THE TENNIS COACH PROBLEM: A GAME-THEORETIC AND EXPERIMENTAL STUDY

AYALA ARAD

7-08

SEPTEMBER, 2008

THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH
AND
THE SACKLER INSTITUTE OF ECONOMIC STUDIES

The Tennis Coach Problem:
A Game-Theoretic and Experimental Study

Ayala Arad*
Tel Aviv University

September 2008

Abstract
The paper introduces a new allocation game, related to the Colonel Blotto game: each tennis coach assigns his four different skilled players to four positions, and then each team plays all other teams in the tournament.

The set of equilibria is characterized and experimental behavior in variants of the game is analyzed in light of an adapted level-*k* model. The results exhibit a systematic pattern- a majority of the subjects used a small number of strategies. However, although level-*k* thinking is naturally specified in this context, only a limited use of low level-*k* thinking was found. Thus, the results illuminate some bounds of the level-*k* approach.

JEL classification: C72, C91
Keywords: level-*k* thinking, tennis game, experimental game theory, colonel Blotto.

* I wish to thank Chaim Fershtman and Ariel Rubinstein for their helpful comments and Eli Zvuluny for his help in programming the experiments.

1. Introduction

This paper introduces a new allocation game called the Tennis Coach problem, which captures the essence of some interesting strategic interactions observed in competitive environments. The game is analyzed both theoretically and experimentally and serves as a platform for studying iterated reasoning and non-equilibrium models based on this concept.

The Tennis Coach problem

Consider a tournament in which each participant plays the role of a tennis coach who is planning to send his team to the tournament. Each team consists of four players with four different skill levels: A+, A, B+ and B, where A+ is the highest level and B is the lowest. The coach's task is to assign his players to positions 1, 2, 3 and 4 (one player to each position). Each team plays against each of the other teams in the tournament.

A battle between two teams includes four matches: a player that was assigned by his coach to a particular position plays once against the player on the other team assigned to the same position. In any match between two players of different levels, the one with the higher level wins and scores one point for his team. When two players with the same level play against each other, the outcome is a tie and each team receives half a point. Thus, a battle between two teams ends with one of the teams winning 3:1 or 2.5:1.5, or in a tie of 2:2. The team's score at the end of the tournament is the total number of points it received in all the battles. The goal of the coaches is to win the tournament, i.e. to achieve the highest score among all the teams.

The strategic interaction between the coaches will be referred to as "the tennis game".

Theoretical motivation

The tennis game is of interest primarily because it is an intuitively appealing version of the popular Colonel Blotto game, introduced by Borel (1921). In the Colonel Blotto game, two players simultaneously allocate a fixed number of troops to N battlefields. A player wins a battle if the number of troops he assigns to a particular battlefield is higher than that assigned by his opponent and whoever wins more battles is the winner of the game.[1]

[1] A number of papers have analyzed the game on a theoretical level though there is still no complete characterization of equilibrium in the continuous case. See some recent progress in Weinstein (2005) and

The game has been widely interpreted as a competition between two players, in which each distributes his limited resources across N tasks and succeeds in a task if he assigns more resources to it than his opponent. A well-known application of the game involves the interaction between vote-maximizing parties in an election campaign, in which the promises made by the parties are modeled as the various ways to divide a homogeneous good and are assumed to determine the outcome of the election. The basic idea is that an individual votes for party X if it has promised him more than party Y and the party that receives more votes wins the election. This scenario could also be interpreted as vote-buying.[2]

Whereas in the Blotto game all partitions (and in some versions only discrete partitions) of the total resources are possible, in the tennis game a player is restricted to a finite number of allocations. This does not make the tennis game a special case of the Blotto game, but rather a different and somewhat simpler version, yet one which captures much of its strategic spirit. Moreover, in many cases, the tennis game reflects more realistic assumptions than the Blotto game. For example, a general might not be able to assign any number of troops to a single battlefield and may be restricted by the internal organization of his army to assigning one division to each battlefield, where the divisions differ in ability and strength. More generally, the tennis game is better suited to competitive scenarios in which human resources are allocated among several tasks.

The tennis game is also able to capture the interaction in the campaign promises game, in which promises are made in the form of a list of priorities (an ordering of projects) that a candidate guarantees to adhere to after being elected. If different projects are associated with different groups (each with equal voting power) then declaring the list of priorities is equivalent to the problem of the tennis coach.

The tennis game is also related to the game discussed in Fershtman and Rubinstein (1997), in which a treasure is hidden in one of N locations. Each of two players tries to be the first to reach the location of the treasure. Each player *i* has resources to sequentially search N_i locations and must choose the order in which to conduct the search (to specify which location he searches in each date). In the case of N_i=N, choosing the order of the

---

Roberson (2006). Hart (2007) provides a complete characterization of equilibrium in the discrete case. I am not aware of experimental studies of the game, apart from a comment by Jonathan Partington (see http://www.geocities.com/j_r_partington/blotto.html) and an unpublished work by Ariel Rubinstein.

[2] See variants of the promises game in Myerson (1993), Laslier and Picard (2002) and Dekel, Jackson and Wolinsky (2008).

search is equivalent to allocating N players to N positions in the tennis game.[3] When two players search according to their ordering, then the probability that a player will find the treasure is equivalent to the number of points earned by a team in the tennis game. Note that Fershtman and Rubinstein did not analyze the set of equilibria in this game. The game with the pair (N_i, N_j) is used as a second stage once each player *i* has chosen N_i in the first stage.

In the spirit of this analogy, the tennis game can be interpreted as an R&D race, in which each firm chooses the order of the routes it will follow in trying to solve a particular problem or the order of the projects it will undertake. This interpretation is appropriate for cases in which each of the various projects or routes is equally promising.

Calculating the value of the symmetric tennis game is straightforward. However, equilibrium payoffs tell us only part of the story and as in the game-theoretic analysis of other strategic scenarios, the interaction is explored by studying the set of equilibria. Characterizing the set of equilibria in the game is quite involved and relies on its special structure, in which any pure strategy has a unique "best response"[4] and the best response function induces a partition of the game's 24 strategies into 6 cycles of 4 strategies each (within a cycle, each strategy is the best response to the preceding strategy in that cycle). The characterization yields some interesting results. For example, it will be shown that the simplest mixed strategy equilibrium (simple in terms of number of strategies in the support of the equilibrium strategies) involves the use of two pure strategies, with the property that each is the best response to the best response of the other strategy.

The game-theoretic analysis of the tennis game ignores the existence of a focal strategy, in which players are allocated according to their correct ranking, and the induced framing effect. Note that the tennis game can capture circumstances in which N different levels of resources need to be distributed across N **labeled** tasks. The labeling may result in task *i* being differentiated from task *j* psychologically. The theoretical analysis is not affected by labeling, as long as the labels do not affect players' payoffs. In the R&D race, for instance, the labels might reflect the **perceived** differences in the attractiveness of the various routes that can be taken. The framing of the case in which the probability to

[3] The players' skill levels are analogous to the search schedule. For example, assigning A+ to the second position in the tennis game is equivalent to searching first in location 2.

[4] A strategy *S* is the "best response" to the strategy *T* if *S* achieves the highest possible score (3 points) when playing against *T*.

succeed in route i is presumed to be slightly higher than in route i+1 resembles the framing in the tennis game.

Experimental motivation

The tennis game's special structure and its psychological properties, which are a result of the existence of a focal strategy, call for addressing solution concepts other than equilibrium, which are based on iterative reasoning. In particular, I discuss the concept of level-*k* thinking which has recently become increasingly popular.[5]

Level-*k* non-equilibrium models assume that the population of players consists of several types, each of which follows a different decision rule. *L0* is a non-strategic type who chooses his action naively by following a particular rule of behavior that depends on the context and is determined by the modeler. *L1* best responds to the belief that all other players are *L0*, *L2* best responds to the belief that all other players are *L1*, and so on. Thus, a type *Lk*, for k>0, is behaving rationally in the sense that he best responds to his belief regarding other players' actions. However, the belief held by *Lk* is not the "correct" belief as required by Nash equilibrium. Level-*k* models were first introduced by Stahl and Wilson (1994, 1995) and Nagel (1995). Since then, they have been developed extensively and used to explain experimental results in a variety of settings. For example, Crawford and Iriberri (2007b) apply the model to explain behavior in auctions.[6]

Papers that use level-*k* models to explain experimental results usually estimate the frequency of each type in a particular context. The appeal of this approach is due to a finding stated clearly in Crawford and Iriberri (2007b, page 1725): "The estimated distribution tends to be stable across games, with most of the weight on *L1* and *L*2. Thus the anchoring *L0* type exists mainly in the minds of higher types."

When analyzing experimental results using a level-*k* approach, one of the principal tasks is to reasonably specify the behavior of *L0* in that particular context. Often (though not always) *L0* is taken to be a uniform randomization over the strategy space. In the tennis game, the specification of *L0* is intuitively appealing due to the existence of a

[5] The term "iterated reasoning" is usually associated with "iterated dominance", although the term is more general and describes a process in which a player applies arguments recursively. In this paper, I do not discuss iterated elimination of dominated strategies since there are no dominated strategies in the tennis game. Thus, throughout the paper I refer to level-*k* thinking as "iterated reasoning".

[6] Some other examples are: Ho, Camerer and Weigelt (1998), Costa-Gomes, Crawford, and Broseta (2001), Crawford (2003), Camerer, Ho, and Chong (2004) and Costa-Gomes and Crawford (2006).

salient strategy or focal point (A+, A, B+, B), which is the natural starting point for iterated reasoning.[7] Decision rules based on level-*k* reasoning are expected to be reflected in subjects' choices also because, given this anchor (starting point), best responding to an *Lk* type is cognitively simple (as I confirmed experimentally). Furthermore, compared to many other level-*k* models, the adapted model in the tennis game assumes weaker and more plausible assumptions on subjects' beliefs. Thus, as will be shown in Section 2.4, the typical choice of *Lk* is not only optimal given the belief that **all** (or almost all) other subjects are *Lk-1* types, but is also the best response to the belief that the **majority** of subjects are *Lk-1*, or to the belief that the **most frequent** type is *Lk-1* and that the rest of the choices are uniformly distributed.

Since level-*k* types are naturally specified in the tennis game, the level-*k* approach appears to be suitable *a priori*. On the other hand, the strategy space in the game is large enough and the structure of the game rich enough to leave room for other kinds of decision rules which are not based on iterated reasoning (examples will be discussed at a later stage). Therefore, the tennis game is an ideal platform for testing the extent to which level-*k* models are capable of explaining behavior in novel settings.

As expected, experimental behavior in the one-shot game was not consistent with any equilibrium predictions. The adapted model of level-*k* reasoning explained only some of the behavior in the tennis game. Patterns based on iterated reasoning were indeed found, but most choices seemed to be driven by other kinds of deliberations. The distribution of strategies reflects a low level of reasoning – even the first step of iterated reasoning was not very common and the second and higher steps were almost totally absent. These frequencies are much lower than those reported in the literature for the parallel steps in other games. The findings are also supported by the results obtained using other experimental techniques, i.e. recording subjects' response time and requesting that subjects provide ex-post explanations of their decisions.

[7] The non-strategic type *L0* in this paper resembles the truthful *L0* type in Crawford and Iriberri (2007b), who bids the value that his own private signal suggests. The specification of *L0* in the tennis game also has features in common with the specification in Crawford and Iriberri (2007a), which adopts a level-*k* approach to explain experimental behavior in hide-and-seek games (presented in Rubinstein, Tversky and Heller, 1996). In particular, both specifications take into account the instinctive attention to focal points.

The rest of the paper is organized as follows: Section 2 presents a game-theoretic analysis of the tennis game and an adapted level-*k* model; Section 3 describes the experimental design; Section 4 reports and discusses the experimental results; and Section 5 concludes.

2. Theoretical Analysis of the Tennis Game

2.1 Formal Presentation of the Game

Players and strategies

The players in the game consist of N tennis coaches who participate in a single round-robin tournament. Coaches choose their strategies simultaneously at the beginning of the tournament. A pure strategy in this game is an assignment of the four players, with skill levels A+, A, B+ and B, respectively, to the four positions. Denote A+ by 1, A by 2, B+ by 3 and B by 4. Formally, denote a pure strategy by a four-tuple, which is a permutation of (1, 2, 3, 4), where the j^{th} component is the level of the player assigned to position *j*. An abbreviation will often be used to represent a strategy, where, for example, 2134 will represent the strategy (2, 1, 3, 4). Since any order of the four players is permissible, there are 24 possible strategies in the game.

Scoring

When two teams play against each other, four points are divided between them. A team receives one point when it assigns a better player to a particular position and no points if the other team assigns a better player. Each team receives half a point when the two players assigned to a position are equally ranked.

Let $score(<x_1,x_2,x_3,x_4>,<y_1,y_2,y_3,y_4>) = \left|\{i \mid x_i > y_i\}\right| + 0.5\left|\{i \mid x_i = y_i\}\right|$ be the total number of points earned by a team that uses a strategy $S=(x_1,x_2,x_3,x_4)$ against a team using the strategy $T=(y_1,y_2,y_3,y_4)$. Thus, $score(S,T)+score(T,S)=4$ for all *S* and *T*.

Note that a team can never score less than one point in a battle against another team since the best tennis player is unbeatable and in the case that he ties, the second-best player cannot lose and at worst will tie. This implies that a team cannot earn more than 3 points in a battle and that there are five possible scores: 3, 2.5, 2, 1.5 and 1.

Payoffs

Each team will play all the other teams in the tournament. The total score of a team that chooses strategy *S* is the sum of points it scores in **all** battles**.** Each team wishes to score the highest number of points among all the teams in order to win the tournament but does not care about its total score per se. This is in fact characteristic of many real-life situations, in which competitors only care about winning and the total points earned or the gap between the winner and runners-up is only of secondary importance. (This was also characteristic of the experiments reported on later in the paper.) Since the prize is shared between the winning teams in the tournament, a team prefers winning together with M other teams over winning with N>M other teams (this assumption prevents the game from having trivial equilibria in which all coaches win by choosing the same assignment). Thus, in a tournament between two players, the payoff structure is simple: unlike the score function which can receive five values, the payoff function can now receive only three (since each coach prefers winning the tournament over a draw and a draw over losing).

2.2 The Score Function

The possible scores in any battle between two strategies can be presented in a matrix. Presenting the score function in an illuminating way (see the appendix) requires an appropriate choice of the strategy order. This sub-section presents some properties of the score function that help direct us to it.

Permutations

Given a strategy *S* and a permutation σ, $\sigma(S)$ is also a strategy. Note that $score(S,T) = score(\sigma(S),\sigma(T))$ since the score is determined by the matching of players from the two teams. The position of a matched pair does not matter.

Partition of strategies into cycles

We say that a strategy *S* **wins** a battle against strategy *T*, if $score(S,T) > 2$. A strategy *S* **defeats** strategy *T* if $score(S,T) = 3$. For any strategy *S*, let *D*(*S*) be the unique strategy that defeats *S*. Given a level $x \in \{1, 2, 3, 4\}$ and an integer $n \in \mathrm{Z}$, denote by $x+n$ the level *y* satisfying $y=x+n$ (mod 4). Then, $D(S) = D(x_1, x_2, x_3, x_4) = (x_1 - 1, x_2 - 1, x_3 - 1, x_4 - 1)$. The function *D* is reversible. Thus, for each strategy *S*, there is exactly one strategy *D*(*S*) that defeats *S* and exactly one strategy $D^{-1}(S)$ that is defeated by *S*.

If we perform D on S four times, we again obtain S. This implies that the function D induces a partition of the game's strategies into six disjoint cycles of four strategies each.

Following are the basic properties of the score function:

Property 1. $score(S,S)=2$, $score(S,D(S))=1$, $score(S,D^2(S))=2$ *and* $score(S,D^3(S))=3$.

The following property, which states that any strategy that confronts a pair of non-sequential strategies in a cycle scores a total of 4 points, is of particular importance.

Property 2. *For any T and S,* $score(T,S)+score(T,D^2(S))=4$.

Cycles 1 and 2

Although the score function is invariant to any permutation of the positions, some strategies are more salient than others. For instance, the strategy 1234 is a focal point because it immediately suggests itself and because of its special characteristics (levels and positions correlate perfectly). Moreover, it is a strategy that can be observed in numerous real-life situations. The cycle that contains 1234 is of particular importance in the experimental part of the study. Denote 1234 by L_0, $D(L_0)=L_1$, $D(L_1)=L_2$, and $D(L_2)=L_3$. Cycle 1 is denoted as $[L_0, L_1, L_2, L_3]$.

Different notations are used for the other cycles. Thus, for any $i \in \{2,..,6\}$, denote Cycle i by $[S_0(i), S_1(i), S_2(i), S_3(i)]$. For Cycle 2, I choose $S_0(2)=4321$, which is another possible focal point. Thus, Cycle 2 is denoted as [4321, 3214, 2143, 1432].[8]

Property 3. *If* $S \in Cycle\ 1$ *and* $T \in Cycle\ 2$*, then* $score(T,S)=2$

Thus, any strategy in Cycle 1 ties with each of the strategies in Cycle 2. A pair of cycles with this property will be called **twin cycles**.

[8] In addition to the focal point property, Cycle 1 and 2 have the special property of being cognitively easy to construct relative to other cycles in the game. The reason is that in these cycles the strategy that defeats *S* is created by a technical "shift" to the right (left) of strategy *S*: each tennis player moves to the position to his right (left) and the last (first) tennis player moves to the first (last) position.

Cycles 3, 4, 5 and 6

Four other cycles will now be identified and the strategies ordered in a manner that will simplify the analysis. The first strategy in each of these cycles is chosen to be a permutation of 1234 that swaps two players at adjacent levels: x and $x+1$. Let $S_0(3)$=1324, $S_0(4)$=4231, $S_0(5)$=1243 and $S_0(6)$=2134.

Property 4. *Cycles 3 and 4 are twin cycles, as are Cycles 5 and 6.*

Property 5. *For* $0 \le k \le 3$ *and* $3 \le i \le 6$: $score(L_k, S_k(i)) = 2$, $score(L_k, S_{k+1}(i)) = 1.5$, $score(L_k, S_{k+2}(i)) = 2,$ *and* $score(L_k, S_{k+3}(i)) = 2.5$.

We define Cycles 3, 4, 5 and 6 as being **parallel** to Cycle 1. This term is appropriate since for i=3,4,5,6 L_k ties with $S_k(i)$ for any k and the score obtained by L_k when played against $S_m(i)$ is close to that obtained by L_k when played against L_m ($\left|score(L_k, L_m) - score(L_k, S_m(i))\right| = 0$ *or* 0.5).

Due to symmetry considerations, any Cycle i can serve as the starting point for identifying parallel cycles (by identifying the order of strategies in four other cycles, which makes these cycles parallel to Cycle i). In this way, the score can be determined for any two strategies.

2.3 Equilibrium

This subsection characterizes the population equilibrium in the tennis game. The Nash equilibrium of a tournament with a large number of teams can be approximated using the following concept of population equilibrium: A distribution of strategies is a population equilibrium if the average score of a strategy in the support of the distribution is at least as high as any other strategy when playing against this distribution.

Denote by $P(S)$ the probability assigned by the distribution P to the strategy S. There is no equilibrium with $P(S)$ =1 since any strategy T for which $score(T,S)$>2 earns a higher score than S. Thus, the support contains at least two pure strategies.

Claim 1. *A probability distribution P is a population equilibrium if and only if the average score for all 24 strategies is 2 points.*

Proof:

→ First, all strategies in the support yield the same average score only if the average is 2 points. Second, the score of any strategy outside the support must be at most 2; however, if some strategy S_k receives strictly less than 2 points, property 2 implies that S_{k+2} receives more than 2 points.

← If all the strategies in the game earn 2 points, then by definition P is an equilibrium. ■

Before moving on to a complete characterization of equilibrium, I present several claims concerning simple forms of equilibrium that will clarify the intuition behind the characterization.

Claim 2. *A probability distribution P, whose support is contained in a single cycle* $[S_0, S_1, S_2, S_3]$ *, is an equilibrium if and only if* $P(S_0)=P(S_2)$ *and* $P(S_1)=P(S_3)$*.*

Proof:

← By property 2, each strategy in the game receives 2 points and thus P is an equilibrium.

→ If for some $S \in [S_0, S_1, S_2, S_3]$, $P(D^2(S))>P(S)$, then $D^3(S)$ earns more than 2 points. To see this, recall that $D^3(S)$ earns 2 points, on average, when played against $D(S)$ and $D^3(S)$ and more than 2 points, on average, when played against S and $D^2(S)$. ■

Note that the only thing that matters in this class of equilibria is that $P(D^2(S))-P(S)=0$ for any S. It does not matter what $P(S)$ is per se. In fact, this understanding leads to a large class of equilibria that can be described compactly by the notion of differences between the probabilities of two non-sequential strategies, S and $D^2(S)$.

Claim 3. *If P satisfies* $P(S)=P(D^2(S))$ *for any strategy S, then P is an equilibrium.*

Proof:

Each strategy T in the game receives an average score of 2 points when played against a pair of non-sequential strategies. Since for all S, $P(S)=P(D^2(S))$, the expected score for any T is 2 points. ■

The analysis of equilibrium remains unchanged if 2 points are subtracted from any possible score in the score matrix. Such a transformation implies that in equilibrium there is no strategy with an average score different from zero. For convenience, what follows is analyzed accordingly.

Claim 4. *Any equilibrium P with a support contained in two cycles satisfies* $P(S)\text{-}P(D^2(S))=0$ *for all S.*

Proof:

Assume the contrary. Consider $S \in Cycle\ i$ for which *$P(S)$-$P(D^2(S))=A$* is maximal. Since $D(S)$ earns a positive score A when played against strategies in Cycle i, it must earn a negative payoff ($-A$) when played against strategies in Cycle j in order to reach the equilibrium score (0 points). This can occur only if *$P(D^2(T))$-$P(T)=2A$* for the strategy $T \in Cycle\ j$, for which *score(D(S),T)*=0.5 (*score(D(S),T)*=2.5 in the original score function). However, A is the maximal difference between the probabilities of non-sequential strategies in a cycle, a contradiction. ■

Claim 2 implies that a minimum of two pure strategies is used in equilibrium. Claim 4 adds that these two strategies must be non-sequential in the same cycle. In other words, the simplest mixed strategy equilibrium involves the use of two strategies, with the property that each is the "best response" to the "best response" of the other strategy.

We now consider the full characterization of the game's equilibrium. Define:

$$X=\left(x_1,x_2,\cdots,x_{12}\right)\equiv\left(p(L_2)-p(L_0),p(L_3)-p(L_1),p(S_2(2))-p(S_0(2))\cdots,p(S_3(6))-p(S_1(6))\right)$$

Proposition 1. *A probability distribution P is an equilibrium if and only if:*

$$\begin{pmatrix} x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} = \begin{pmatrix} -(x_1+x_3+x_5) \\ -(x_2+x_4+x_6) \\ 0.5\cdot(-x_2-x_4-2x_6+x_3-x_1) \\ 0.5\cdot(x_1+x_3+2x_5+x_4-x_2) \\ 0.5\cdot(x_2+x_4+2x_6+x_3-x_1) \\ 0.5\cdot(-x_1-x_3-2x_5+x_4-x_2) \end{pmatrix}$$

Outline of the proof: In equilibrium, the score earned by any strategy must be zero. Using Property 2, it is sufficient to verify that in any cycle, two arbitrary adjacent strategies both earn 0 points (which implies that each of the other two adjacent strategies also earns 0 points). The next step is to understand that the points earned by a strategy *S* are determined only by differences between the probabilities of two non-sequential strategies that do not tie with *S*. Solving the system of 12 linear equations (see the appendix) yields the solution given in the proposition. ■

Comments

(I) The only equilibria with a support contained in three cycles belong to the class suggested in Claim 3. This is because there are 6 degrees of freedom in the system. Therefore, if we substitute zero for the 6 variables, we obtain a single solution: X=0. This claim does not hold for equilibria with a support contained in four cycles. By Proposition 1, the following distributions are equilibria for which the condition in Claim 3 is not satisfied:

$$P = (0,0,\frac{1}{4},0,\quad 0,0,\frac{1}{4},0,\quad \frac{1}{4},0,0,0,\quad \frac{1}{4},0,0,0,\quad 0,0,0,0,\quad 0,0,0,0)$$

and $P = (\frac{1}{12},0,\frac{1}{6},0,\quad \frac{1}{12},0,\frac{1}{6},0,\quad \frac{1}{12},0,0,0,\quad \frac{1}{4},0,\frac{1}{6},0,\quad 0,0,0,0,\quad 0,0,0,0)$.

Note that these two examples induce different vectors of the type $X = (a,b,a,b,-a,-b,-a,-b,0,0,0,0)$, which reflects the structure of equilibria with a support contained in the first four cycles.

(II) The analysis in this sub-section is equivalent to that of a symmetric mixed-strategy Nash equilibrium in a two-player game, in which the payoff matrix is the score matrix of the tennis game. In other words, the analysis also captures scenarios in which each of the two players aims at maximizing his objective score and not just to obtain a higher score than his opponent. In fact, *P* is a population equilibrium if and only if it is an equilibrium mixed strategy (possibly asymmetric) in this two-player game.

(III) Consider the **two-player tournament**, in which the players' payoffs are 1 for winning the tournament, 0 for a draw and -1 for losing. It is straightforward to show that in this game, a probability distribution *G* is a Nash equilibrium mixed strategy if and only if $G(S)=G(D^2(S))$ for any *S*.

2.4 Best Response Function

This sub-section focuses on finding the best responses to some interesting distributions of choices. In particular, I identify the best responses for distributions that I consider to be natural beliefs and which may be those actually held by coaches. Examples of natural beliefs include: "All other coaches will choose S", "Most of the coaches will choose S" and "The most frequent choice will be S".

As intuition suggests, the best response to the belief that "**almost all** other coaches will choose S_k(i)" is $D(S_k(i))$. However, given the belief that **all** other coaches will choose S_k(i), any $D(S_k(j))$ for a parallel Cycle j is also a best response (a coach who chooses $D(S_k(j))$ earns an average score of 2.5 points but wins the tournament since it is the highest score among the coaches). The next proposition refers to the natural belief that "most of the coaches will choose S". The adapted level-k model that will be constructed in Section 2.6 relies on this proposition.

Proposition 2. *If $1>P(S)>0.5$ for some S, then D(S) is the unique best response to P.*

Outline of the proof: Assume without loss of generality that $1>P(L_0)>0.5$. We need to show that no strategy earns as much as L_1. It is enough to show that for any $X \neq L_1$, if $score(X, L_0) = (3-t)$, then $score(X,Y) - score(L_1,Y) \leq t$ for any Y. In other words, X cannot compensate for its inferiority to L_1 when played against L_0 by its superiority when played against some other strategies. The proof covers all the possible strategies X and confirms that the condition on the score is satisfied (see the appendix). ■

Now consider the belief that "all choices will be in Cycle i and the most frequent choice will be S". For such a belief, the optimal choice is not necessarily $D(S)$. For example, if $P(S_0)=0$, $P(S_1)=0.4$, $P(S_2)=0.3$ and $P(S_3)=0.3$, then the optimal choice is S_3, and not S_2. The reason is that the optimal choice, when choices are in a single cycle, is determined by the differences between two non-sequential strategies. The optimal choice in this case is S_{k+1}, for k that maximize $P(S_k)-P(S_{k+2})$.

This last example also demonstrates why $D(S)$ is not necessarily the optimal strategy given the belief that "the most frequent strategy is S". However, it is easy to see, as an implication of Property 2, that $D(S)$ is the optimal strategy for the belief that the most popular choice is S and that the rest of the chosen strategies are uniformly distributed. Essentially, this claim states that $D(S)$ is the best response to a belief that attributes high probability to the strategy S and takes into account some level of noise.

2.5 A Variant of the Game

In the experimental part of the paper, a second version of the game is discussed, which is denoted as Version 2. It differs from the first version only in the method of scoring. Thus, in Version 2, a team receives one point **only if it wins three matches out of four** against another team. At any other case, it does not receive any points.

In this version of the game, and given a probability distribution P, it is always optimal to choose $D(S^*)$, where S^* is the strategy for which $P(S)$ is maximal. Therefore, Proposition 2 becomes trivial in this context and can be extended to the following proposition: If none of the strategies are chosen more often than S, then $D(S)$ is a best response. If, in addition, none of the strategies are chosen as often as S, $D(S)$ is the only optimal strategy. Equilibrium analysis also becomes simpler in this version. Thus, the probability distribution P constitutes an equilibrium if and only if, for any S and T in the support, $P(S)=P(T)$ and in any Cycle i, $P(S_0(i))= P(S_1(i))= P(S_2(i))= P(S_3(i))$.

2.6 The Adapted Level-*k* Model

In this sub-section, the equilibrium solution concept is abandoned and an alternative approach is considered in an attempt to account for the experimental behavior in the tennis game. The game's structure and its psychological properties call for applying the concept of level-*k* thinking, which is based on iterative reasoning.

Level-*k* non-equilibrium models assume that the population consists of several different types of decision makers and that each type uses a different level of iterated reasoning. *L0* is a non-strategic type who chooses his action naively. *L1* best responds to the belief that all other players are *L0*; *L2* best responds to the belief that all other players are *L1*; and so on.[9] In each game, the specification of *L0* determines the definition of the other *Lk* types in that particular context. Type *L0* is often assumed to choose a strategy by performing a uniform randomization over the strategy space, but there are cases in which *L0* is specified differently. A relevant example is presented by Crawford and Iriberri (2007a) who construct an adapted level-*k* model to explain behavior in hide-and-seek games with non-neutral framing. Their *L0* type instinctively recognizes focal points[10] and his typical decision rule is taken to be a mixed strategy which puts greater weight on focal points. Their specification of the naive *L0* type accurately captures a psychological effect

[9] In some cases, *Lk* is assumed to best respond to a combination of lower types. See, for example, Camerer et al. (2004).

[10] Bacharach and Stahl (1997) propose a general framework that captures this idea.

that is also relevant in the tennis game. Another related specification is that used by Crawford and Iriberri (2007b) in the context of auctions, in which the truthful *L0* bids the value that his own private signal suggests.

Note that any distribution of choices can be explained trivially by specifying *L0* as a decision maker who chooses according to that particular distribution. A level-*k* model attempts to explain the data primarily through the behavior of *L1*, *L2* or higher types and by considering only a small number of **natural** non-strategic types. In other words, the explanatory power of level-*k* models is based on the typical behavior of the strategic types.

Specification of *L0* in the tennis game

The main assumption I make in this subsection is that the natural starting point for iterated reasoning in the tennis game is the focal strategy 1234 (L_0), which is associated with the non-strategic type *L0*. Since this naive strategy is a natural choice, a sophisticated coach might choose to best respond to such a strategy by choosing 4123 (L_1). Forming a belief concerning the opponent's strategy and best responding to it is the first step of iterated reasoning and thus the type who chooses this strategy is denoted as *L1*. An iteration of this process involves best responding to the belief that other coaches will choose L_1. Therefore, *L2* will typically choose the strategy 3412 (L_2) which reflects the second step of iterated reasoning. The highest level of iterated reasoning that this model takes into account is the third iteration[11] which leads to type *L3* choosing 2341 (L_3).

Note that if a coach simply wants to win the tournament and believes that **all** other coaches will choose L_0, then he actually has five possible best responses: L_1, $S_1(3)$, $S_1(4)$, $S_1(5)$ and $S_1(6)$, though the score for $S_1(i)$ against L_0 is less than that for L_1 against L_0. The justification for my definition of types is Proposition 2, which states that if "the majority of the coaches choose T" (rather than all the coaches), then the only optimal strategy is $D(T)$. This kind of belief reflects a rough estimation of the opponents' choices and is likely to be more common than the belief that all other coaches will choose a specific strategy. Therefore, the assumption made here concerning coaches' beliefs is more plausible than

[11] This is because the fourth level of iterated reasoning and the choice of *L0* cannot be distinguished. Tennis teams were defined as consisting of 4 rather than 3 players because in previous experimental studies of other games, the fourth level of iterated reasoning was rarely observed, whereas the third level was more commonly observed. This finding justifies the assumption that *L3* is the highest type.

those made in other level-*k* models.[12] In fact, the typical choices of types defined in the model can be sustained even under a weaker assumption, according to which type *Lk* best responds to the belief that the **most frequent** choice is L_{k-1} and that the rest of the choices are uniformly distributed.

Another strategy to be considered as an anchor for iterated reasoning is 4321 ($S_0(2)$). Allocating the players in the reverse order can be viewed as a focal strategy, though a weaker one than 1234. It is likely that non-strategic types would choose this strategy while strategic types might treat it as an anchor for iterative reasoning. Thus, the choice of $S_k(2)$ is considered as a possible outcome of another level-*k* decision rule, based on a different anchor. Clearly, allowing for another kind of level-0 type can only improve the fit of the level-*k* model.

The experimental results will be analyzed in light of the above specification, thus allowing for two possible anchors and two possible types that use each level of reasoning. In other words, all the strategies in Cycle 1 and Cycle 2 are associated with level-*k* reasoning.

Comment: The notion of level-*k* reasoning does not necessarily contradict the concept of Nash equilibrium, although it may lead to outcomes that are essentially different from equilibrium outcomes. In this game, a subset of equilibria can be achieved if the population consists of various level-*k* types using different levels of iterated reasoning. For example, if the proportion of each *Lk* type is 0.125, then the resulting distribution of strategies will constitute an equilibrium.

Alternative specifications of *L0*

There are other intuitively appealing specifications of level-0 types. For example, consider a non-strategic type who chooses each strategy in the game randomly and equally often, excluding the strategy 1234 which he chooses more frequently. Given this alternative specification, *L1*, who best responds to *L0*, would choose 4123 as before and hence higher types would also behave as before. Note that from *L1*'s point of view, the interpretation of this *L0* is the same as in the original model, under the assumption that type *Lk* best responds to the belief that the most frequent choice is L_{k-1} and that the rest of the choices

[12] In many other games appearing in the literature (for example, Costa-Gomes et al. 2001), the definition of level-*k* types would be affected dramatically by a transition to this assumption.

are uniformly distributed. The non-strategic type could be specified in a similar manner under the assumption that the strategy 4321 is chosen more frequently than the rest or under the assumption that both 1234 and 4321 are chosen more frequently than other strategies. In this last case, as long as 1234 receives more weight than 4321, the best response to this type would be 4123. Allowing the existence of two non-strategic types, one who gives more weight to 1234 and another who gives more weight to 4321, implies that the two types who use the first step of iterated reasoning (based on the two possible anchors) choose L_1 and $S_1(2)$, respectively. Note that the alternative specifications of $L0$ above would not change the typical behavior of higher types and hence should not affect the explanatory power of the model. The only possible change that could result is an increase or decrease in the proportion of behavior that can be explained by the level-0 types.

Taking $L0$ to be a type who chooses a strategy randomly and uniformly is also intuitively appealing; however, it does not produce any constraint on the k-level types for any $k>0$. In fact, all 24 strategies are best responses to this strategy and thus, for any strategy and for any k, one can say that the strategy is the choice of a level-k type (see Crawford and Iriberri (2007a) for an explanation of why they avoid specifying $L0$ as a type who practices uniform randomization). In addition, the best response to this $L0$ type guarantees a tie and thus differs fundamentally from best responses that guarantee winning the tournament (such as the best responses to the $L0$ types discussed above). Therefore, I do not treat the uniform randomization decision rule as an outcome of level-k thinking.

3. Experimental Design

Three experiments were designed with the following goals in mind: to test whether the adapted level-k model can explain behavior in the game, to ascertain the depth of iterated reasoning in this context and to explore the triggers of this kind of reasoning. The experiments were conducted through the website: http://gametheory.tau.ac.il, which was created by Ariel Rubinstein and provides tools for conducting choice and game theoretic experiments. The original text used for the questions in the experiments appears in the appendix. All the experiments are based on the Tennis Coach problem introduced in Section 1. Each experiment was carried out in the form of a tournament in which subjects choose a strategy and then automatically play against their classmates (using that strategy). A total of 1,624 students participated in the experiments, most of them undergraduates in game theory and other economics courses.

3.1 Experiments 1 and 2

The subjects in Experiment 1 consisted of 641 students in 14 different courses, originating from 7 countries.[13] Subjects in Experiment 2 consisted of 704 students in 22 different courses, originating from 14 countries.[14] The lecturers in these courses assigned the Tennis Coach problem as a compulsory homework problem. The website's server recorded the time each subject spent on making the decision (response time) together with the strategy that he chose.[15] Following the decision, subjects were asked to explain why they had chosen the strategy they did. The subjects did not know in advance that they would be asked to explain their choice or that their response time would be recorded. Lecturers were not able to observe the individual decisions made by their students. However, they did have access to the distribution of choices made, the three winning strategies and the identities of the three winning students. The winners in the tournament did not receive a monetary prize. Nevertheless, they had an incentive to treat the tournament seriously in order to have the honor of being announced in class as one of the winners.

The game played in Experiment 1 was the original version of the game, which is presented in Section 1 and denoted by **Version 1**. In Experiment 2 subjects played a variant of the game denoted by **Version 2**, which is presented in Section 2.4. Recall that the only difference between the two versions is in the system of scoring. In Version 2, a team scores 1 point only if it wins three matches out of four against another team.

According to the adapted level-*k* model presented in Section 2.5, the process of iterated reasoning in the tennis game is based on two ordered components: (I) Forming a **concrete** belief of the type "Most subjects will choose strategy *S*" and (II) Best responding to that belief by choosing *D*(*S*). Since in Version 2, *D*(*S*) is the only best response to the belief that *S* is the **most frequent choice**, the adapted level-*k* model is

[13] The US, the UK, Colombia, the Slovak Republic, Argentina, Canada and Brazil.

[14] The US, Mexico, Brazil, Chile, India, Switzerland, Moldova, Ecuador, France, Brunei Darussalam, Germany, Portugal, Spain and Israel.

[15] Strategies were not presented in a list in order to avoid order effects. Subjects faced a matrix with four columns representing players' levels and four rows representing the different positions. They allocated the tennis players on their team by marking one box in each row.

even more appropriate in Version 2 than it is in Version 1. In a sense, the scoring system in Version 2 provides some guidance in the second component of the process and perhaps triggers the first component. By comparing behavior in the two versions, it is possible to determine whether the changes in the score function and the resulting guidance increase the use of iterated reasoning.

3.2 Experiment 3

This experiment responds to the concern that subjects in Experiment 1 and 2 were not motivated by monetary incentives. In addition, it further investigates the subjects' understanding of the best response function by testing whether they optimally respond to a concrete belief.

Students from three undergraduate economics courses in Israel (at Tel Aviv University, Haifa University and Ben-Gurion University, respectively) were invited by email to take part in the online experiment. 279 subjects who decided to participate were randomly assigned to play either Version 1 or Version 2. The winner of the tournament in each class won NIS 200 (around $60). After explaining their choices, subjects answered three questions that tested their understanding of the best response function. They were asked to provide an optimal response to each of the following beliefs: "All other subjects will choose (A, B, A+, B+)", "All other subjects will choose (B+, B, A+, A)" and "Most of the subjects will choose (B, A+, A, B+)". Subjects who played Version 1 were told that there is at least one correct answer to each question while subjects in Version 2 were told that there is only one correct answer to each question. They were told that those who answered the questions correctly would win some CD's.

Recall that in Experiment 1 and 2, lecturers asked their students to participate and hence subjects treated it as a compulsory exercise. In this experiment, lecturers were not involved and did not have access to any of their students' answers. Since the number of students who entered the website and only then decided not to participate was negligible, I conclude that a subject's decision to participate in the experiment was no different in character than the decision to participate in a laboratory experiment. Therefore, there is no reason to think that the recruiting method used here attracted a subject pool different from that of any other experiment.

3.3 Response Time and Explanations

As the analysis in Costa-Gomes et al. (2001) suggests, it is possible to draw incorrect conclusions concerning the frequencies of types based on observed choice alone. They used subjects' patterns of information search to interpret their choices in normal-form games. The approach in this paper is to use subjects' response time and explanations to interpret their observed choices.

A subject's explanation of his choice may reveal the decision rule he used and in particular whether it was based on an iterated reasoning process. Recall that subjects were asked to explain their choices only after making the decision and therefore their choices could not have been affected.

Response time is defined as the number of seconds from the moment that the server receives the request for the problem until the moment that an answer is returned to the server. This additional information is used to classify strategies in the game as intuitive choices or as an outcome of cognitive deliberation. This method is discussed in Rubinstein (2007), whose main claim is that the response time of choices made using cognitive reasoning is longer than that of choices made instinctively, i.e. on the basis of emotional response.[16] This is in line with dual-system theories, such as that in Kahneman and Frederick (2002).

4. Experimental Results

4.1 Experiment 1 (Version 1)

Table 1 presents the aggregate data for all 641 subjects. I focus on analyzing the aggregate data and comment only briefly on the distribution of choices for each of the classes.

Main results

Each of the 24 strategies was chosen by at least 1.25% of the subjects. About 57% of subjects' choices were strategies in the first two cycles (see the table below), where 41% of the subjects chose one of the following three strategies: L_0 (22%), L_1 (10.1%) or $S_0(2)$

[16] Of particular relevance are Rubinstein's findings concerning the 2/3-beauty contest, which has been intensively studied in the level-*k* literature. He found that the median response time of the second step of iterative reasoning in this game was much longer than that of choices representing the first step of reasoning, which in turn was much longer than the response time of other (perhaps less strategic) choices.

(8.7%). Strategies in other cycles were chosen far less frequently – almost always by less than 4% of the subjects. The main features of the distribution of choices are preserved in the four large classes (77, 80, 92 and 115 students) and even in the small classes L_0 is relatively common and L_1 is rarely absent.

Strategies	L_0	L_1	L_2	L_3	$S_0(2)$	$S_1(2)$	$S_2(2)$	$S_3(2)$	*Other*
Percentage	22%	10.1%	3.3%	3.6%	8.7%	3.6%	2.8%	2.7%	43%

Following are the main findings concerning level-*k* thinking, taking into account subjects' response time and explanations:[17]

1. 22% of the subjects chose the naive strategy L_0, which confirms its focality and its role as a potential anchor for iterated reasoning. Its significantly lower response time (median=125s) relative to other strategies suggests that it is typically an instinctive choice or an outcome of a low level of sophistication.

2. 10.1% of the subjects chose L_1. Their explanations and significantly higher response time (median=194s) suggest that most of them actually used the first level of iterated reasoning with L_0 as an anchor.

3. 3.3% of the subjects chose L_2 while 3.6% chose L_3, strategies that are supposed to reflect the second and third steps of iterated reasoning, respectively. Subjects' explanations suggest that many (though not all) of those who chose this category used alternative decision rules rather than high levels of iterated reasoning.

4. 8.7% of the subjects chose $S_0(2)$, the reverse order strategy. The response time of this strategy (median=158.5s) was significantly higher than L_0's, suggesting that subjects who chose it were not confused and had not intended to choose L_0.

5. $S_1(2)$ was chosen by 3.6%. Subjects' explanations suggest that only a small fraction of the choices were the result of an iterated reasoning process with $S_0(2)$ as the anchor. $S_2(2)$ and $S_3(2)$ were chosen even less often and, according to subjects' explanations, do not seem to have been the result of such a process.

[17] See the appendix for a detailed discussion of subjects' response times and explanations.

Equilibrium

The distribution of strategies is not consistent with any equilibrium prediction. One way to see this is by examining the expected score of the strategies presented in Table 1. The strategy L_1 is the clear leader and the only strategy that comes close to it is $S_1(3)$. These strategies were chosen by only 14% of the subjects. Thus, the vast majority of the subjects could have significantly improved their chances of winning by deviating to L_1.

It is possible that small changes in the distribution of strategies would significantly affect the strategies' score. In such a case, the previous argument would not be convincing. Thus, a different method is used to show that the distribution is far from being an equilibrium. An equal number of subjects was subtracted from each pair of non-sequential strategies in a cycle – the choices of 283 subjects remained. A consequence of Property 2 is that subtracting an equal number of coaches who choose S and $D^2(S)$ does not change the best response. Hence, the resulting distribution is an equilibrium if and only if the original distribution is as well. In the resulting distribution, $P(L_0)$=0.42 and $P(L_2)$=0 (see the distribution after normalization in Table 1). This implies that L_1 would earn more than the equilibrium score even if all other choices were concentrated around S_2(i), for i=3,4,5,6 since the weight on these strategies in equilibrium needs to be at least twice as much as the weight on L_0. The argument is strengthened by the fact that $P(S_2(i))$=0, for i=3,4,5,6.

Explanations

70% of the subjects (out of 526 who were asked) provided an explanation of their choices. Each of the explanations is classified according to one of the following categories and the proportion of each category is estimated:

1. ***Intuitive choice*** (18%)

This category includes explanations such as: "It was a guess"; "I don't know why"; "It felt right" and "Intuition". 45% of subjects who provided intuitive explanations chose L_0.

2. ***Random choice*** (18%)

This category includes explanations that mentioned the word "random". Some of them explained the randomization as an attempt to choose a different strategy from that of other players or to surprise their opponent. The category also includes explanations such as: "It does not matter what I choose because the distribution of choices is practically uniform if I don't know it". Among subjects in this category, 10% chose L_0 and explained that it did not matter what they chose. The other 90% said that they randomized and 19 strategies were chosen by them.[18]

3. ***First step of iterated reasoning*** (10%)

This category includes explanations that describe best responding to the belief that most of the choices will be *X* (primarily L_0 or $S_0(2)$). 80% of the subjects in this category chose L_1 and 8% chose $S_1(2)$.

4. ***Second step of iterated reasoning*** (three subjects, less than 1%)

This category includes explanations that describe best responding to the belief that most of the choices will be L_1.

5. ***Other strategic decision rules*** (53%)

This category includes explanations such as: "I was trying to be original", "I am mixing good and bad players", "I am sacrificing the weak player in order to win in other positions", "My choice was based on my life experience", "The best players of my opponent were likely to be in the middle positions and therefore I put mine on the edges" and "The player in the first position should be the best one since my opponent will put A in the first position" (or something similar based on some other partial belief). It also includes explanations based on incorrect reasoning (such as "I am trying to achieve a tie") or irrelevant considerations (such as taking into account order effects). Each of the 24 strategies was chosen by subjects in this category.

Comment: Only four subjects mentioned the concept of Nash equilibrium in their explanation, although many of the subjects had studied game theory.

Discussion of Experiment 1

As stated by Crawford and Iriberri (2007b): "The estimated distribution tends to be stable across games, with most of the weight on *L1* and *L*2. Thus, the anchoring *L0* type exists mainly in the minds of higher types." The results of Experiment 1 reflect a low level of sophistication in terms of level-*k* reasoning. Moreover, many choices do not reflect level-*k* reasoning at all and are the result of other types of considerations.

The frequency of non-strategic types (level-0) is higher and the frequency of level-1 types relatively lower than in other studies; higher types are in fact almost totally absent.

[18] Unchosen strategies: 4231, 2143, 1243, 1432 and 1342. Most frequently chosen strategies: 1234 (24%), 2413 (14%), 4321 (13%) and 1324 (6%).

The proportion of subjects that actually use a high level of iterated reasoning might be even smaller than that indicated by observed choice since subjects who chose randomly or use decision rules other than iterated reasoning may also have chosen L_2, L_3, $S_2(2)$ or $S_3(2)$. I do not consider the choice of L_0 to be an outcome of four steps of iterated reasoning since in previous studies this level of reasoning was not evident. This approach is also supported by L_0's low response time and the fact that no one who made this choice explained it as being a best response to L_3.

Response times and explanations provide support not only for the interpretation that the observed choices reflect a low level of iterated reasoning, but also for the specification of level-*k* reasoning in this context. The subjects' explanations indicate that the only common starting point for iterated reasoning in players' minds was L_0. A secondary and less common anchor for iterated reasoning was $S_0(2)$. Furthermore, strategies not in Cycle 1 or Cycle 2 have shorter response times than L_1, suggesting that there are no other pure strategies with the same role as L_1.

The distribution of strategies in the experiment was far from being an equilibrium. However, if subjects were to play the game repeatedly and in each round would internalize the distribution of strategies in the previous round, they might converge to one of the equilibria of the one-shot game. Since subjects may notice the patterns based on iterated reasoning in earlier rounds, they might modify their choices in later rounds accordingly. In particular, I conjecture that in later rounds subjects' choices would be concentrated in the first cycle. Thus, level-*k* reasoning may turn out to influence not only outcomes of one-shot games, but also the selection of equilibrium in the long run.

4.2 Experiment 2 (Version 2)

Table 2 presents the aggregate data for all 704 subjects.

Main Results

The aggregate data show that each of the 24 strategies was chosen by at least 1.14 % of the subjects. About 55% of subjects' choices were strategies in the first two cycles, where 37% were one of the following three strategies: L_0 (18.6%), L_1 (12.9%) or $S_0(2)$ (5.8%). Strategies in other cycles were chosen less frequently, with most of them chosen by only 1-4% of the subjects. The main features of the distribution are preserved in the two large classes (99 and 209 students) and even in the smaller classes L_0 is relatively common and L_1 is rarely absent.

The following table presents the frequencies of strategies that may reflect iterated reasoning and compares them to those in Experiment 1.

	L_0	L_1	L_2	L_3	$S_0(2)$	$S_1(2)$	$S_2(2)$	$S_3(2)$	*Other*
Version 2	**18.6%**	**12.9%**	**5.3%**	**2.8%**	**5.8%**	**5.4%**	**2.4%**	**1.7%**	**45%**
Version 1	22%	10.2%	3.5%	3.6%	8.6%	3.6%	2.8%	2.6%	43%

The distributions of strategies in the two versions are similar though not identical. In Version 2, fewer subjects chose $S_0(2)$ and L_0 while more chose $S_1(2)$, L_1 and L_2. These differences suggest that Version 2 leads to less intuitive choices and somewhat more choices based on iterated reasoning. Applying the chi-square test with respect to nine categories (one for each strategy in the first two cycles and another for the rest), it was found that the difference between the frequencies of categories in the two versions is significant at the 5% level (chi-square=16; df=8; p=0.04).[19] It is also of interest that the response time of choices in Version 2 was significantly longer than in Version 1.[20]

Equilibrium

The distribution of chosen strategies is not consistent with any equilibrium prediction. Only 13% chose the winning strategy L_1 while the score earned by other strategies is far below that of L_1. Thus, the vast majority of the subjects could have significantly improved their chances of winning by deviating to L_1 and therefore the distribution of the results is far from an equilibrium.

To see this in a different way, note that the necessary conditions for equilibrium discussed in Section 2.4 are violated. We have $P(L_0)$=0.185, $P(L_1)$=0.129, $P(L_2)$=0.053 and $P(L_3)$=0.028, while in equilibrium the probability of each strategy in a cycle must be equal. Moreover, the weight on Cycle 1 is more than twice that on any other cycle, whereas in equilibrium the weight on each cycle in the support must be equal.

[19] If we treat each strategy in the game as a category, the distributions of choices in the two versions are not significantly different according to a chi-square test (chi-square=25.85 df=23 p=0.31). However, we are primarily interested in the differences between choices in the first two cycles and hence the partition into nine categories is more appropriate here.

[20] The median RT is 147 in Version 2 and 140 in Version 1.The average RT of observations under 600s is 170 in Version 2 and 156 in Version 1.

Explanations

70% of the subjects provided an explanation of their choice. The explanations are classified into the same categories as in Version 1.

1. ***Intuitive choice*** (15%)

30% of the subjects chose 1234.

2. ***Random choice*** (20%)

16% of the subjects in this category explained that it did not matter what they chose (only half of them chose 1234). The rest (84%) said that they had tried to randomize. Each of the 24 strategies was chosen by these subjects.[21]

3. ***First step of iterated reasoning*** (15%)

84% of the subjects chose L_1 and 12% chose $S_1(2)$.

4. ***Second or third step of iterated reasoning*** (3%)

Twelve explanations were based on level-2 reasoning. Nine best responded to L_1 and three best responded to S_1. Two explanations were based on level-3 reasoning.

5. ***Other strategic decision rules*** (47%)

Each of the 24 strategies was chosen. Subjects used the same decision rules as in Version 1.

The proportions of the various categories were similar in the two experiments, with somewhat more weight on the categories that support the use of iterated reasoning in Version 2 and less weight on intuitive explanations and other strategic rules. Although equilibrium is simpler in this case, only two subjects mentioned this concept in their explanations.

Discussion of Experiment 1 and 2

The data from the two experiments confirms that the specification of level-k types was appropriate in this setting. However, iterated reasoning was not triggered as often in the tennis game as in other games studied in the literature. In both versions, iterated reasoning was observed only in Cycles 1 and 2, with most of the weight on level-0 and level-1. Version 2 induces slightly more choices and explanations that involve iterated reasoning. The increased use of iterated reasoning might be a result of the "guidance" provided by the scoring system (which requires winning three out of four matches in order to earn a point).

[21] Most frequent choices: 1234 (12%), 2314 (10%) and 3214 (7%).

It may also indicate that the assumptions of the adapted level-*k* model in Version 2 are more plausible than those in Version 1. In particular, the assumption that *Lk* best responds to the belief that the **most frequent choice** is L_{k-1} has greater plausibility.

The conventional definition of strategic thinking requires forming a belief on the opponent's strategy and best responding to it. The choices and explanations of subjects in the two experiments suggest that this kind of thinking is not prevalent. However, the results do reveal partial strategic thinking. Many of the decisions are apparently based on a partial belief over the opponents' choices and thus exhibit an attempt to forecast features of other players' choices.

4.3 Experiment 3 (Version 1 and 2)

Table 3 presents the observed frequencies of strategies in Version 1 and 2. The total number of subjects is 279, where 131 subjects received Version 1 of the game and 148 received Version 2.

Main results

Each of the 24 strategies was chosen. In both versions, 12% of the choices were in Cycle 2 and 44% in Cycle 1. The main features of the distributions of strategies were similar in all three classes. The following table presents the frequencies of strategies in the first two cycles for Version 1 and 2.

	L_0	L_1	L_2	L_3	$S_0(2)$	$S_1(2)$	$S_2(2)$	$S_3(2)$	*Others*
Version 1	10.7%	19.1%	4.6%	2.3%	7.6%	5.3%	6.1%	0.8%	43.5%
Version 2	10.1%	18.9%	6.8%	2.0%	4.1%	10.8%	2.7%	1.4%	43.2%

There are no significant differences between the distributions of choices in the two versions, whether we treat each strategy as a category (chi-square=31.75; df=23; p=0.11) or treat all strategies in Cycles 3,4,5 and 6 as a single category (chi-square=6.79 df=8 p=0.56).

In both versions, the first step of iterated reasoning was reflected more often in choices and explanations (i.e. L_1 and $S_1(2)$ were more frequent) than in Experiment 1 and 2 while the naive choice of L_0 was less frequent than in Experiment 1 and 2. The distribution of choices in Version 1 (in this experiment) differs significantly from Version

1 in Experiment 1 (chi-square=45.1; df=23; p=0.004) and the distribution of choices in Version 2 differs significantly from that in Experiment 2 (chi-square=43.1; df=23; p=0.007).

Equilibrium

As in the previous experiments, it is straightforward to confirm that the distribution of strategies in each of the versions is far from being an equilibrium. The best response to the distribution in both versions is clearly L_2, which was chosen by less than 7% of the subjects.

Best responding to a concrete belief

In the second part of the experiment, subjects were asked to provide an optimal response to each of the following beliefs: 1. All other subjects will choose (A, B, A+, B+) 2. All other subjects will choose (B+, B, A+, A) and 3. Most of the subjects will choose (B, A+, A, B+). 125 out of 131 subjects in Version 1 and 143 out of 148 subjects in Version 2 participated in this part of the experiment. The following table summarizes the results:

% that answered correctly:	1	2	3	1 & 2 & 3	At least two out of three
Version 1	93%	90%	89%	81%	93%
Version 2	91%	94%	93%	89%	93%

Comment: Among those who chose 1234 or 4321 in both versions, only 12% (5 students out of 41) did not answer the three best response questions correctly. In other words, their possibly naive choice does not indicate that they did not understand the game or did not know how to best respond to a concrete belief.

Discussion of Experiment 3

The finding that naive choices were less frequent in Experiment 3 than in Experiment 1 and 2 and that the first level of reasoning was more common, can be explained by three factors: (1) Subjects in Experiment 3 were asked to respond to only one problem whereas in Experiment 1 and 2 subjects were asked to respond to additional (unrelated) problems as a part of their homework (2) In Experiment 3, the subjects had monetary incentives and (3) Participation in Experiment 1 and 2 was mandatory and it is likely that some subjects were not motivated to invest effort in the task. The greater seriousness of subjects in

Experiment 3 was reflected in longer response times, as well as in the proportion of subjects that provided explanations (80% in Experiment 3 vs. 70% in Experiment 1 and 2).

However, note that the use of higher levels of iterated reasoning is not dramatically higher than in Experiment 1 and 2 and not as frequent as in other games studied in the literature.

In the second part of the experiment, at least 81% of the subjects answered all three questions correctly in both versions. This indicates that subjects understood the game and are cognitively able to best respond to a concrete belief, such as the belief that all other choices will be *S*. The high percentage of correct answers to Question 3 implies that subjects also have the correct intuition regarding the optimal response to the belief that most of the subjects (rather than all) will choose *S*. This result is important since the belief that most of the subjects will choose *S* sounds more plausible than the belief that all of them will choose *S*.

In answering Question 1 and 2, almost all subjects in Version 1 chose the best response that **defeats** the strategy (i.e. wins 3 out of 4 matches) assumed to be chosen by other coaches. Only a few chose one of the four pure strategies that earn 2.5 points. These findings provide support for the definition of iterated reasoning used in this game (i.e. that the typical choice of *Lk* defeats the strategy chosen by *Lk-1*). It also suggests that the first component of the process of iterated reasoning is lacking in this context. In other words, most of the subjects do not hold a concrete belief, such as the belief that most of the subjects will choose L_k.

5. Concluding Remarks

The tennis game captures various strategic real-life interactions. Examples include: allocating troops among a number of battlefields, choosing the order of R&D projects to be undertaken, promises in election campaigns, assigning workers to projects in a competitive environment and, of course, assigning players in sports games. The paper's theoretical analysis provides a complete characterization of equilibria in the tennis game. In an attempt to explain the experimental behavior in the game, the equilibrium solution concept is replaced by an adapted level-*k* model, which is based on a natural specification of iterated reasoning in this setting.

Although level-*k* thinking seems to be highly appropriate in the tennis game, the adapted model explains only part of the experimental results and many of the choices seem to be the result of other decision rules not based on level-*k* thinking. Perhaps the most striking result is the low frequency of types that use high levels of iterated reasoning. Even the first step of iterated reasoning is not very common in the two versions of the game and higher steps of reasoning are almost totally absent. These findings are supported by the subjects' explanations. Furthermore, their explanations hint that many of them do not hold a concrete belief over other subjects' choices and certainly do not best respond to the belief that most of the subjects are level-*k* types.

The results in this paper differ from those obtained in previous studies, which found high frequencies of level-*k* reasoning among subjects in various games. I suggest two reasons for this: First, the pure strategies attributed to level-*k* reasoning in the tennis game are only a small fraction of the possible choices in the game. Second, there is a natural tendency in the tennis game to form partial beliefs over the opponents' strategies. In other words, the rich structure of the game triggers other kinds of strategic thinking. Further research is needed in order to more clearly identify the circumstances in which the level-*k* approach is successful at explaining the data.

6. References

Bacharach, M, and Stahl, D. "Variable-Frame Level-*n* Theory," *Games and Economic Behavior*, 2000, 32, 220–246. Previous Version (1997): http://www.eco.utexas.edu/~stahl/vflnt4a.pdf

Borel, E. " La th´eorie du jeu les ´equations int´egrales `a noyau sym´etrique". *Comptes Rendus de l'Acad´emie.* 1921, 173, 1304–1308; English translation by Savage, L. "The Theory of Play and Integral Equations with Skew Symmetric Kernels". *Econometrica,* 1953**,** 21**,** 97–100.

Camerer, C. Ho, T, H. and Chong, J, K. "A Cognitive Hierarchy Model of Games." *Quarterly Journal of Economics*, 2004, 119(3), 861–898.

Costa-Gomes, M. A., Crawford, V. P. and Broseta, B. "Cognition and Behavior in Normal-Form Games: an Experimental Study." *Econometrica*, 2001, 69(5): 1193–1235.

Costa-Gomes, M. A. and Crawford, V. P. "Cognition and Behavior in Two-Person Guessing Games: An Experimental Study", *American Economic Review,* 2006, 96, 1737-1768.

Crawford, V. P. “Lying for Strategic Advantage: Rational and Boundedly Rational Misrepresentation of Intentions,” *American Economic Review*, 2003, 93, 133–149.

Crawford, V. P. and Iriberri, N [a]. "Fatal Attraction: Salience, Naivete, and Sophistication in Experimental Hide-and-Seek Games", *American Economic Review,* 2007, 97, 1731–1750.

Crawford, V. P. and Iriberri, N [b]. "Level-*k* Auctions: Can a Non-Equilibrium Model of Strategic Thinking Explain the Winner's Curse and Overbidding in Private-Value Auctions?," *Econometrica*, 2007, 75, 1721–1770.

Dekel, E., Jackson, M. and Wolinsky, A. "Vote Buying: General Elections", *Journal of Political Economy*, 2008, 116(2), 351-380.

Fershtman, C. and Rubinstein, A. "A Simple Model of Equilibrium in Search Procedures", *Journal of Economic Theory*, 1997, 72, 432-441.

Hart, S. " Discrete Colonel Blotto and General Lotto Games". *Working paper*. 2007.

Ho, T, H. Camerer, C. and Weigelt, K. “Iterated Dominance and Iterated Best Response in Experimental ‘*p*-Beauty Contests’.” *American Economic Review*, 1998, 88(4), 947–969.

Kahneman, D. and Frederick, S. "Representativeness Revisited: Attribute Substitution in Intuitive Judgment". *Heuristics & Biases: The Psychology of Intuitive Judgment*, Eds: T. Gilovich, T. , Griffin, D. and Kahneman, D, New York: Cambridge University Press, 2002, 49-81.

Laslier, J, F. and Picard, N. "Distributive Politics: Does Electoral Competition Promote Inequality?" *Journal of Economic Theory*, 2002

Myerson, R.B. "Incentives to Cultivate Minorities under Alternative Electoral Systems". *The American Political Science Review*. 1993, 87, 856–869.

Nagel, R. "Unraveling in Guessing Games: An Experimental Study." *American Economic Review*, 1995, 85 (5), 1313-1326.

Roberson, B. "The Colonel Blotto Game", *Economic Theory*, 2006, 29, 1–24.

Rubinstein, A. Tversky, A. and Heller, D. "Naive Strategies in Competitive Games," *Understanding Strategic Interaction - Essays in Honor of Reinhard Selten*, Eds: Albers, W. et al, Springer-Verlag, 1996, 394-402.

Rubinstein, A. "Instinctive and Cognitive Reasoning: Response Times Study", *Economic Journal,* 2007, 117, 1243-1259.

Stahl, D. and Wilson, P. "Experimental Evidence on Players' Models of Other Players", *Journal of Economic Behavior and Organization*, 1994, 25 (3), 309-327.

Stahl, D. and Wilson, P. "On Players' Models of Other Players: Theory and Experimental Evidence", *Games and Economic Behavior*, 1995, 10 (1), 218-254.

Weinstein, J. "Two notes on the Blotto game". *Northwestern University (mimeo)* 2005.

7. Appendix:

The score matrix

	Cycle 1				Cycle 2				Cycle 3				Cycle 4				Cycle 5				Cycle 6			
Strategies by cycles (score of the row player)	**1234**	**4123**	**3412**	**2341**	**4321**	**3214**	**2143**	**1432**	**1324**	**4213**	**3142**	**2431**	**4231**	**3124**	**2413**	**1342**	**1243**	**4132**	**3421**	**2314**	**2134**	**1423**	**4312**	**3241**
1234	2	1	2	3	2	2	2	2	2	1.5	2	2.5	2	1.5	2	2.5	2	1.5	2	2.5	2	1.5	2	2.5
4123	3	2	1	2	2	2	2	2	2.5	2	1.5	2	2.5	2	1.5	2	2.5	2	1.5	2	2.5	2	1.5	2
3412	2	3	2	1	2	2	2	2	2	2.5	2	1.5	2	2.5	2	1.5	2	2.5	2	1.5	2	2.5	2	1.5
2341	1	2	3	2	2	2	2	2	1.5	2	2.5	2	1.5	2	2.5	2	1.5	2	2.5	2	1.5	2	2.5	2
4321	2	2	2	2	2	1	2	3	2	1.5	2	2.5	2	1.5	2	2.5	2	2.5	2	1.5	2	2.5	2	1.5
3214	2	2	2	2	3	2	1	2	2.5	2	1.5	2	2.5	2	1.5	2	1.5	2	2.5	2	1.5	2	2.5	2
2143	2	2	2	2	2	3	2	1	2	2.5	2	1.5	2	2.5	2	1.5	2	1.5	2	2.5	2	1.5	2	2.5
1432	2	2	2	2	1	2	3	2	1.5	2	2.5	2	1.5	2	2.5	2	2.5	2	1.5	2	2.5	2	1.5	2
1324	2	1.5	2	2.5	2	1.5	2	2.5	2	1	2	3	2	2	2	2	1.5	2	2.5	2	2.5	2	1.5	2
4213	2.5	2	1.5	2	2.5	2	1.5	2	3	2	1	2	2	2	2	2	2	1.5	2	2.5	2	2.5	2	1.5
3142	2	2.5	2	1.5	2	2.5	2	1.5	2	3	2	1	2	2	2	2	2.5	2	1.5	2	1.5	2	2.5	2
2431	1.5	2	2.5	2	1.5	2	2.5	2	1	2	3	2	2	2	2	2	2	2.5	2	1.5	2	1.5	2	2.5
4231	2	1.5	2	2.5	2	1.5	2	2.5	2	2	2	2	2	1	2	3	2.5	2	1.5	2	1.5	2	2.5	2
3124	2.5	2	1.5	2	2.5	2	1.5	2	2	2	2	2	3	2	1	2	2	2.5	2	1.5	2	1.5	2	2.5
2413	2	2.5	2	1.5	2	2.5	2	1.5	2	2	2	2	2	3	2	1	1.5	2	2.5	2	2.5	2	1.5	2
1342	1.5	2	2.5	2	1.5	2	2.5	2	2	2	2	2	1	2	3	2	2	1.5	2	2.5	2	2.5	2	1.5
1243	2	1.5	2	2.5	2	2.5	2	1.5	2.5	2	1.5	2	1.5	2	2.5	2	2	1	2	3	2	2	2	2
4132	2.5	2	1.5	2	1.5	2	2.5	2	2	2.5	2	1.5	2	1.5	2	2.5	3	2	1	2	2	2	2	2
3421	2	2.5	2	1.5	2	1.5	2	2.5	1.5	2	2.5	2	2.5	2	1.5	2	2	3	2	1	2	2	2	2
2314	1.5	2	2.5	2	2.5	2	1.5	2	2	1.5	2	2.5	2	2.5	2	1.5	1	2	3	2	2	2	2	2
2134	2	1.5	2	2.5	2	2.5	2	1.5	1.5	2	2.5	2	2.5	2	1.5	2	2	2	2	2	2	1	2	3
1423	2.5	2	1.5	2	1.5	2	2.5	2	2	1.5	2	2.5	2	2.5	2	1.5	2	2	2	2	3	2	1	2
4312	2	2.5	2	1.5	2	1.5	2	2.5	2.5	2	1.5	2	1.5	2	2.5	2	2	2	2	2	2	3	2	1
3241	1.5	2	2.5	2	2.5	2	1.5	2	2	2.5	2	1.5	2	1.5	2	2.5	2	2	2	2	1	2	3	2

Proofs

Proposition 1: *A probability distribution P constitutes an equilibrium if and only if:*

$$\begin{pmatrix} x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{pmatrix} = \begin{pmatrix} -(x_1 + x_3 + x_5) \\ -(x_2 + x_4 + x_6) \\ (-x_2 - x_4 - 2x_6 + x_3 - x_1)\cdot 0.5 \\ (x_1 + x_3 + 2x_5 + x_4 - x_2)\cdot 0.5 \\ (x_2 + x_4 + 2x_6 + x_3 - x_1)\cdot 0.5 \\ (-x_1 - x_3 - 2x_5 + x_4 - x_2)\cdot 0.5 \end{pmatrix}$$

Proof: The following system of 12 linear equations characterizes the game's set of equilibria:

$$\begin{cases} X_5 + X_7 + X_9 + X_{11} + 2X_1 = 0 \\ X_6 + X_8 + X_{10} + X_{12} + 2X_2 = 0 \\ X_5 + X_7 - X_9 - X_{11} + 2X_3 = 0 \\ X_6 + X_8 - X_{10} - X_{12} + 2X_4 = 0 \\ X_1 + X_3 - X_{10} + X_{12} + 2X_5 = 0 \\ X_2 + X_4 + X_9 - X_{11} + 2X_6 = 0 \\ X_1 + X_3 + X_{10} - X_{12} + 2X_7 = 0 \\ X_2 + X_4 - X_9 + X_{11} + 2X_8 = 0 \\ X_1 - X_3 + X_6 - X_8 + 2X_9 = 0 \\ X_2 - X_4 - X_5 + X_7 + 2X_{10} = 0 \\ X_1 - X_3 - X_6 + X_8 + 2X_{11} = 0 \\ X_2 - X_4 + X_5 - X_7 + 2X_{12} = 0 \end{cases}$$

and its solution is the 6-dimension space that appears in the proposition. ■

Proposition 2. *If 1>P(S)>0.5 for some S, then D(S) is the best response to P.*

Proof:

Assume without loss of generality that 1>$P(L_0)$>0.5. We need to show that no strategy earns as high a score as L_1. It is sufficient to show that for any $X \neq L_1$, $score(X, L_0) = (3-t)$ implies that $score(X,Y) - score(L_1,Y) \leq t$ for any *Y*. In other words, *X* cannot compensate for its inferiority to L_1 against L_0 by its superiority when playing against some other strategies. The proof continues by considering all the possible strategies *X* and confirms that the condition on the payoffs is satisfied for all of them:

(I) The case of X=L_3 is straightforward: $score(L_3, L_0) = 1$, and $score(L_3,Y) - score(L_1,Y) \leq 2$ since the lowest possible score is 1 point and the highest is 3 points.

(II) If $score(X, L_0) = 2$, assume to the contrary that $score(X,Y) - score(L_1,Y) > 1$. This implies that $score(L_1,Y) = 1$ or 1.5 and thus *Y* can only be L_2 or S_2(i), for i=3,4,5,6. However, the only strategies that score 2.5 or 3 points against L_2 or S_2(i) are S_3(i) and L_3, which do not tie with L_0, a contradiction.

(III) In the case of X=S_1(i), for *i*=3,4,5,6, $score(X, L_0) = 2.5$. Since S_1(i) is parallel to L_1, it scores at most half a point more than L_1 against $Y \in Cycle\ i$ or *Cycle 1*. $S_1(i)$ can score at most 2.5 points against $Y \notin cycle\ i\ \ or\ \ cycle\ 1$, while L_1 scores at least 1.5 points. $Score(L_1,Y) = 1.5$ only if Y=$S_2(j)$ for *j*=3,4,5,6, and $score(S_1(i), S_2(j)) < 2.5$.

(IV) In the case of X=S_3(i), for *i*=3,4,5,6, $score(X, L_0) = 1.5$. S_3(i) cannot score 2 points more than L_1 against some other strategy *Y*: $score(S_3(i),Y) = 3$ only for Y=$S_2(i)$, and $score(L_1, S_2(i)) = 1.5$ and not 1. ■

Experimental results

Table 1 and 2 below present the aggregate quantitative data for Version 1 and 2. The columns from left to right are: row number; all 24 possible strategies in the game; median response time (RT) of each strategy; the number and then proportion of subjects who chose the strategy; the average score of that strategy in the general tournament; the notation used for each strategy; and in Table 1 an additional column presents the distribution following the normalization discussed on page 23.

Table 1

Version 1

N=641

	Strategies	RT	#	%	Score	Notation	Normalization
1	**1234**	125s	141	**22%**	1.94	L_0	42.40%
2	**4123**	194s	65	**10.14%**	2.22	L_1	14.84%
3	**3412**	165s	21	**3.28%**	2.06	L_2	0%
4	**2341**	196s	23	**3.59%**	1.78	L_3	0%
5	**4321**	158.5s	56	**8.74%**	1.96	$S_0(2)$	13.43%
6	**3214**	172s	23	**3.59%**	2.06	$S_1(2)$	2.12%
7	**2143**	167s	18	**2.81%**	2.04	$S_2(2)$	0%
8	**1432**	221s	17	**2.65%**	1.94	$S_3(2)$	0%
9	**1324**	80s	33	**5.15%**	1.94	$S_0(3)$	7.77%
10	**4213**	146s	25	**3.9%**	2.16	$S_1(3)$	5.30%
11	**3142**	109s	11	**1.72%**	2.06	$S_2(3)$	0%
12	**2431**	165s	10	**1.56%**	1.84	$S_3(3)$	0%
13	**4231**	96s	23	**3.59%**	1.95	$S_0(4)$	0%
14	**3124**	181.5s	18	**2.81%**	2.12	$S_1(4)$	1.41%
15	**2413**	102s	23	**3.59%**	2.05	$S_2(4)$	0%
16	**1342**	120s	14	**2.18%**	1.88	$S_3(4)$	0%
19	**1243**	106s	15	**2.34%**	2.01	$S_0(5)$	2.47%
20	**4132**	140s	13	**2.03%**	2.08	$S_1(5)$	0%
17	**3421**	126.5s	8	**1.25%**	1.99	$S_2(5)$	0%
18	**2314**	128s	27	**4.21%**	1.92	$S_3(5)$	4.95%
21	**2134**	96s	19	**2.96%**	1.96	$S_0(6)$	3.18%
22	**1423**	101s	11	**1.72%**	2.07	$S_1(6)$	0%
23	**4312**	112.5s	10	**1.56%**	2.04	$S_2(6)$	0%
24	**3241**	141s	17	**2.65%**	1.93	$S_3(6)$	2.12%

Table 2

Version 2

N=704

	Strategies	RT	#	%	Score	Notation
1	**1234**	116s	131	**18.61%**	0.03	L_0
2	**4123**	194s	91	**12.93%**	0.19	L_1
3	**3412**	170s	37	**5.26%**	0.13	L_2
4	**2341**	157s	20	**2.84%**	0.05	L_3
5	**4321**	161s	41	**5.82%**	0.02	$S_0(2)$
6	**3214**	187s	38	**5.40%**	0.06	$S_1(2)$
7	**2143**	179s	17	**2.41%**	0.05	$S_2(2)$
8	**1432**	158s	12	**1.70%**	0.02	$S_3(2)$
9	**1324**	90.5s	24	**3.41%**	0.02	$S_0(3)$
10	**4213**	173s	33	**4.69%**	0.03	$S_1(3)$
11	**3142**	128s	15	**2.13%**	0.05	$S_2(3)$
12	**2431**	106s	11	**1.56%**	0.02	$S_3(3)$
13	**4231**	124s	30	**4.26%**	0.01	$S_0(4)$
14	**3124**	130s	17	**2.41%**	0.04	$S_1(4)$
15	**2413**	193.5s	22	**3.12%**	0.02	$S_2(4)$
16	**1342**	141.5s	8	**1.14%**	0.03	$S_3(4)$
19	**1243**	136s	18	**2.56%**	0.05	$S_0(5)$
20	**4132**	171s	17	**2.41%**	0.03	$S_1(5)$
17	**3421**	122s	8	**1.14%**	0.02	$S_2(5)$
18	**2314**	115s	33	**4.69%**	0.01	$S_3(5)$
21	**2134**	136s	28	**3.98%**	0.03	$S_0(6)$
22	**1423**	208s	21	**2.98%**	0.04	$S_1(6)$
23	**4312**	139s	11	**1.56%**	0.03	$S_2(6)$
24	**3241**	128s	21	**2.98%**	0.02	$S_3(6)$

Table 3

Comparison of the two versions in Experiment 3

N=279, 131 in Version 1 and 148 in Version 2.

Strategies	Version 1 (%)	Version 2 (%)	Score in Version 1	Score in Version 2
1234 - L_0	10.7%	10.1%	1.87	0.02
4123 - L_1	19.1%	18.9%	2.1	0.1
3412 - L_2	4.6%	6.8%	2.23	0.19
2341 - L_3	2.3%	2.0%	2	0.07
4321 - $S_0(2)$	7.6%	4.1%	2.02	0.01
3214 - $S_1(2)$	5.3%	10.8%	2.06	0.04
2143 - $S_2(2)$	6.1%	2.7%	2.07	0.11
1432 - $S_3(2)$	0.8%	1.4%	2.04	0.03
1324	1.5%	1.4%	1.98	0.03
4213	1.5%	6.8%	2.06	0.01
3142	3.1%	2.0%	2.12	0.07
2431	3.8%	2.7%	2.04	0.02
4231	2.3%	3.4%	1.91	0.02
3124	3.1%	5.4%	2.1	0.03
2413	2.3%	8.1%	2.18	0.05
1342	3.1%	2.0%	2	0.08
1243	0%	0.7%	----	0.03
4132	3.8%	0.7%	2.04	0.01
3421	2.3%	0%	2.14	----
2314	1.5%	2.7%	2.06	0
2134	5.3%	3.4%	1.99	0.02
1423	3.1%	0%	2.1	----
4312	3.8%	2.0%	2.11	0
3241	3.1%	2.0%	2	0.02

Details on response time and explanations in Experiment 1

Subjects' explanations and response times are used to support the interpretation of the main strategy choices in Experiment 1 (Version 1).

The response time of L_0 suggests that it is typically an instinctive choice. Its response time is lower than that of each of the strategies in cycle 1 and lower than that of the rest of the strategies taken as a whole, even when L_1 is excluded. Furthermore, its response time is lower in comparison to all strategies that are not in the first cycle taken as a whole.[22] 28% of those who chose L_0 did not explain their choice. Of those who did (83): 39% belong to the intuitive category, 26% belong to the random choice category and 35% belong to the category of other strategic rules.

The choice of L_1 is clearly an outcome of cognitive reasoning: It has higher response time than the rest of the strategies taken as a whole, even when L_0 is excluded. Only 11% of those who chose this strategy did not provide an explanation. Among those who did provide an explanation (41), 75% of the explanations belong to the category of iterated reasoning.

The response time of L_1 is not significantly different from those of L_2 or L_3, or the class that includes both. Among the subjects who chose L_3, none of their explanations included a process of iterated reasoning. Only three explanations (out of 15) for the choice of L_2 explicitly described the use of two levels of iterated reasoning. No one chose a strategy other than L_1 and L_2 while explaining that he had used two levels of iterated reasoning or higher. Considering the low frequencies of L_2, L_3, $S_2(2)$ and $S_3(2)$, and taking into account that various decision rules can lead to these choices, I conclude that level-2 and level-3 types are negligible in this game.

The response time of $S_0(2)$ is higher than that of L_0 and not significantly lower than that of L_1, or the response time of all other strategies taken as a whole. This finding suggests that the strategy $S_0(2)$ is not as instinctive as L_0 and does not play the same role as L_0. 27% of the subjects who chose it did not explain their choice. Among those who did provide an explanation (32), 25% of the explanations belong to the random choice category, around 60% are based on other strategic rules and around 15% are intuitive.

---

[22] The Mann-Whitney U test, also known as the Wilcoxon Two-Sample Test, was used to test the differences in response time. The significance level for all the results was at least 5%. 52 observations with response times higher than 600 seconds were omitted (the RT was higher than 1000 in 50% of these observations). It is likely that these observations do not reflect real response times and omitting them reduces the noise.

Version 1 and 2 as they appear on the didactic website

Games and Behavior - The Problems

The Problem

You are a tennis team coach, planning to send your team to a tournament. Each team in the tournament has four players: one of level **A+** (the highest level), one of level **A**, one of level **B+**, and one of level **B** (the lowest level).

The coach's task is to assign his players to **"position 1"**, **"position 2"**, **"position 3"** and **"position 4"** (one player in each position).
Each team will play against each of the other teams in the tournament. A game between two teams includes four matches: a player that was assigned by his coach to "position X" will play once against the player in "position X" of the other team. You don't know how the other coaches assign their players.

In any match between two players of different levels, the one with the higher level wins. When two players with the same level play, the outcome is a tie.

[In version 1 - A winner in a match brings his team **1 point**, and a player who ends the match with a tie brings his team **½ a point**. A loss yields **0 points**.]

[In version 2 - **At the end of any game between two teams, a team gets 1 point only if it won three matches out of four**. In such a case, the other team gets 0 points. In case of any other result, none of the teams gets points.]

The team's score at the end of the tournament is the number of points it gained in all the games against other teams.
The winning team is the one with the highest score, and the prize is $10,000. [In case of several winning teams, the prize is divided between them.]
The only goal of players and coaches (including you) is to have their team getting the highest score among the teams.

How will you allocate your players in order to achieve this goal?**

** Note that other students in your class play the role of other coaches in the tournament, so your total score in this game will be your team's total score, after playing against each of the other students' teams.

	A+	A	B+	B
pos 1				
pos 2				
pos 3				
pos 4				

THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH and
THE SACKLER INSTITUTE FOR ECONOMIC STUDIES
The Eitan Berglas School of Economics
Tel Aviv university

Recent list of Working Papers

1-2008	Limor Hatsor	The Allocation of Public Education Resources
2-2008	Anat Bracha Donald J. Brown	Affective Decision Making: A Behavioral Theory of Choice
3-2008	Saul Lach Gil Shiff Manuel Trajtenberg	Together but Apart: ICT and Productivity Growth in Israel
4-2008	Ariel Rubinstein	Comments on Neuroeconomics
5-2008	Tomer Blumkin Yoram Margalioth Efraim Sadka	Incorporating Affirmative Action into the Welfare State
6-2008	Chemi Gotlibovski Yoram Weiss	Retirement Decisions of Married Couples
7-2008	Ayala Arad	The Tennis Coach Problem: A Game-Theoretic and Experimental Study

----

Foerder Institute for Economic Research, Tel-Aviv University, Tel-Aviv, 69978 Israel, Tel: 972-3-640-9255; fax: 972-3-640-5815; e-mail: foerder@post.tau.ac.il
Papers from 2000 onwards (and some from 1998 and1999) can be downloaded from our website as follows: **http://econ.tau.ac.il/research/search_workingPapers.asp**