - T$, as well as the total welfare level, $W$. As the cost of hiring declines with unemployment benefits, the contribution of the differentiated sector rises throughout. But net income rises initially as long as the rise in $\omega L$ is larger than the rise in taxes $T$ needed to finance

\(^6\)The following parameters were used in the simulations described in Figures 2 and 3: $\lambda = 1.2$, $\alpha = 1$, $\beta = 2/3$, $\eta = 1/2$, $\zeta_0 = 0.6$, $\zeta = 0.66$, $f_d = 1$, $L = 1.5$. In addition, $f_c$, $\theta_{min}$ and $z$ were chosen to yield $\theta_d = 1$. 
the unemployment benefits, and declines eventually. As a result, the welfare curve $W$ has a hump shape. The North-East panel shows that not only do taxes rise with unemployment benefits, they also rise as a fraction of net income, $T/E$. In addition, the fraction of workers searching for jobs in the differentiated sector, $N/L$, rises. Finally, rising unemployment benefits reduce tightness in both sectors’ labor markets, as shown in the South-West panel of the figure (which also shows the rise in $\omega$).\footnote{Our numerical example illustrates more general patterns. It can be shown analytically that wages and expected wages rise in both sectors with $b_u$ while the levels of tightness in both sectors’ labor markets decline with $b_u$. In addition, the hiring cost $b$ decreases in $b_u$ if and only if $\zeta > \zeta_0$, as we show in footnote 5. The only analytical ambiguity in the derivation of the optimal unemployment benefits stems from the response of $E = \omega L - T$.} As a result, higher unemployment benefits raise sectoral rates of unemployment. Since workers also move from the homogeneous to the differentiated sector, which is the higher unemployment rate sector, aggregate unemployment rises with unemployment benefits.

5.3 Product and Labor Market Distortions

An interesting implication of the example depicted in Figure 3 is that unemployment benefits are beneficial up to a point despite the fact that they raise unemployment. Yet if we were to reduce $\alpha \lambda$ in this example to a sufficiently low level, we would find that unemployment benefits raise unemployment and reduce welfare. The question is why. To understand the answer, first note that in this type of economy there are multiple distortions. To begin with, the differentiated sector
is too small, because it prices goods with a markup above marginal cost and there is too little entry into the industry. For this reason unemployment benefits that reduce the cost of hiring in the differentiated sector and induce a reallocation of workers from the homogeneous to the differentiated sector, benefit the economy. On the other side, in this example tightness in the labor market is too high initially and unemployment benefits bring it down. This is illustrated in the South-East panel of Figure 3 for the homogeneous sector, in which the horizontal dashed line $x^H_0$ describes the optimal level of tightness, and the vertical dashed line shows the welfare-maximizing unemployment benefits policy. For low levels of unemployment benefits $x_0$ is too high, while for high levels of unemployment benefits it is too low. For this reason raising unemployment benefits from an initially low level reduces distortions in labor markets by reducing labor market tightness, and this raises welfare. But when initial unemployment benefits are high, the levels of tightness in the labor markets are too low and further increases in unemployment benefits aggravate the labor market distortions, which may reduce welfare.

There are no distortions in the labor market when the Hosios (1990) condition is satisfied, which in our case is $\alpha \lambda = 1$ (i.e., the elasticity of the matching function with respect to the number of vacancies equals the weight of employers in the bargaining game). While the Hosios condition was derived in models with linear revenue and single-job firms, we extend this condition to a model with monopolistic competition, multiple-job firms, and Sole and Zwiebel (1996) style multilateral bargaining. When the Hosios condition holds, or $\alpha \lambda \leq 1$, unemployment benefits always magnify the distortions in the labor markets, which reduces welfare. But because they reduce the distortion in the intersectoral allocation of labor (when they increase $N$), unemployment benefits may initially raise welfare on net. When $\alpha \lambda$ is very low, however, the distortions in the labor markets are so high that even small unemployment benefits reduce welfare on net.

To understand the link between the Hosios condition and labor market distortions in this model, consider the following experiment. Suppose we want to employ $H$ workers in the differentiated sector, but we cannot allocate them directly to firms; all we can do is instruct $N$ workers to search for jobs in the differentiated sector and the remaining $L - N$ workers to search for jobs in the homogeneous sector. How many vacancies do we need to open in each sector in order to secure the employment of $H$ workers in the differentiated sector at minimum cost to the economy?

Instead of working directly with vacancies we can instead choose levels of tightness in the
sectoral labor markets, $x_0$ and $x$. Naturally, in this case we need to send $N = H/x$ workers to search for jobs in the differentiated sector, which leaves $L - H/x$ workers searching for jobs in the homogeneous sector. However, only a fraction $x_0$ of the latter workers find employment in the homogeneous sector, producing $(L - H/x)x_0$ units of the homogeneous good. The cost of filling up $(L - H/x)x_0$ vacancies in the homogeneous sector is $(L - H/x)x_0\zeta_0x_0^\alpha$, because the cost of hiring is $\zeta_0x_0^\alpha$ per worker. And the cost of filling up $H$ vacancies in the differentiated sector is $H\zeta x^\alpha$, because the cost of hiring is $\zeta x^\alpha$ per worker in the differentiated sector. Consequently, the net output of homogeneous goods—which can be used for consumption or for entry of firms in the differentiated sector—equals

$$(L - H/x)x_0(1 - \zeta_0x_0^\alpha) - H\zeta x^\alpha.$$ 

Given $H$, the optimal levels of $x$ and $x_0$ maximize this measure of net output. The solution to this problem does not depend on $H$, however, and it can be characterized by

$$\zeta_0x_0^\alpha = \frac{1}{1 + \alpha}$$

and (17). When these conditions are satisfied, the ratio $x/x_0$ is at the optimal level independently of $H$ or the level of unemployment benefits. Comparing (16) with (19), we see that levels of labor market tightness are optimal in the absence of unemployment benefits if and only if $\alpha\lambda = 1$. Moreover, if $\alpha\lambda < 1$, $x_0$ is too small without unemployment benefits and it moves further away from the optimal level the larger the unemployment benefits are. If, on the other hand, $\alpha\lambda > 1$, there exists a positive level of unemployment benefits at which the levels of tightness in the labor markets are optimal. In the South-East panel of Figure 3 this happens at the intersection point with the horizontal dashed line, where $x_0 = x^H_0$. However, the level of unemployment benefits that secures the optimal levels of labor market tightness does not maximize welfare, because it leaves distortions in the allocation of labor across sectors (the optimal levels of unemployment benefits are depicted in this figure by the dashed vertical lines). If, for example, the differentiated sector has a higher rate of unemployment than the homogeneous sector, then it is optimal to raise $b_u$ above the level

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See the Appendix for more details.
That maximizes the net output of homogeneous goods, because this would attract more workers to the differentiated sector and thereby partially offset the monopolistic distortion that reduces the size of the differentiated sector. Evidently, since this economy has multiple distortions, multiple instruments are needed to attain efficiency. These instruments are discussed in the next section.

5.4 Optimal Policies

We now consider policies that implement a constrained Pareto optimal allocation. The objective is to maximize the joint welfare of countries $A$ and $B$, which—in view of the utility function (12)—is given by

$$
\sum_{j=A,B} \left( q_{0j} + \frac{1}{\eta} Q_j^{\eta} \right).
$$

The constraint is that the planner can allocate workers to industries but not to firms. However, the planner can post vacancies for every firm and thereby determine the probability with which vacancies are filled in every industry.

The Appendix contains an explicit formulation and solution to the planner’s problem. This solution satisfies the labor market tightness conditions (17) and (19) in every country, for the reasons explained in the previous section. Therefore, if $\alpha \lambda = 1$, no intervention is required in the labor markets, despite the fact that the frictions $\zeta_0$ and $\zeta$ differ across countries. If, however, $\alpha \lambda \neq 1$, then it is necessary to design labor market policies in the country in which the Hosios condition is not satisfied in order to implement the optimal allocation. Importantly, a country’s optimal labor market policies depend only on its labor market parameters $\alpha$ and $\lambda$. A direct policy that eliminates the labor market distortions is a subsidy or tax to the hiring cost, which is equivalent to a subsidy or tax to the cost of posting vacancies. When the subsidy rate to hiring in the homogeneous sector is $s_{b0}$ (possibly negative), the resulting tightness in this labor market

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9 The impact of unemployment benefits on the relative size of sectors is similar in our case to Acemoglu and Shimer’s (1999) analysis of policies that maximize output, except that in their case the distortion results from the reluctance of risk-averse workers to search for jobs in high-unemployment sectors. In their case aggregate output is too small without policy intervention, while unemployment benefits encourage workers to take the risk of searching for jobs thereby raising output.

10 In the main text we assume that the relative bargaining weight $\lambda$ is the same in both sectors, although it may vary across countries. In the Appendix we allow $\lambda$ to vary across sectors also.

11 Note that a correction of this distortion requires a labor market policy that encourages job creation through lower costs of matching. For example, it cannot be a direct employment subsidy, because this policy does not reduce the matching costs to firms. Moreover, an employment subsidy is equivalent to a subsidy to the firms’ revenues.
satisfies

\[(1 - s_{bo}) \xi_0 x_0^{\alpha} = \frac{\lambda}{1 + \lambda}.\]

Comparing this condition to (19) we see that \(x_0\) is optimal if and only if

\[s_{bo} = \frac{1 - \alpha \lambda}{1 + \lambda}.\]

Evidently, hiring has to be subsidized in the homogeneous sector when \(\alpha \lambda < 1\) and taxed when \(\alpha \lambda > 1\). For \(\alpha \lambda < 1\), firms post too few vacancies and labor market tightness is too low. The required hiring subsidy is decreasing in the relative weight of job-seekers in the matching technology, \(\alpha\), and in the relative weight of the employer in wage bargaining, \(\lambda\). For \(\alpha \lambda > 1\), firms post too many vacancies and labor market tightness is too high, which implies that a hiring tax is required. In this latter case, optimal labor market tightness can also be achieved with unemployment benefits, but unemployment benefits cannot correct the labor market distortion when \(\alpha \lambda < 1\). A similar labor market policy is required in the differentiated sector, with the rate of subsidy the same in the two sectors: \(s_b = s_{bo}\).

With the optimal labor market subsidies in place, there are no remaining distortions in labor markets. But the relative size of the two sectors is not optimal. To correct the distortions in the relative size of the two sectors the planner can subsidize sales of the differentiated product. A subsidy of \(s_r\) per unit of sales in terms of the numeraire raises the revenue of every manufacturer, and the optimal subsidy is

\[s_r = \frac{1 - \beta}{\beta} \frac{\lambda}{1 + \lambda}.\]

The first term on the right hand side, \((1 - \beta) / \beta\), represents the subsidy that offsets the distortion that results from the markup of price over the marginal cost (the monopolistic distortion), while the second term, \(\lambda / (1 + \lambda)\), represents the subsidy that offsets the distortion that results from wage bargaining. The total subsidy is increasing in the relative weight of employers in wage bargaining (\(\lambda\)) and decreasing in the elasticity of substitution between varieties of the differentiated product (\(\beta\)). A higher elasticity of substitution reduces the manufacturers’ market power and thereby their markups above marginal costs, which leads to expansion of output and employment and implies that

\[12\text{In this case } b_u < 0 \text{ is required, which means taxing the unemployed.}\]
a lower subsidy to sales is required. The relative weight of employers in wage bargaining affects the value of the subsidy, because in the Stole-Zwiebel bargaining game firms have an incentive to hire more workers than is socially optimal in order to reduce the wage paid to infra-marginal workers, i.e., this bargaining mechanism leads to overemployment. This overemployment distortion partially offsets the effect of monopolistic pricing that makes the differentiated sector too small. The subsidy increases with the relative weight of employers in wage bargaining, because a larger value for this weight reduces the overhiring distortion, and hence reduces the size of the differentiated sector, which implies that a larger subsidy is required to restore the size of the differentiated sector to its socially optimal level.

In addition to the subsidy to sales, in the differentiated sector the fixed costs of production, export, and entry have to be subsidized at a common rate, equal to

$$sf = \frac{1}{1 + \lambda}.$$ 

This subsidy is decreasing in the relative weight of employers in wage bargaining ($\lambda$), because of the overhiring distortion in the Stole-Zwiebel bargaining game discussed above. Note that this subsidy does not depend on $\beta$, because the markup does not distort entry.

Importantly, the same optimal policies in the differentiated sector apply to all firms. In other words, they equally apply to low- and high-productivity firms, and to exporters and nonexporters alike. This means that the optimal policies do not discriminate between firms based on productivity, size, or export status.

We show in the Appendix that the optimal policies in product markets depend on whether subsidies or unemployment benefits are used in the labor market. If the social planner uses unemployment benefits in the labor market, which are feasible when $\alpha \lambda > 1$, then the subsidies to sales and to fixed plus entry costs in the differentiated sector depend not only on $\lambda$ and $\beta$ but also on the frictions in the labor markets, $\zeta_0$ and $\zeta$. For this reason, the optimal policies based on subsidies, discussed above, require less information than policies that rely on unemployment benefits.
6 Conclusion

The impact of trade liberalization on wage inequality and unemployment and the role of labor market institutions in shaping the effects of trade liberalization are areas of intense policy debate. Until recently, the ability of research in international trade to engage with this policy debate has been hampered by the widespread assumption of flexible labor markets and the associated prediction of full employment at a common wage.

In this paper, we have reviewed a new framework that combines firm heterogeneity in the product market with search and matching frictions in the labor market to examine the economy's response to trade. The resulting framework highlights a new mechanism for international trade to affect wage inequality: when only some firms export, the increase in wages that occurs at the productivity threshold for exporting raises wage inequality across firms. This mechanism accounts for the empirical findings of rising wage inequality in both developed and developing countries following trade liberalization and rationalizes rising wage inequality among groups of workers with the same observed characteristics. While the opening of trade can raise social disparity through both higher wage inequality and higher unemployment, expected welfare necessarily rises.

The introduction of labor market frictions into a general equilibrium model of trade permits the study of interdependence in labor market institutions across countries and the analysis of interactions between labor market institutions and trade liberalization. While labor market reforms that reduce search and matching frictions in the differentiated sector increase a country's own welfare, they reduce welfare in its trade partners. The aggregate unemployment rate depends on both the unemployment rate within each sector and the composition of the labor force across sectors. In consequence, policies that reduce the unemployment rate within the differentiated sector need not reduce aggregate unemployment if they also change the composition of the labor force across sectors. One important implication is that relative aggregate unemployment rates across countries are not, in general, fully informative about relative levels of labor market frictions.

In our setting with multiple product and labor market distortions, the market allocation is not constrained efficient. Only if the Hosios condition is satisfied, which requires the relative bargaining weight of employers to equal their relative weight in the matching technology, is the efficient level of labor market tightness attained. More generally, if the Hosios condition is not satisfied, subsidies
to hiring costs or the costs of posting vacancies or unemployment benefits can be used to achieve
the efficient level of labor market tightness. However, with several distortions in product and
labor markets, unemployment benefits alone cannot achieve the constrained efficient allocation and
their introduction can either raise or reduce welfare. To achieve the constrained efficient allocation
requires a combination of these interventions in the labor market and subsidies to revenue and fixed
costs in the product market. Notably, the efficient subsidies in the product market take the same
value for both exporters and nonexporters. Finally, the use of direct subsidies or taxes to hiring
requires less information than unemployment benefits and avoids the limitation that unemployment
benefits have to be non-negative.
Appendix

This appendix sets up the model for Sections 4 and 5 and derives the results reported in the text; the model is based on Helpman and Itskhoki (2010), with the addition of policy instruments. We start by describing the decentralized equilibrium. We then set up the planner’s problem and compare its solution with the decentralized allocation.

A Decentralized Equilibrium

We consider a decentralized equilibrium of the Helpman and Itskhoki (2010) model, allowing for nonsymmetric bargaining power of firms and workers, unemployment benefits, subsidies and taxes to hiring costs, entry costs, fixed production costs, and firm revenues in the differentiated sector.

A.1 Labor market equilibrium

In Helpman and Itskhoki (2010) we showed how a Cobb-Douglas matching function results in the following hiring cost function in the homogeneous-good sector:

\[ b_0 = \zeta_0 x^\alpha_0, \]

where \( \zeta_0 \) is the ratio of vacancy costs to the productivity of the matching technology, \( x_0 \) is the endogenous labor market tightness (equal to the probability of a worker finding a job), and \( \alpha \) is the ratio of the Cobb-Douglas parameters on unemployment and vacancies. In words, \( b_0 \) is the (expected) cost for a homogeneous-good producer of matching with (hiring) one worker. Similarly, the cost of matching in the differentiated sector is \( \zeta x^\alpha \), where we allow \( \zeta \) to differ from \( \zeta_0 \).

Consider the homogeneous-good sector. Upon matching, the firm and the worker produce one unit of the homogeneous good, which is our numeraire. They split this surplus via Nash bargaining. The outside option is zero for the firm and it equals unemployment benefits, \( b_u < 1 \), for the worker. In the process of bargaining, the surplus \( (1 - b_u) \) is divided between the firm and the worker according to their relative bargaining power, which we denote by \( \lambda_0 \). This results in an operating
profit level $\pi_0$ and a wage rate $w_0$:

$$\pi_0 = \frac{\lambda_0}{1 + \lambda_0} (1 - b_u) \quad \text{and} \quad w_0 = b_u + \frac{1}{1 + \lambda_0} (1 - b_u).$$

With free entry, equilibrium profits equal zero, and therefore the operating profits equal hiring costs. That is,

$$\pi_0 = (1 - s_{b_0}) b_0,$$

where $s_{b_0}$ is the hiring-cost subsidy. Combining this with the previous expressions, we obtain

$$(1 - s_{b_0}) \zeta_0 x_0^\alpha = \frac{\lambda_0}{1 + \lambda_0} (1 - b_u),$$

which reduces to (16) when $s_{b_0} = b_u = 0$ and $\lambda = \lambda_0$. Finally, the expected income of workers in the homogeneous-good sector is given by

$$\omega_0 \equiv x_0 w_0 + (1 - x_0) b_u$$
$$= b_u + x_0 \frac{1}{1 + \lambda_0} (1 - b_u)$$
$$= b_u + \frac{1}{\lambda_0} (1 - s_{b_0}) \zeta_0 x_0^{1+\alpha}.$$

Next consider the differentiated sector. We show below that the equilibrium wage in this sector is

$$w = b_u + \frac{1}{\lambda} (1 - s_b) \zeta x^\alpha,$$

where $\lambda$ is the relative bargaining power of firms in this sector, $s_b$ is the sector-specific hiring-cost subsidy to firms, $\zeta$ is the sectoral labor market friction parameter (i.e., the ratio of vacancy costs to the productivity level of the matching technology), and $x$ is the sector’s labor market tightness (equal to the matching probability for workers). Therefore, a worker’s expected income in the differentiated sector is

$$\omega \equiv xw + (1 - x)b_u = b_u + \frac{1}{\lambda} (1 - s_b) \zeta x^{1+\alpha}.$$

In equilibrium workers have to be indifferent between searching for jobs in the homogeneous or
differentiated sectors, which requires $\omega_0 = \omega$. The latter implies

$$\frac{1}{\lambda_0} (1 - s_b) \zeta_0 \omega_0^{1+\alpha} = \frac{1}{\lambda} (1 - s_b) \zeta x^{1+\alpha}. \quad (21)$$

Naturally, (17) is a special case of this condition, for $\lambda_0 = \lambda$ and no hiring subsidies. Also note that as long as unemployment benefits are common to the unemployed in both sectors, they do not affect relative labor market tightness in the two sectors. Conditions (20) and (21) pin down labor market tightness in the two sectors.

A.2 Product market equilibrium

In the differentiated sector firms solve the following maximization problem:

$$\pi(\Theta) = \max_{h \geq 0, I_x \in \{0,1\}} \left\{ (1 + s_r) R(h, \Theta) - w(h, \Theta) h - (1 - s_b) \zeta x^\alpha h - (1 - s_d) f_d - I_x (1 - s_x) f_x \right\},$$

where $h$ is employment, $I_x$ is the firm’s export status indicator, $\Theta \equiv \theta^{\beta/(1-\beta)}$ is a measure of the firm’s productivity, $s_r$ is the revenue subsidy rate, $\zeta x^\alpha$ is the hiring (matching) cost per worker, $s_b$ is the subsidy rate to hiring costs, $f_d$ is the fixed cost of production, $f_x$ is the fixed cost of exporting, $s_d$ is the subsidy rate to the fixed cost of production, and $s_x$ is the subsidy rate to the fixed cost of exporting. As shown in Helpman and Itskhoki (2010), the revenue function of a firm in country $j$ is

$$R(h, \Theta) = \left[ Q^{-\frac{\beta - n}{1-\beta}} I_x \cdot \tau^{-\frac{n}{1-\beta}} Q^{-\frac{\beta - n}{1-\beta}}_{(\cdot j)} \right]^{1-\beta} \Theta^{1-\beta} h^\beta,$$

where $(\cdot j)$ denotes the foreign country and we drop the subscript $j$ from country $j$’s variables. Importantly, revenue is a power function of employment.

Wages are set via bargaining over revenue between the firm and its workers. At the bargaining stage, entry costs, the export status, the fixed costs of production and export, and the hiring costs, are all sunk. We adopt Stole and Zweibel’s (1996) bargaining game, which implies that the wage function satisfies the following differential equation:

$$\frac{\partial}{\partial h} \left[(1 + s_r) R(h, \Theta) - w(h, \Theta) h\right] = \lambda \left[w(h, \Theta) - b_w\right].$$
In words, the incremental surplus of the firm from an additional worker equals the surplus of the worker weighted by the firm’s relative bargaining power. This differential equation has the following solution:

\[ w(h, \Theta) = \frac{\beta}{\beta + \lambda} \frac{(1 + s_r)R(h, \Theta)}{h} + \frac{\lambda}{1 + \lambda} b_u. \]

Anticipating this bargaining outcome, the firm’s profit maximization problem becomes

\[
\pi(\Theta) = \max_{h \geq 0, f \in (0, 1)} \left\{ \frac{\lambda}{\beta + \lambda} (1 + s_r)R(h, \Theta) - bh - (1 - s_d)f_d - I_x(1 - s_x)f_x \right\},
\]

where its effective hiring cost is

\[ b = (1 - s_b)\zeta x^\alpha + \frac{\lambda}{1 + \lambda} b_u. \]

The solution of the firm’s problem can now be characterized in the following way. Optimal employment can be expressed as

\[ h(\Theta) = h_d(\Theta) + I_x(\Theta)h_x(\Theta), \]

where

\[
h_d(\Theta) = \left( \frac{\beta \lambda}{\beta + \lambda} \right) \frac{\alpha}{1 - \beta}(1 + s_r) \frac{1}{1 - \beta} \left[ (1 - s_b)\zeta x^\alpha + \frac{\lambda}{1 + \lambda} b_u \right]^{-\frac{1}{1 - \beta}} Q^{-\beta - \eta} \Theta, \tag{22}
\]

\[
h_x(\Theta) = \left( \frac{\beta \lambda}{\beta + \lambda} \right) \frac{\alpha}{1 - \beta}(1 + s_r) \frac{1}{1 - \beta} \left[ (1 - s_b)\zeta x^\alpha + \frac{\lambda}{1 + \lambda} b_u \right]^{-\frac{1}{1 - \beta}} \tau^{-\beta} Q^{\beta - \eta} \Theta. \tag{23}
\]

Here \( h_d(\Theta) \) represents employment needed to supply the home market while \( h_x(\Theta) \) represents employment needed to supply the foreign market. The profit level can be similarly decomposed into profits from domestic sales and profits from export sales, \( \pi(\Theta) = \pi_d(\Theta) + I_x(\Theta)\pi_x(\Theta) \), where

\[
\pi_d(\Theta) = \frac{1 - \beta}{\beta} \frac{\beta \lambda}{\beta + \lambda} (1 + s_r) Q^{-(\beta - \eta)} \Theta^{1-\beta} h_d(\Theta)^{\beta} - (1 - s_d)f_d,
\]

\[
\pi_x(\Theta) = \frac{1 - \beta}{\beta} \frac{\beta \lambda}{\beta + \lambda} (1 + s_r) \tau^{-\beta} Q^{-(\beta - \eta)} \Theta^{1-\beta} h_x(\Theta)^{\beta} - (1 - s_x)f_x.
\]

See Helpman and Itskhoki (2010) for more detail. An immediate implication of (22)-(23) is that
the solution to the wage-bargaining problem yields the equilibrium wage:

\[ w = \frac{1}{\lambda} b + \frac{\lambda}{1+\lambda} b_u \]

\[ = b_u + \frac{1}{\lambda} (1 - s_b) \zeta x^\alpha, \]

which is the same for all firms, independently of export status or productivity.

A firm’s decisions as to whether to stay in the industry and whether to export can be characterized by two cutoff productivity levels \( \Theta_d \) and \( \Theta_x \), which are implicitly defined by \( \pi_d(\Theta_d) = 0 \) and \( \pi_x(\Theta_x) = 0 \). That is, firms with productivity below \( \Theta_d \) exit, firms with productivity \( \Theta \in [\Theta_d, \Theta_x) \) serve the domestic market only, and firms with productivity above \( \Theta_x \) serve both the domestic and export markets. The two conditions \( \pi_d(\Theta_d) = 0 \) and \( \pi_x(\Theta_x) = 0 \) can be expressed as

\[
\frac{1 - \beta}{\beta} \left( \frac{\beta \lambda}{\beta + \lambda} (1 + s_r) \right)^{\frac{1}{1-\beta}} \left[ (1 - s_b) \zeta x^\alpha + \frac{\lambda}{1+\lambda} b_u \right]^{\frac{\beta}{1-\beta}} Q^{\frac{\beta-\gamma}{1-\beta}} \Theta_d = (1 - s_d) f_d, \quad (24)
\]

\[
\frac{1 - \beta}{\beta} \left( \frac{\beta \lambda}{\beta + \lambda} (1 + s_r) \right)^{\frac{1}{1-\beta}} \left[ (1 - s_b) \zeta x^\alpha + \frac{\lambda}{1+\lambda} b_u \right]^{\frac{\beta}{1-\beta}} \tau^{\frac{\beta-\gamma}{1-\beta}} Q^{\frac{\beta-\gamma}{1-\beta}} (-j) \Theta_x = (1 - s_x) f_x. \quad (25)
\]

Finally, free entry requires the entry cost net of the entry subsidy to equal expected profits from domestic and export sales:

\[
\int_{\Theta_d}^{\infty} \pi_d(\Theta) dG(\Theta) + \int_{\Theta_x}^{\infty} \pi_x(\Theta) dG(\Theta) = (1 - s_e) f_e.
\]

Using the expressions for optimal profits and cutoffs, this condition can be expressed as

\[
(1 - s_d) f_d \int_{\Theta_d}^{\infty} \left( \frac{\Theta}{\Theta_d} - 1 \right) dG(\Theta) + (1 - s_x) f_x \int_{\Theta_x}^{\infty} \left( \frac{\Theta}{\Theta_x} - 1 \right) dG(\Theta) = (1 - s_e) f_e. \quad (26)
\]

Conditions (24)-(26) characterize product market equilibrium in the home country. Specifically, given \( x \) and \( Q(-j) \), they allow us to solve for \((\Theta_d, \Theta_x, Q)\) for country \( j \) (recall that we have dropped the index \( j \) from the home country’s variables). Similar conditions describe the foreign country’s product market equilibrium. Jointly, the two countries’ equilibrium conditions allow us to solve the cutoffs and real consumption indexes \((\Theta_{dj}, \Theta_{xj}, Q_j)\) and \((\Theta_{d(-j)}, \Theta_{x(-j)}, Q_{(-j)})\).

Finally, conditions (20)-(26), together with the parallel conditions for the foreign country, de-
scribe the decentralized equilibrium allocation for the world economy, given labor and product market policies in the two countries. In this equilibrium the governments are assumed to have access to lump-sum taxes and transfers in order to finance their policies. Under the circumstances we need not worry about the government’s budget constraint as long as there is positive consumption of the homogeneous good, which is assured when $L$ is large enough.

B Optimal Policies

The world planner’s problem—constrained by search frictions—can be formulated as follows:

$$\max_{\{x_{0j}, x_j, \Theta_{dj}, \Theta_{xj}, M_j, h_{dj}() , h_{xj}()\}_{j=A,B}} \sum_{j=A,B} \left( q_{0j} + \frac{1}{\eta} Q^\beta_j \right),$$

where

$$Q^\beta_j = M_j \int_{\Theta_{d}} \Theta^{1-\beta} h_{dj}(\Theta)^\beta dG(\Theta) + M_{(-j)} \int_{\Theta_{x(-j)}} \tau^{-\beta}(\Theta)^{1-\beta} h_{x(-j)}(\Theta)^\beta dG(\Theta),$$

$$q_{0j} = x_{0j}(L_j - H_j/x_j)(1 - \zeta_{0j} x_{0j}^{\alpha_j}) - \zeta_j x_j^{\alpha_j} H_j - M_j \left[ f_e + f_d(1 - G(\Theta_{dj})) + f_x(1 - G(\Theta_{xj})) \right],$$

$$H_j = M_j \int_{\Theta_{d}} h_{dj}(\Theta)dG(\Theta) + M_j \int_{\Theta_{xj}} h_{xj}(\Theta)dG(\Theta),$$

and $M_j$ denotes the measure of differentiated product firms that enter in country $j$, and the other variables have been previously defined. The equation for $Q_j$ comes from the CES aggregator once we notice that $\Theta^{1-\beta} h_{dj}(\Theta)^\beta = q_{dj}(\Theta)^\beta$, where $q_{dj}(\Theta)$ is consumption of a home variety produced in country $j$ by a firm with productivity $\Theta$, and similarly for imported varieties. The last term on the right hand side of the expression for $q_{0j}$ represents total entry, production, and export fixed costs in terms of the homogeneous good, where $1 - G(\Theta_{d})$ is the fraction of surviving firms and $1 - G(\Theta_{x})$ is the fraction of exporting firms out of all entrants. The second term on the right hand side is the total hiring cost paid in the differentiated sector, where $H_j$ is the total number of matches (employment) in this sector and $\zeta_j x_j^{\alpha_j}$ is the search cost per match. Finally, the first term is the output of homogeneous goods less search costs in the homogeneous-good sector (see the main text for a detailed discussion of the first two terms on the right hand side of the equation for $q_{0j}$).

Consider the planner’s optimal allocation in a world of two symmetric countries (most of the
following results generalize to a world of asymmetric countries). In this case we need to consider the optimality conditions for \((x_0, x, h_d(\cdot), h_x(\cdot), \Theta_d, \Theta_x, M)\), which are common to both countries. The first-order conditions for this problem yield\(^\text{13}\)

\[
\begin{align*}
\zeta_0 x_0^\alpha &= \frac{1}{1 + \alpha}, \\
\zeta x^{1+\alpha} &= \zeta_0 x_0^{1+\alpha}, \\
h_d(\Theta) &= \left(\frac{\zeta}{\zeta_0}\right)^{\frac{1}{1+\alpha} - \frac{1}{\alpha}} Q^{-\frac{\beta-\eta}{1-\beta}} \Theta, \\
h_x(\Theta) &= \left(\frac{\zeta}{\zeta_0}\right)^{\frac{1}{1+\alpha} - \frac{1}{\alpha}} \tau^\frac{\beta-\eta}{1-\beta} Q^{-\frac{\beta-\eta}{1-\beta}} \Theta, \\
\frac{1 - \beta}{\beta} \left(\frac{\zeta}{\zeta_0}\right)^{\frac{1}{1+\alpha} - \frac{1}{\alpha}} Q^{-\frac{\beta-\eta}{1-\beta}} \Theta_d &= f_d, \\
\frac{1 - \beta}{\beta} \left(\frac{\zeta}{\zeta_0}\right)^{\frac{1}{1+\alpha} - \frac{1}{\alpha}} \tau^\frac{\beta-\eta}{1-\beta} Q^{-\frac{\beta-\eta}{1-\beta}} \Theta_x &= f_x, \\
f_d \int_{\Theta_d}^{\infty} \left(\frac{\Theta}{\Theta_d} - 1\right) dG(\Theta) + f_x \int_{\Theta_x}^{\infty} \left(\frac{\Theta}{\Theta_x} - 1\right) dG(\Theta) &= f_e.
\end{align*}
\]

Equations (20\(P\))-\(26\(P\)) characterize the planner’s allocation, and they are a direct counterpart to the decentralized equilibrium system (20)-(26). To characterize optimal policies, we simply need to find policy parameters \((b_u, s_b, s_b, s_r, s_d, s_x, s_e)\) that implement the planner’s allocation \((x_0, x, h_d(\cdot), h_x(\cdot), \Theta_d, \Theta_x, Q)\) as a decentralized equilibrium, i.e., policies with which the solution of

\[\text{Equation (20\(P\)) obtains from the first-order condition with respect to } x_0; \text{ (21\(P\)) obtains from the first-order condition with respect to } x, \text{ combined with (20\(P\)). Equations (22\(P\))-\(23\(P\)) obtain from the first-order conditions with respect to } h_d(\cdot) \text{ and } h_x(\cdot) \text{ after substituting in } \]

\[
\zeta x_0^\alpha + (1 - \zeta_0 x_0^\alpha) \frac{x_0}{x} = \left(\frac{\zeta}{\zeta_0}\right)^{\frac{1}{1+\alpha}},
\]

which is implied by (20\(P\))-\(21\(P\)). We can use the above equation in similar fashion to derive the other conditions. Equations (24\(P\))-\(25\(P\)) obtain from the first-order conditions with respect to \(\Theta_d\) and \(\Theta_x\) after substituting in the expressions for \(h_d(\Theta_d)\) and \(h_x(\Theta_x)\) from (22\(P\))-\(23\(P\)). Equation (26\(P\)) obtains through manipulation of the first-order condition with respect to \(M\), which can be written as

\[
\frac{1}{\beta} Q^\eta - \left(\frac{\zeta}{\zeta_0}\right)^{\frac{1}{1+\alpha}} H = \frac{1}{\beta} M \{f_x + f_d [1 - G(\Theta_d)] + f_x [1 - G(\Theta_x)]\}.
\]

Next substitute (22\(P\))-\(23\(P\)) into the definition of \(Q\) and \(H\) which implies

\[
Q^\eta = \left(\frac{\zeta}{\zeta_0}\right)^{\frac{1}{1+\alpha}} H = M \left(\frac{\zeta}{\zeta_0}\right)^{\frac{1}{1+\alpha} - \frac{1}{\alpha}} Q^{-\frac{\beta-\eta}{1-\beta}} \left[\int_{\Theta_d}^{\infty} \Theta dG(\Theta) + \tau^{-\frac{\beta-\eta}{1-\beta}} \int_{\Theta_x}^{\infty} \Theta dG(\Theta)\right].
\]

Finally, combine the above two expressions with (24\(P\))-\(25\(P\)) to obtain (26\(P\)).
(20)-(26) coincides with the solution of (20P)-(26P).\footnote{We replace $M$ with $Q$ in the final equations, because it is simpler to discuss allocations in these terms.}

**B.1 Optimal policy with hiring-cost subsidies**

We first consider the case without unemployment benefits ($b_u = 0$), but with hiring-cost subsidies ($s_{b_0}, s_b$) used to offset distortions in the labor market. Comparing (20)-(26) with (20P)-(26P), we obtain the following characterization of the optimal policy:

$$s_{b_0} = \frac{1 - \alpha \lambda_0}{1 + \lambda_0}, \quad s_b = \frac{1 - \alpha \lambda_0}{1 + \lambda_0} - \frac{1 + \alpha}{1 + \lambda_0} (\lambda - \lambda_0),$$

$$s_r = \frac{1 - \beta \lambda / \lambda_0 - \beta \lambda_0}{\beta(1 - \beta) + \beta + \lambda}, \quad s_d = s_x = s_e = \frac{1}{1 + \lambda_0}.$$

The policies simplify when the firm’s relative bargaining power is the same in the two sectors. In this case, we have

$$s_{b_0} = s_b = \frac{1 - \alpha \lambda}{1 + \lambda}, \quad s_r = \frac{1 - \beta \lambda}{\beta(1 + \lambda)}, \quad s_d = s_x = s_e = \frac{1}{1 + \lambda}.$$

This corresponds to the expression in Section 5.4 where we provide interpretation.

**B.2 Optimal policy with unemployment benefits**

We now consider the case when hiring-cost subsidies are unavailable, and the government uses unemployment benefits in order to offset labor market distortions. As long as unemployment benefits are common in the two sectors, it would be impossible to decentralize the planner’s allocation when $\lambda \neq \lambda_0$. We therefore consider the case with $\lambda_0 = \lambda$. In this case the comparison of (20)-(26) and (20P)-(26P) yields the following optimal policies:

$$b_u = \frac{\alpha \lambda - 1}{(1 + \alpha) \lambda},$$

$$s_r = \frac{1 - \beta \lambda}{\beta(1 + \lambda)} - \frac{\alpha \lambda - 1}{(1 + \alpha)(1 + \lambda)} \left[ 1 - \left( \frac{\zeta}{\zeta_0} \right)^{\frac{1}{1+\alpha}} \right],$$

$$s_d = s_x = s_e = \frac{1}{1 + \lambda} - \frac{\alpha \lambda - 1}{(1 + \alpha)(1 + \lambda)} \left[ 1 - \left( \frac{\zeta}{\zeta_0} \right)^{\frac{1}{1+\alpha}} \right].$$
In comparison to direct hiring subsidies, unemployment benefits attract more workers to the sector with the higher labor market frictions and therefore with the higher rate of unemployment. This effect needs then to be neutralized by the adjustment of the optimal product market policies, as the above equations demonstrate.

B.3 Single instrument: unemployment benefits

From the above discussion it is clear that the constrained efficient allocation is not feasible when the countries can use unemployment benefits as the only policy instruments. Therefore, the method used above to characterize optimal policies no longer applies. For this reason we directly search for the level of unemployment benefits that maximizes world welfare in a decentralized equilibrium.

Consider again a world of symmetric countries. The indirect utility function for a country is given by

$$V = E + \frac{1 - \eta}{\eta} Q^\eta,$$

where $E$ is disposable household income (for details see Helpman and Itskhoki, 2010). Unemployment benefits are financed by a lump-sum tax $T$ on households, so that disposable income is

$$E = \omega_0 \bar{L} - T,$$

where $\omega_0 = x_0 b_0$ is expected income in the economy. Finally, the government budget constraint can be written as lump-sum taxes equal total spending on unemployment benefits, or

$$T = b_u \left[ (1 - x_0)(\bar{L} - N) + (1 - x) N \right]. \quad (27)$$

Therefore, we numerically maximize welfare $V$ with respect to $b_u$,

$$\max_{b_u} \left\{ x_0 b_0 \bar{L} - T + \frac{1 - \eta}{\eta} Q^\eta \right\},$$

subject to the government budget constraint (27), and with $(b_0, x_0, x, Q, N)$ determined as functions
of $b_u$ from the equilibrium system (20)-(26).\footnote{Note that $N = H/x$ and
\[
H = \frac{\beta \lambda}{\beta + \lambda} \left( \zeta x^\alpha + \frac{\lambda}{1 + \lambda} b_u \right)^{-1} Q' \cdot
\]
To see this, note from (22)-(23) that $h(\Theta) = \frac{\beta \lambda}{\beta + \lambda} \left( \zeta x^\alpha + \frac{\lambda}{1 + \lambda} b_u \right)^{-1} R(\Theta)$ when $s_r = s_b = 0$. Furthermore, aggregate revenue equals $Q' = M \int R(\Theta) dG(\Theta)$, and aggregate employment equals $H = M \int h(\Theta) dG(\Theta)$. For more details see Helpman and Itskhoki (2010).}
References


