Private Investments in Higher Education: Comparing Alternative Funding Schemes

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Abstract: We consider an OLG economy with endogenous human capital formation. Young individuals make decision about their investment in higher education only after they obtain some signal, correlated to their ability. We examine three different funding regimes, each requires governmental intervention but not funding, available to students if they choose to invest in higher education. The economic implications of such funding schemes in equilibrium are studies, in particular the effects on accumulation of human capital and income inequality.

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Key words: higher education, funding, human capital, income inequality.

1. Introduction

The important role played by higher education in the development process of countries and its significant impact on the distribution of income has long been recognized by economists. Researchers demonstrated cross-country evidence that show positive association between investment in education and economic growth [see, e.g., Barro (1998) and Bassinini and Scarpenta (2001)]; particularly, those OECD countries who exapanded higher education more rapidly from the 1960's experienced faster growth. The level of education has been shown to have positive effect on physical capital investments. Higher education has been expanded considerably in the OECD countries during the second half of the twentieth century; this is in terms of aggregate numbers of students as well as the total funding coming from public and private sources. The most striking example is the UK's higher education system: In 1960 there were 400,000 full time students compared to 2 million in the year 2000 [see, Greenaway and Haynes (2003)]. As a result of this expansion, all major industralized countries in Europe and elsewhere have been grappling with financing the rising costs of of higher education/ training systems. Due to fiscal pressures, we now observe a process of shifting part of this financial burden from public funding towards the students.[1] We also see a shift from income support transfers to programs based on students loans, which has resulted in a significant decline in the public funding per student. Clearly, the cost of higher education extends well beyond payments of tuitions, students in universities do not live with their parents anymore. Access to attarctive loans, to be invested in education, which are used for tuition, housing, equipment etc. also allow students to devote more time to improving their scholar achievements.

[1] Even in Russia 47% of the students in the higher education system must finance their tuition fees (which are significant), as well as other related costs from their own resources [see Kaganovich (2005)].

Although, there are good reasons for governmental intervention in the provision of higher education, there are some justifications to proceed with shifting part of the financial load to personal funding. In certain countries, due to the socio-economic background of the students, subsidizing heavily the higher education results in adverse effect on the distribution of income. On the other hand, it is desirable to remove the financial barriers to enhance participation of the younger generation in the higher education system. Due to the well-documented market failure in financing higher education, some other alternatives should be examined. Milton Friedman (1955, 1962) was the first to raise this issue and to suggest some solutions. Friedman was the first to offer income-contingent financing of students' investments in higher education. After pointing out the empirical evidence that indicates the underinvestment phenomena in higher education, he proposes to create financial instruments that allow investors to "buy" part of a student's future income: "....for education would be to "buy" a share of an individual's earning prospects" [Friedman (1962), p. 103]. These methods of financing schemes are designed to reduce the risk that students face, since such an arrangement will provide a hedge against low or no income in the future. In other words, we need to consider not a 'mortgage-type' contracts, or graduate taxes (even though it is income-dependent), but rather income-contingent paytment scheme. Such ideas were described in Barr and Crawford (1998), in Greenaway and Haynes (2003) and, in a more extensive way, by Lleras (2004) who focuses on the implementation of various ways of funding higher education via the private sector. Lleras considers few possibilities to carry out the income contingent funding schemes, including the case where we take into account ability and future labor possibilities among students. The design of such student loan program, repayment terms and debt default, as well as the international experience, has been discussed by Woodhall (1988). The problems involving cost of such programs and evidence related to loans collectibilty has been discussed by Albrecht and Ziderman (1993). Do such ideas really work?

let us describe specific programs for private funding of higher education implemented in some countries. The role of student financial aid in the US has been studied by Dynarski (2001), where it was shown that in the last two decades of the twentieth century "offering $1000 of grant aid increased the probability of attending college by 3.6% and years of completed schooling by a tenth of a year".

In 1989 Australia was the first to initiate income contingent students loans program [for details, see Chapman (1997)], which turned out to be a successful experiment. Following Australia came Ghana, Sweden, Chile, New Zealand. and the UK [see Lleras (2004)]. The uniqueness of the Australian model was not only in converting the paybacks of the education loans to be income-contingent, but also in using the existing tax authorities as a collection agency, which was unprecedented. The Australian system has been successful, in part, due to the well organized collection system that operates at low marginal cost. More precisely, the collection cost of loan repayments is 0.5% in this case, which is , clearly, a low cost. Moreover, taking legal actions agaist individuals is, relatively cheap under this collection mechanism. A reliable low cost loan repayment system is an essential component in such funding program.

These ideas were adopted by the other countries, including the UK. In the UK, as pointed out by Greenaway and Haynes [(2003), p.F162], "after 1998 income contingency applies and students become liable for repayment of maintenance loans once their gross income exceeds £10,000 per annum. Beyond this, graduates pay 9% of their marginal income, collected by the Inland Revenue and passes on to the Student Loans Company". Clearly, such a mechanism also minimizes debt default. In the US there has been recently some changes in the sudent loans repayment program. The ecactment of the Higher Education Reconciliation Act (HERA) of 2005 modifies the Direct Loan program which offers contingent repayment plans as well as 'incomesensitive' repayments. In some special cases interest rates are subsidized, and under some circustances HERA strengthens the teacher loan forgiveness plan.

Our aim in this paper is to use a general equilibrium framework in which we can analyze the economic consequences of various schemes to funding private expenditures during the higher education period. The framework used in this work is an overlapping generations model with endogenous human capital formation. This type of models were used, for example, by Azariadis and Drazen (1990), Orazem and Tesfatsion (1997), Viaene and Zilcha (2002). Heterogeneity in our case is introduced via random innate abilities in each generation. Throughout our analysis we take the investment made by the government in higher education to remain fixed. However, individual's own investment in higher education takes place after observing a signal correlated to his/her ability (or the random future income).

We shall consider three different funding schemes and examine their equilibrium effects on aggregate investments in human capital, on accumulation of human capital (and hence the impact on growth) and the compare their implication to income inequality. The three funding schemes we consider here are:

(a) Access to Credit Markets: guaranteeing that young individuals, who wish to invest in higher education, have unrestricted access to the existing credit markets.

(b) Full Risk Sharing: payments of education loans are income contingent where risk about future income (due to random abilities) is shared by all students of the same cohort,

(c) Partial Risk Sharing: future payments are contingent on the random future income, but only within groups of students who are of the 'same type' when they enter the higher education system.

Case (a) does not contain any element of repayment being income-dependent; in fact, each student needs to pay back his/her loans under the credit market terms. Thus, implementation in this case requires overcoming hurdles that prevent free access to credit during the studying period. Case (b) is a mechanism of risk sharing of the uncertainty about future income within the students population of the same cohort, where loan paybacks are fully linked to the (random) future income. Thus no external funds are needed to subsidize this program, it only requires implementation via certain mechanism that guarantees revelation of information about income and debt collection (see the Australian example). Case (c) contains partial risk sharing : each group (or 'type') will have its own risk sharing: payments of debt is income contingent within that type of students. There is no risk sharing among students with "different ability charecretistics"; this restricts the 'cross subsidization' between groups characterized by different 'signals' (which are correlated to incomes).

The paper is organized as follows. We present our model and the above mentioned three financing regimes in Section 2. Section 3 examines the implications of these financing schemes to human capital accumulation. Section 4 compares the welfare implications these funding schemes. The implications to income inequality are studied in section 5. Section 6 concludes the paper. Some proofs are relegated to the Appendix.

2. The Model

We consider an overlapping-generations economy with a single commodity, say, physical capital, which can be consumed or invested in production. Individuals live for three periods: the 'youth period', where each individual is supported by parents and acquires education and skills, has no consumption decisions to make. In the 'middle period', where individuals work, obtain income, consume and save. Finally, the 'retirement period' in which individuals consume their total savings. There is no population growth and each generation G_t (i.e., all individuals born at date $t-1$), $t = 0, 1, 2...$, consists of a continuum of agents with (Lebesgue-) measure 1, say the interval $[0, 1]$.

Our framework is characterized by heterogenous individuals in each generation, where heterogeneity is generated by random innate ability. While nature assigns abilities to individuals at birth, no individual knows exactly his/her own ability while 'young'. We denote by $\nu(A)$ the time-independent density function of agents with ability A , where $A \in \mathcal{A} = [\underline{A}\ ,\ \bar{A}] \subset \mathbb{R}_{++}$. Thus, agents are exposed to uncertainty about their ability during their youth period since it becomes known only at the outset of the second period. However, since the probability distribution function over $\mathcal{A}$ is time-independent, there is no aggregate risk: our modeling approach follows the technique suggested in Feldman and Gilles [1985, Proposition 2], where risk exists at the individual level but in the aggregate there is no uncertainty.

The production function of human capital is, in general, a complex function which depends on individual, family and exogenous parameters. We shall restrict the structure of the human capital formation process, in order to make our equilibrium comparative dynamics analytically manageable. We assume that the level of human capital, or skills, of an individual $i \in G_t$, denoted by h_t^i, depends on the (random) innate ability A^i , the private investment in education x^i and the *average* human capital of the older generation, denoted by H_{t-1} (which may represent the human capital of 'teachers'). Namely,

$$h_t^i = \varphi(A^i) g(x_t^i, H_{t-1}). \tag{1}$$

This process, characterized by the function $g(x, H)$, implicitly contains the *public investment* in education, assumed to be equal to all and constant over time. We make the following assumption about this process:

(A1) As a function of x, $g(x, H)$ is strictly increasing, concave and differentiable. Also, $g_1(x, H)$ is nondecreasing in H; $\varphi(A)$ is an increasing and differentiable function.

We have used the notaion $g_1(x, H) = \frac{\partial g(\cdot)}{\partial x}$. The choice of the private investment in education x_t^i (in the youth period) is done by individual i after observing some signal $y^i \in Y \subset \mathbb{R}$, assumed to be correlated with his/her innate ability. The signals assigned to agents with ability A are distributed according to the density $\nu_A(y)$. Also, the distribution of signals received by agents in the same generation has the density: $\mu(y) = \int_{\mathcal{A}} \nu_A(y)\nu(A)\, dA$. Let $\nu_y(A)$ be the density of the conditional distribution of A given the signal y. The average ability of all agents who have received the signal y is given by: $\bar{\varphi}(y) = E[\varphi(\tilde{A}) \mid y] = \int_{\mathcal{A}} \varphi(A)\nu_y(A)\, dA$.

In our economy the signals are public information and the investments made by individuals in their education are observable. We assume throughout the paper that the Monotone Likelihood Ratio Property (MLRP) holds, namely : for any prior distribution of the random variable A, $y' \geqslant y$ implies that the posterior distribution conditional on y' dominates the posterior distribution conditional on y in the first-degree stochastic dominance [see, Milgrom (1981)]. In particular, if $y' \geqslant y$ then for any increasing function $f(z)$ we have: $E[f(A) \mid y'] \geqslant E[f(A) \mid y]$. Thus, higher signals are 'good news'.

Assume that the investment in education by the young individual should be financed by loans. Consider now the optimization that determines the optimal level of this investment. Notice that this choice is made after the signal y^i is observed; however, future income is still random due to the uncertainty about human capital. We also assume, implicitly, that the *public* investments in education are reflected by the technology of production of human capital $g(x, H)$ (this process of human capital formation is assumed to be time-invariant, for simplicity). We shall consider three different alternative interventions by the government in financing students' higher education investments:

(1) **Access to Credit Markets**: Students have access to the existing financial markets to finance their education investment. In particular, in this regime the government guarantees unrestricted access to the credit markets to each student, as

well as enforcement of debt collection. This financing scheme will be called funding **regime I**.

(2) **Full-insurance of loans**: In this regime each individual has to pay back is **linked to the realization of future income** (hence, linked to the realization of human capital). The pooling of risks in this insurance arrangement contains *all* young individuals who choose to invest in education. The governmental intervention contains releasing information about individual income, as well as guaranteeing the collection of debt. This financing scheme will be called funding **regime II.**

(3) **Restricted-insurance of loans**: The amount to be paid back by each individual is linked to realization of his/her income; however, the pooling of risks in this insurance arrangement is among the group of individuals who have the *same signal*!! The role played by the government in this case is similar to that in case (2). We call this financing scheme **regime III**.

Given certain funding regime we assume that all agents must use this channel of financing. Namely, we do not allow students to "diversify" between the insured-loans channel and the credit market funding channel. Given the evidence about borrowing constraints that students face in the financial markets (see, e.g., Galor and Zeira (1993)), this seems a reasonable assumption. Clearly, some government intervention is needed in order to implement funding regime I; namely, to guarantee access to credit markets for students. Since investments in human capital formation are important to the creation of human capital we shall study and compare the economic implications that the various funding schemes have upon the equilibrium aggregate human capital and the intragenerational income distribution.

We assume that agents are expected utility maximizers with von-Neumann Morgenstern lifetime utility function: $u_1(c_1) + u_2(c_2)$, where $u_1(c_1)$ and $u_2(c_2)$ where c_1 and c_2 denote the consumption in the second and third period of lifetime. Note that agents do not make any consumption decisions while they are young. We assume that these utility functions are increasing and strictly concave.

Production in our economy is carried out by competitive firms using two production factors: physical capital K and human capital H. We assume that the aggregate production function is $F(K,H)$ exhibits constant returns to scale, full depreciation of physical capital and that individuals supply labor (in their 'working period') inelastically, normalized to be 1. Given the human capital level of agent i, denoted by h^i, and the wage rate w per unit of human capital, the wage income is equal to $h^i \cdot w$. We assume that:

(A2) $F(K,H)$ is concave, homogenous of degree 1 and satisfies: $F_K > 0$, $F_H > 0$, $F_{KK} < 0$ and $F_{HH} < 0$.

We assume throughout this paper that **physical capital** is internationally mobile while **human capital is immobile**. The economic implications of capital mobility are clear: interest rate at each date t, $\bar{r}_t$, is exogenously given. Thus, given the stock of human capital H_t at date t, the stock of physical capital K_t must adjust in a way that:

$$R_t = 1 + \bar{r}_t = F_K(K_t, H_t), \qquad t = 1, 2, 3, ... \tag{2}$$

In a competitive labor market the wage rate is given by the marginal product of labor, i.e.,

$$w_t = F_L\left(\frac{K_t}{H_t}, 1\right) \qquad t = 1, 2, 3, ... \tag{3}$$

2.1. Financing Regime I. Let us consider first the optimal investment in education and saving behavior under the funding regime I. Agents choose optimal investment in generating human capital under uncertainty due to the random ability that realizes only later; however, all agents with the same signal y will choose the same investment $x(y)$. Consider agent i's ($i \in G_t$) optimization when 'young' in this case. Given the signal y , denote by c_t^{1i} and c_t^{2i} consumption in the 'working period' and the 'retirement period' of agent i, and let $x_t^i(y)$ be the agent's choice of investment, made under uncertainty. The optimal saving s_t^i , which is determined in the working period, is made *after* the uncertainty about income has been resolved. For simplicity, let us drop the individual index i. Given the market interest rates and wages, $\{\bar{r}_t, w_t\}_{t=1}^{\infty}$, the level of H_{t-1} and the agent's signal y the optimal **saving** decision at date t is determined by,

$$\max_{s_t} \; u_1(c_t^1) + u_2(c_t^2) \tag{4}$$

$$\text{s.t.} \qquad \begin{aligned} c_t^1 &= w_t \varphi(\tilde{A}) g(x_t(y), H_{t-1}) - R_t x_t(y) - s_t \\ c_t^2 &= R_{t+1} s_t. \end{aligned}$$

Denote optimal savings by $s_t(I_t(A, y))$ where $I_t(A, y)$ is the *net* income in the working period,

$$I_t(A, y) = w_t \varphi(\tilde{A}) g(x_t(y), H_{t-1}) - R_t x_t(y) \tag{5}$$

The necessary and sufficient condition for problem (4) is:

$$u_1'(I_t(A, y) - s_t) = R_{t+1} u_2'(R_{t+1} s_t) \qquad A \in \mathcal{A}. \tag{6}$$

Note that, by our assumptions about the utility functions, equation (6) implies that the optimal saving $s_t(I_t)$ and $I_t - s_t(I_t)$ are increasing functions in net lifetime income I_t. Now, when the 'young' agent chooses an optimal level of investment, taking into account the optimal savings behavior $s_t(I_t(A, y))$, the investment in education is determined by:

$$\max_{x_t} \; E[u_1(c_t^1) + u_2(c_t^2)| \;\; y] \tag{7}$$

$$\text{s.t.} \qquad \begin{aligned} c_t^1 &= I_t(A, y) - s_t \\ c_t^2 &= R_{t+1} s_t, \end{aligned}$$

where $I_t(A, y)$ is given by equation (5) and s_t satisfies equation (6). By the Envelope theorem and the strict concavity of the utility functions, this optimization has a unique solution determined by the first order condition:

$$E\{[w_t \varphi(\tilde{A}) g_1(x_t, H_{t-1}) - R_t] u_1'(c_t^1) \mid y\} = 0 \tag{8}$$

Let us define competitive equilibrium under the funding regime I. The same definition generalizes directly to the other (forthcoming) regimes as well. Our economy start at date 0 with a given initial stocks of physical capital K_0 and human capital H_0 . The dynamic equilibrium will depend upon the evolution of the human capital stock, hence, upon the investment opportunities of the younger generation in human capital. This will affect the accumulation and the distribution of human capital along the equilibrium path. We assume throughout this paper that labor (employees) is internationally **immobile**, while physical capital is fully **mobile**. This is in line with the globalization process that we witness in recent decades where international capital mobility has been increasing tremendously. Given the the aggregate production function $F(K, H)$ and the (international) interest rates $\{r_t\}_{t=1}^{\infty}$ (exogenous), we derive the competitive market **wages** $\{w_t\}_{t=1}^{\infty}$, where each w_t is given by the marginal product of effective labor.

We say that $\{(c_t^1, c_t^2), x_t(y), s_t \; ; \; (r_t, w_t)\}_{t=1}^{\infty}$ is a **Competitive equilibrium** (under financing scheme I), if for all dates t, $t = 1, 2, ...$

(a) **Optimum**: each agent with a signal y attains the optimum, in optimization problem (4), in $[(c_t^1, c_t^2), x_t(y), s_t]$.

(b) **Market Clearing conditions** hold: Given that the aggregate capital stock satisfies: $R_{t+1} = 1 + r_{t+1} = F_K(K_{t+1}, H_{t+1})$, The wages are determined by the marginal product of effective labor,

$$w_{t+1} = F_L(K_{t+1}, H_{t+1}) \quad , \text{where} \quad H_{t+1} = \int \overline{\varphi}(y) g(x_t(y), H_t) \mu(y) dy \qquad (9)$$

In our comparative dynamics analysis we assume that competitive equilibria (under various regimes) start from the same initial stocks K_0, H_0 and compare the allocations along these dynamic paths period by period.

2.2. Financing Regime II. Consider the behavior of young individuals when funds needed to financing their investment in higher education are some type of 'insured loan'. Namely, the payback of the loans are linked to the individual's income (thus, linked to the realization of his/her random income in the future). Moreover, this mechanism contains no subsidization of this program by the government: th pooling of risks includes all the young students of the current generation who wish to invest in education. In particular, as we have assumed, the regular credit markets cannot be used for funding educational expenditure in this case. Denote by $\bar{\varphi} = E_y \bar{\varphi}(y)$. An agent who receives a loan to finance investment x_t is obliged to pay back $R_t x_t \frac{\varphi(A)}{\bar{\varphi}}$ in the working period of lifetime. In this case net income in the working period is given by:

$$I_t(A, y) = w_t \varphi(A) g(x_t(y), H_{t-1}) - R_t x_t(y) \frac{\varphi(A)}{\bar{\varphi}}.$$

Proceeding as in Section 2.1, optimal investment and savings are chosen according to (6) and

$$E\left\{\left[w_t g_1(x_t, H_{t-1})\varphi(\tilde{A}) - R_t\frac{\varphi(\tilde{A})}{\bar{\varphi}}\right]u_1'(c_t^1)\middle|y\right\} \begin{array}{ll} \leq & 0 \\ = & 0, \quad \text{if} \quad x_t > 0. \end{array} \tag{10}$$

This condition implies that,

$$x_t^* > 0 \qquad \Longleftrightarrow \qquad \bar{\varphi}g_1(x_t^*, H_{t-1}) = \frac{R_t}{w_t} \tag{11}$$

Moreover, (11) implies that all individuals will invest the same amount, regardless of the signal y each has received, i.e., $x_t^*(y) = \hat{x}_t$. Clearly, $\hat{x}_t$ will depend on H_{t-1} and, by our assumptions, it is nondecreasing in H_{t-1}. Due to the 'fair insurance' arrangement provided under financing scheme II, coupled with the risk aversion assumption, the optimal investment in education $\hat{x}_t$ maximizes the expected lifetime income as if the maximum was attained prior to the revelation of the signal; namely, $\hat{x}_t$ solves

$$\max_x E_y\left\{E\left[w_t\varphi(\tilde{A})g(x_t, H_{t-1}) - R_t x_t\frac{\varphi(\tilde{A})}{\bar{\varphi}}\middle|y\right]\right\} \tag{12}$$

and hence it is independent of y. We shall proceed with the assumption that $\hat{x}_t > 0$ for all t. These conditions are guaranteed if $g_1(0, H_{t-1}) > \frac{R_t}{w_t}\frac{1}{\bar{\varphi}}$ holds for all t.

2.3. Financing Regime III. In this case there is no cross-subsidization among groups with different signals so that the insured loans are provided on different terms for agents in different 'signal groups'. In some cases private fundings are based on grouping of students either by universities (e.g., at Yale, Harvard) or by field of career; this is justified due to "Grouping students by fields reflects similarity in the risks and the expected returns within the same group" (Lleras (2004), page 66). However, in our framework, all individuals in the same signal have access to the same terms of borrowing: a loan of x to be invested in education by a young individual with a signal y involves an obligation to pay back $R_t \frac{\varphi(A)}{\bar{\varphi}(y)} x$ dollars in the next period (given that A has been realized). Thus, risks of the cost of education are shared only within the same 'signal group'. As before, this income-linked loan program does not require any funding from the government: The agency providing the loans pays a gross interest rate R_t in the capital market which is just equal to the rate realized on total loans within each signal group, i.e., $\int_{\mathcal{A}} R_t \frac{\varphi(A)}{\bar{\varphi}(y)} \nu_y(A)\, dA = R_t$. In this case, the net income in the working period depends on the agent's signal, his realized ability and the terms of the loan,

$$I_t(A, y) = w_t \varphi(A) g(x_t(y), H_{t-1}) - R_t x_t(y) \frac{\varphi(A)}{\bar{\varphi}(y)}.$$

The first-order conditions for optimal investment and savings decisions are (6) and

$$\begin{aligned} E\left\{ \left[w_t g_1(x_t, H_{t-1}) \varphi(\tilde{A}) - R_t \frac{\varphi(\tilde{A})}{\bar{\varphi}(y)} \right] u_1'(c_t^1) \middle| y \right\} &\leq 0 \\ &= 0, \quad \text{if} \quad x_t > 0. \end{aligned} \qquad (13)$$

This condition implies that,

$$x_t^* > 0 \qquad \Longleftrightarrow \qquad \bar{\varphi}(y) g_1(x_t^*, H_{t-1}) = \frac{R_t}{w_t} \qquad \text{for all } y. \qquad (14)$$

Observe that the signal y enters the condition (14) only via the term $\bar{\varphi}(y)$. Thus we may express the optimal investment decision as a function of $\bar{\varphi}(y)$ rather than as a function of the signal itself, i.e., $x_t^* = x_t^*(\bar{\varphi}(y), H_{t-1})$. Since $g(x, H)$ is strictly concave in x and since $\bar{\varphi}(y)$ is strictly increasing in y (due to the MLRP) we derive from equation (14) that:

Corollary: Under financing regime III the optimal investment in education x_t^* is increasing in the signal y, and nondecreasing in H_{t-1} ; thus, good news stimulate investment in education.

3. Human Capital Accumulation

Our economy start at date 0 with a given initial stocks of physical capital K_0 and human capital H_0 . The dynamic equilibrium will depend upon the evolution of the human capital stock and, hence, upon the investment opportunities of the younger generation in human capital. It is our aim to compare the equilibria under the various financing schemes of educational investements. Does a change in financial terms of funds, used for investment in higher education, affect the equilibrium accumulation of human capital? how does it affect intrageneartional income distributions? does risk-sharing improve welfare in this case? We shall study these issues in the coming sections.

Given some exogenous interest rates, let us denote the (equilibrium) optimal investment and saving under the **financing scheme I** by $x_t(y, H_{t-1})$, $s_t(I_t(A, y))$. Under the financing **scheme II** it is denoted by $\widehat{x}_t(H_{t-1})$ and $\widehat{s}_t(\widehat{I}_t(A, y))$, and under the financing **scheme III** it is denoted by $x_t^*(\overline{\varphi}(y), H_{t-1})$, $s_t^*(I(A, y))$. The equilibrium aggregate human capital under scheme k in date t is denoted by H_t^k . Now we prove,

1. Each agent will choose higher investment in education under the funding scheme III compared to funding scheme I; namely, $x_t(y) < x_t^*(\overline{\varphi}(y))$ for all signals y.
2. In equilibrium the stock of human capital under financing scheme III is larger than that under the financing scheme I, i.e., $H_t^{III} \geq H_t^I$ for $t = 1, 2, ...$

This result demonstrates that the partially-insured credit market, with paybacks linked to future income, enhances investments in the formation of human capital, compared to the non-insured funding via credit markets.

Proof: Consider first optimal investments under scheme I , i.e., individuals have access to loans in the banks with the market interest rates R_t. For each given y and some fixed H_{t-1} we have,

$$w_t g_1(x_t(y), H_{t-1}) Cov[(\varphi(\widetilde{A}) \mid y), u_1'(c_t^1(\widetilde{A}, y))] < 0 \tag{15}$$

Since $c_t^1(A, y)$ is increasing in realizations of A , u_1' is a decreasing function and $\varphi(\widetilde{A})$ is increasing in A. From equation (10) and equation (24) we derive that,

$$E\ [w_t g_1(x_t(y), H_{t-1})\varphi(\widetilde{A}) - R_t \mid y] > 0 \tag{16}$$

Namely, we obtained: $g_1(x_t(y), H_{t-1})\overline{\varphi}(y) > \frac{R_t}{w_t}$ under scheme I. We already have shown that under scheme III , assuming that $\overline{H}_{t-1}$ is fixed, we have $g_1(x_t(\overline{\varphi}(y)), H_{t-1})\overline{\varphi}(y) = \frac{R_t}{w_t}$; thus under the assumption that H_{t-1} is fixed for both regimes, due to the concavity of $g(x, H)$ in x and the assumption of free capital mobility, we obtain that $x_t(y) < x_t^*(\overline{\varphi}(y))$.

Now let us proceed with induction. Since K_0, H_0 are given at the outset, the above result about the investment in education yields that H_1 under scheme III is higher than H_1 under scheme I. Now we proceed with the induction step: Let H_t^{III} be larger than H_t^I for some t. For each individual in generation $t+1$ we have: $g_1(x_{t+1}(y), H_t^I)\overline{\varphi}(y) > \frac{R_{t+1}}{w_{t+1}}$ and $g_1(x_{t+1}^*(\overline{\varphi}(y)), H_t^{III})\overline{\varphi}(y) = \frac{R_{t+1}}{w_{t+1}}$.

But our assumption that $g_1(x, H)$ is nondecreasing in H, hence, we obtain that : $g_1(x_{t+1}(y), H_t^{III})\overline{\varphi}(y) > \frac{R_{t+1}}{w_{t+1}}$; hence $x_{t+1}(y) < x_{t+1}^*(\overline{\varphi}(y))$. Therefore, for all individuals h_{t+1} under scheme III is larger than under regime I , which means that $H_{t+1}^I < H_{t+1}^{III}$. This proves the induction step and hence (b).■

Now, let us compare the aggregate human capital levels under the funding schemes II and III. To that end, to simplify our analysis we shall impose additional assumption about the human capital production process. When H_{t-1} is fixed during certain analysis we shall ignore it to simplify our notations. For example, we shall write $x_t^*(\overline{\varphi}(y))$ for the optimal investment in education under scheme III, without referring to the human capital level H_{t-1}. Define the following concavity measures for the accumulation function $g(x, H)$:

$$K(x,H) = -\frac{g''(x,H)}{g'(x,H)}$$

$$\widehat{K}(x,H) = -\frac{g''(x,H)}{g'(x,H)^2} = \frac{K(x,H)}{g'(x,H)} \tag{17}$$

(A3) The human capital accumulation function $g(x, H)$ exhibits **decreasing concavity**, i.e., $K(x, H)$ is decreasing in x.

Note that Assumption (A3) holds if $g'(x, H)$ is convex in x . We say that $g(x, H)$ exhibits a **moderately decreasing concavity** if $\widehat{K}(x, H)$ is increasing in x (given H). It exhibits a **strongly decreasing concavity** if $\widehat{K}(x, H)$ is decreasing in x (given H). Clearly, when $\widehat{K}(x, H)$ is decreasing in x (A3) holds. Also, note that 'moderately decreasing concavity' and 'strongly decreasing concavity' are mutually exclusive properties.

Let $g'(x, H)\varphi = \frac{R}{w}$ =constant , and let H be fixed; define $x = \phi(\varphi)$. Let us show that the **average** investment in human capital, as we change financing schemes depends upon the convexity property of $\phi(\varphi)$.

Lemma 1. *Let $x = \phi(\varphi)$ be defined from $g'(x, H)\varphi = \frac{R}{w}$, where R, w and H are given; Then:*

$\widehat{K}(x, H)$ is (strictly) decreasing $\Rightarrow$ $\phi(\varphi)$ is (strictly) convex

$\widehat{K}(x, H)$ is (strictly) increasing $\Rightarrow$ $\phi(\varphi)$ is (strictly) concave

We shall relegate some of the proofs (including this Lemma) to the Appendix to facilitate the reading. Lemma 1 implies the Proposition:

Proposition 1. *Under moderately (strongly) decreasing concavity of the human capital accumulation function the average investment in education under financing scheme II is* **higher** *(lower) than that under scheme III.*

To gain some insight about this result note that the more able agents are subsidizing the less able ones more heavily under financing scheme II, where *all risks* are pooled, compared to scheme III . Thus, the rate of 'interest' paid on investment in education by agents with high levels of y is higher under financing regime II. Thus, it is beneficial to increase investment (under funding scheme II) with higher levels of y only if the rate of return on this investment is sufficiently high. Apparently, our Proposition claims that the monotonicity of $\widehat{K}(x, H)$ determines that. Note that when $\widehat{K}(x, H)$ is decreasing, hence $\phi(\varphi)$ is strictly *convex,* we obtain:

$$E_y[x_t^*(\bar{\varphi}(y))] = E_y[\phi(\bar{\varphi}(y))] > \phi[E_y\bar{\varphi}(y)] = \hat{x}$$

Define the function: $\xi(z, H) = zg[x_t^*(z, H), H]$. Let $\xi' = \frac{\partial \xi}{\partial z}$, then, it is easy to verify that $sign\ \xi'' = sign\ \{\frac{g''}{g'} - \frac{g'''}{g''}\} = -sign[K'(x, H)]$.Moreover, under (A3) we have, $\frac{g''(x)}{g'(x)} - \frac{g'''(x)}{g''(x)} > 0$. Thus, the function $\xi(z, H)$ is strictly convex function of z.

Proposition 2. *Assume that (A1)-(A3) hold, then the equilibrium aggregate human capital levels under the funding scheme III are* **higher** *than that under the funding scheme II in all dates, namely:* $H_t^{III} > H_t^{II}$ *for all* t.

Note that the comparison of the aggregate stock of human capital, between funding regimes II and III, assumes that $K(x, H)$ is decreasing in x , regardless of the monotonicity property of $\widehat{K}(x, H)$. Thus, higher levels of aggregate human capital does not necessarily require higher average investment in human capital.

Proof: We shall prove it by induction in each case. Using our earlier finding, the investment of each agent under scheme II is the constant $\widehat{x}$,

$$H_t^{II} = \int_Y \overline{\varphi}(y) g(\widehat{x}(H_{t-1}^{II}), H_{t-1}^{II}) \mu(y) \, \mathrm{d}y = \overline{\varphi} g(\widehat{x}(H_{t-1}^{II}), H_{t-1}^{II}) \tag{18}$$

Consider now the avarage human capital under scheme III. Under our assumption $\xi(z, H)$ is strictly convex in z. Assume, by induction that $H_{t-1}^{III} \geq H_{t-1}^{II}$. Then, since $x_t^*(\overline{\varphi}, H)$ is non-decreasing in H and $x_t^*(\overline{\varphi}, H) = \widehat{x}(H)$ for all H. Hence, $H_t^{III} > H_t^{II}$ for all dates t. ■

Thus, the funding scheme III is more efficient than the scheme II in terms of generating economic growth. Uniform risk sharing of random incomes, which include the lower ability individuals, may result in an inefficient investment profile in human capital. On the other hand, the monotonicity of the function $\widehat{K}(x, H)$ in x determines whether the aggregate investment in human capital is higher under scheme II than under financing scheme III. In other words, higher *average investment* in human capital does not imply higher accumulation of human capital.

3.1. Special Cases. Let us consider the above results when we choose the human capital formation process to be in a certain family of functions. We shall concentrate on the properties with respect to x , hence let us ignore H altogether here.

Case 1: Let $g(x)$ belong to the CRRA family, i.e.,

$$g(x) = \frac{x^{1-\gamma}}{1-\gamma} \ , \quad 0 < \gamma \leq 1 \tag{19}$$

Then, for any $\gamma > 0$ we have : sign $K'(x) < 0$. Namely, for all $1 > \gamma > 0$ the stock of human capital in equilibrium is higher under the funding scheme III than under the funding scheme II , i.e., $H_t^{III} > H_t^{II}$ for all t.

Consider now the monotonicity of the function $\hat{K}(x)$: $\hat{K}(x) = \frac{\gamma x^{-\gamma-1}}{x^{-2\gamma}} = \gamma x^{\gamma-1}$. Namely, for all $1 > \gamma > 0$, $\hat{K}(x)$ is strictly decreasing. In other words, at this range $g(x)$ exhibits **strongly decreasing concavity** when $1 > \gamma > 0$. Denote the aggregate investment in education by generation t , under the various regimes, by X_t^I , X_t^{II} , X_t^{III} . Based on our earlier results:

Corollary 1. *If $g(x)$ belongs to the CRRA functions then: (a) For $0 < \gamma < 1$ we obtain that $x_t^*(\bar{\varphi}(y))$ is strictly convex in y, hence : $X_t^{III} = E_y[x_t^*(\bar{\varphi}(y))] > X_t^{II} = \hat{x}$, while $H_t^{III} > H_t^{II}$ for all t. (b) If $\gamma = 1$, then: $X_t^{III} = X_t^{II}$, and $H_t^{III} = H_t^{II}$ holds for all t.*

Case 2: Let $g(x)$ belong to the CARA family, i.e.,

$$g(x) = 1 - e^{-\gamma x}, \quad \gamma > 0. \tag{20}$$

In this case, it is easy to see that $\hat{K}(x) = e^{\gamma x}$ and $K(x) = \gamma = cons$. Thus $g(x)$ exhibits moderately decreasing concavity:

Corollary: Let $g(x)$ be a CARA function. Then, (a) $X_t^{III} < X_t^{II}$ and $X_t^{III} > X_t^I$ for all t, (b) $H_t^{II} = H_t^{III}$ for all t.

4. Welfare Implications

Let us compare the welfare in equilibrium under various funding schemes of investement in education in our economy. For comparison of the various regimes we apply the **ex-ante** expected lifetime utility of consumers, i.e., prior to the revelation of their own signals. Let us consider the optimum, under the funding scheme I , for a given signal y , i.e., the optimization (4), and integrate it over all feasible signals; this will be denoted by: $W_I = E_y\{E_A[u_1(c_1^I) + u_2(c_2^I) \mid y]\}$, the ex-ante lifetime expected utility under scheme I.

Similarly, we define $W_{III} = E_y\{E_A[u_1(c_1^{III})+u_2(c_2^{III}) \mid y]\}$, the welfare under the financing scheme III. We say that *welfare is higher* under the financing scheme III, compared to the financing regime I, if the ex-ante lifetime expected utility is higher (for all agents) under scheme III, namely, if $W_{III} > W_I$. Now we state,

Proposition 3. *In equilibrium, all agents are better off under funding scheme III than under the funding scheme I.*

Thus, any political process with voting about the preferred funding scheme the arrangement provided by funding scheme III, i.e., with the partially-insured financing of private investment in education, will prevail.

Proof: Let us write the prooof of this Proposition under the assumption that the optimal investment functions are *independent* of the human capital stock (of the earlier generation). Since by Proposition 1 we have $H_t^{III} \geq H_t^I$ for all t, the case where higher stock of human capital (of the older generation) only increases the total output will only reinforce our claims in this proof.

Let $x_t(y)$ and $s_t\big(I_t(A, y)\big)$ be optimal investment and optimal saving under the financing regime I. Let,

$$I_t(A, y) = w_t g\big(x_t(y)\big)\varphi(A) - R_t x_t(y).$$

Consumption in t and $t + 1$ can then be stated as

$$\begin{aligned} c_t^1(A, y) &= I_t(A, y) - s_t\big(I_t(A, y)\big) \\ &= \big[w_t g\big(x_t(y)\big)\varphi(A) - s_t\big(I_t(A, y)\big)\big] - R_t x_t(y) \qquad (21) \\ c_t^2\big(I_t(A, y)\big) &= R_{t+1} s_t\big(I_t(A, y)\big). \qquad (22) \end{aligned}$$

Denote by $\bar{s}_t(y)$ expected savings conditional on the signal y, i.e., $\bar{s}_t(y) := E\big[s_t\big(I_t(\tilde{A}, y)\big)\big]$. Expected utility conditional on y is

$$E\big\{u_1\big(c_t^1(\tilde{A}, y)\big) + u_2\big(c_t^2\big(I_t(\tilde{A}, y)\big)\big)\big|y\big\}. \qquad (23)$$

We show below that under regime III expected utility conditional on y is higher than in (29). Since this assessment is valid for *any* signal y, we conclude that welfare is higher under regime III than under regime I. Under regime III, the following $\wedge$-allocation is admissible (but not necessarily optimal):

$$\hat{x}_t(y) = x_t(y) \tag{24}$$

$$\hat{s}_t(A,y) = s_t\big(I_t(A,y)\big)\left[1 - \frac{R_t x_t(y)}{w_t\bar{\varphi}(y)g\big(x_t(y)\big)}\right] + R_t x_t(y)\frac{\bar{s}_t(y)}{w_t\bar{\varphi}(y)g\big(x_t(y)\big)} \tag{25}$$

$$\hat{I}_t(A,y) = \left[w_t g\big(x_t(y)\big) - R_t\frac{x_(y)}{\bar{\varphi}(y)}\right]\varphi(A) \tag{26}$$

$$\begin{aligned}\hat{c}_t^1(A,y) &= \hat{I}_t(A,y) - \hat{s}_t(A,y) = \left[1 - \frac{R_t x_t(y)}{w_t\bar{\varphi}(y)g\big(x_t(y)\big)}\right]\big[w_t g\big(x_t(y)\big)\varphi(A) - s_t\big(I_t(A,y)\big)\big] \\ &- R_t x_t(y)\frac{\bar{s}_t(y)}{w_t g\big(x_t(y)\big)\bar{\varphi}(y)}\end{aligned} \tag{27}$$

$$\hat{c}_t^2(A,y) = R_{t+1}\hat{s}_t(A,y) \tag{28}$$

To complete the proof we show that the $\wedge$-decision leads to higher expected utility conditional on y than the optimal decision under Regime 1. From (27) and (33) it is immediate that $E\{\hat{c}_t^1(\tilde{A},y)|y\} = E\{c_t^1(\tilde{A},y)|y\}$. Also, $\big[w_t g\big(x_t(y)\big)\varphi(A) - s_t\big(I_t(A,y)\big)\big]$ is increasing in A (see equation (9)). Therefore, $c_t^1(\tilde{A},y)$ differs from $\hat{c}_t^1(\tilde{A},y)$ by a mean preserving spread which implies $E\big\{u_1\big(\hat{c}_t^1(\tilde{A},y)\big)\big|y\big\} \geq E\big\{u_1\big(c_t^1(\tilde{A},y)\big)\big|y\big\}$. Similarly, $E\big\{u_2\big(\hat{c}_t^2(\tilde{A},y)\big)\big|y\big\} \geq E\big\{u_2\big(c_t^2(I_t(\tilde{A},y)\big)\big|y\big\}$ because $s_t\big(I_t(\tilde{A},y)\big)$ is a mean preserving spread of $\hat{s}_t(\tilde{A},y)$. ■

Although the coming propositions hold (with minor restrictions) under the more general human capital accumulation function, where g depends on the average human capital of the earlier generation, we shall simplify our notations and proofs by assuming from now on that:

(A4) The function $g(x, H)$ does not depend on H.

The main implication of (A4) is that the optimal investment functions in human capital do not depend on the stock of human capital of the earlier generation. Let us compare the welfare under regimes II and III. We shall show that under some conditions (related to the accumulation process of human capital) welfare is higher under funding scheme III, but under certain cases welfare is higher under funding scheme II. To simplify the notations we shall ignore the date index t.

Proposition 4. *If the human capital accumulation function $g(x)$ exhibits moderately decreasing concavity, the economy attains higher welfare under funding scheme II than under funding scheme III.*

Proof: The value functions under funding schemes III and II are given by:

$$V_{III}(\bar{\varphi}(y)) = E_A\{u_1[w\varphi(A)g(x^*(\bar{\varphi}(y))) - R_t \tfrac{x^*(\bar{\varphi}(y))}{\bar{\varphi}(y)}\varphi(A) - s^*] + u_2[R_{t+1}s^*]\}$$

and

$$V_{II}(\bar{\varphi}) = E_A\{u_1[w\varphi(A)g(\widehat{x}(\bar{\varphi})) - R_t \frac{\widehat{x}(\bar{\varphi})}{\bar{\varphi}}\varphi(A) - \widehat{s}] + u_2[R^*_{t+1}\widehat{s}]\} \tag{29}$$

In addition, $x^*(\bar{\varphi}) = \widehat{x}(\bar{\varphi})$ and $s^*_t(\bar{\varphi}, A) = \widehat{s}_t(\bar{\varphi}, A)$, for all A, follow from the first order conditions. Therefore, $V_{II}(\bar{\varphi}) = V_{III}(\bar{\varphi})$ holds. Also, by definition we have $\bar{\varphi} = E_y[\bar{\varphi}(y)]$. Below we show that under the assumptions of the proposition V_{III} is a strictly concave function of $\bar{\varphi}(y)$ .Hence, the claim in this Proposition follows from:

$$W_{II} = V_{II}(\bar{\varphi}) = V_{III}(\bar{\varphi}) = V_{III}(E_y[\bar{\varphi}(y)]) > E_y[V_{III}(\bar{\varphi}(y))] = W_{III}$$

Thus, it remains to verify the strict concavity of $V_{III}(\bar{\varphi}(y))$ as a function of $\bar{\varphi}(y)$. Now, differentiating $V_{III}(\bar{\varphi}(y))$ with respect to $\bar{\varphi}(y)$ and using the Envelope theorem we obtain that:

$$V'_{III}(\bar{\varphi}(y)) = E_A\{u'_1(\bullet)R_t \frac{x^*(\bar{\varphi}(y))}{\bar{\varphi}(y)^2}\varphi(A)\} \quad (30)$$

However, the optimal consumption in the first period c_1^* is increasing in $\bar{\varphi}(y)$, therefore, $u'_1(c_1^*)$ is decreasing in $\bar{\varphi}(y)$. Furthermore, since $g(x)$ exhibits moderately decreasing concavity, by Lemma 1 $x^*(\bar{\varphi}(y))$ is strictly concave hence, $\frac{x^*(\bar{\varphi}(y))}{\bar{\varphi}(y)}$ is decreasing [note that from equation (11) we have $\lim x^*(\varphi) = 0$ as $\varphi \to 0$]. This implies that $V'_{III}(\bar{\varphi}(y))$ is decreasing and, hence, the value function under the funding scheme III is strictly concave. ■

Now let us show that when $g(x)$ exhibits **strongly decreasing** concavity then, under certain conditions, the welfare is higher under funding regime III; namely, we obtain that $W_{III} > W_{II}$. Since the funding scheme III contains "less" risk sharing than funding scheme II, we derive in such a case that it is not only more efficient with respect to human capital accumulation, but also (ex-ante) more desirable !

Corrolary: Let $g(x) = x^\alpha$, where $0 < \alpha < 1$, and the utility functions u_1 and u_2 are the CRRA type. Then, $W_{III} > W_{II}$ if $2\alpha - \alpha\gamma > 1$, where the parameter γ corresponds to u_1.

Proof: In this case, using the FOC's we have, for some $k^* > 0$:

Thus, we reach that

$$x^*(\bar{\varphi}(y)) = k^*[\bar{\varphi}(y)]^{\frac{1}{1-\alpha}} \quad \text{and} \quad \frac{x^*(\bar{\varphi}(y))}{\bar{\varphi}(y)^2} = k^*[\bar{\varphi}(y)]^{\frac{2\alpha-1}{1-\alpha}} \quad (31)$$

By choosing $0.5 < \alpha < 1$ this function is increasing. Moreover, the closer α is to 1 the faster it increases. Now, the utility functions are from the CRRA family, hence we can write: $u'_1(c) = [c]^{-\gamma}$ where $0 < \gamma$. In this case the first period optimal consumption is proportional to the net income, i.e.,

$$
\begin{aligned}
c_1^{III}(y,A)) &= \lambda^* I^{III}(y,A) = \lambda^*\{w\varphi(A)[\bar{\varphi}(y)]^{\frac{\alpha}{1-\alpha}} - R\varphi(A)\; k^*[\bar{\varphi}(y)]^{\frac{\alpha}{1-\alpha}}\} \\
&= \Lambda(A)[\bar{\varphi}(y)]^{\frac{\alpha}{1-\alpha}} \qquad (32)
\end{aligned}
$$

for some $0 < \lambda^* < 1$. Thus, from the above expression (32) we obtain that,

$$
\begin{aligned}
u_1'(c_1^{III}(I^{III}(y,A)))\frac{x^*(\bar{\varphi}(y))}{\bar{\varphi}(y)^2} &= [\Lambda(A)[\bar{\varphi}(y)]^{\frac{\alpha}{1-\alpha}}\;]^{-\gamma} k^*[\bar{\varphi}(y)]^{\frac{2\alpha-1}{1-\alpha}} \\
&= B(A)[\bar{\varphi}(y)]^{\frac{2\alpha-1-\alpha\gamma}{1-\alpha}} \quad \text{for all } y \text{ and } A \qquad (33)
\end{aligned}
$$

Choosing, $2\alpha - 1 - \alpha\gamma > 0$, the RHS of the equality in (33) is strictly increasing, which shows, by (32), that $V'_{III}\ [\bar{\varphi}(y)]$ is a decreasing function of $\bar{\varphi}(y)$, i.e., V_{III} $[\bar{\varphi}(y)]$ is a strictly convex function. In this case we obtain that $W_{III} = E_A\{E_y V_{III}$ $[\bar{\varphi}(y)]\} > E_A\{V_{III}\ [\bar{\varphi}]\} = E_A\{V_{II}[\bar{\varphi}]\} = W_{II}$.

Let us consider the circumstances under which agents prefer ex-ante the funding scheme with less risk sharing. The condition that guarantees such preference, $2\alpha - \alpha\gamma > 1$, requires that the exponent in $g(x)$, α , is not small while the relative measure of risk aversion γ is not too large. Such decision makers will prefer the more efficient funding scheme III (in the sense of generating human capital) to the scheme with more risk sharing, scheme II. When α is small the 'efficiency advantage' of regime III, comared to regime II, declines, so the balance between these two effects is obtained when the condition $2\alpha - \alpha\gamma > 1$ holds.

5. Income Inequality

One important aim of goverment intervention in financing investments in human capital formation is to affect the intragenerational income distribution. The literature contains abundant evidence, as well as theoretical results, demonstrating the importance of public education in reducing income inequality and enhancing mobility. We concentrate here on the institutions that government can provide to facilitate private investment in education in order to reduce income inequality. Let us compare the three financing regimes we are considering here with respect to their effect on income inequality. In our framwork one can consider lifetime **income** at different points of time. We take 'income' to be the expected (conditional on signal) net income. The reason is that, in our framework, we are comparing various regimes of governmental intervention at a time where agents have acquired the signals about their future income.

The comparison of inquality between income distributions applies the usual Lorentz-ordering [or, actually, the **second degree** stochastic dominance (SDSD), where one Lorentz curve is strictly above the other one; see Atkinson (1970). See also Rothschild and Stiglitz (1970) for various characterizations of SDSD]. Denoted by $I(y, A)$ the income of an individual who obtained a signal y, at a time where the signal y is known but the income is yet unknown. We compare income distributions as follows: For each signal y define the mean income for this 'type' by $\bar{I}(y) = E_A[I(y, \tilde{A})]$.

Definition 1. *Given two income distributions $I(y, A)$ and $I^*(y, A)$. We say that $I^*(y, A)$ is* ***more unequal*** *than $I(y, A)$ if:*

a) (intra-group inequality) Within each group of agents with the same signal y, $\frac{I^*(y,A)}{E_A I^*(y,\tilde{A})}$ is a MPS of $\frac{I(y,A)}{E_A I(y,\tilde{A})}$

b) (inter-groups inequality) The (normalized) mean income $\frac{\bar{I}^*(y)}{E_y \bar{I}^*(y)}$ is a MPS of the mean income $\frac{\bar{I}(y)}{E_y \bar{I}(y)}$.

Our definition requires that the more unequal distribution to satisfy: the incomes

within each 'type' are more dispersed and **across types** it is more unequal. Before we state the following proposition let us claim:

Lemma 2. *Assume that the utility functions* $u_1(c)$ *and* $u_2(c)$ *are CRRA with parameters* γ_1 *and* γ_2 *and let* $\gamma_1 \leq 1$ *. Then, the optimal investment in education under the funding scheme I,* $x_t(y)$ *, is nondecreasing in the signal* y*.*

Proposition 5. *Assume that under the funding scheme I the investment in education* $x_t(y)$ *is nondecreasing in the signal* y *. Then, at each date, the income distribution under funding scheme I is more unequal than the income distribution under the funding scheme II.*

This result provides a justification for the risk sharing intervention given in the funding scheme II : the loans for investment in education are fully linked to the realized income of each individual, providing this way 'fair insurance' by linking the payback level to the random income. As a result, individuals with lower signals y will have an advantage in borrowing funds to be invested in human capital formation (compared to the case where credit market is used). Since funding scheme II contains certain subsidization to the investments in education of those with lower ability by the more able individuals (who receive higher signals, and **may** invest less under the funding scheme II). As a result, we are unable to compare these two regimes with respect to their aggregate stocks of human capital.

Proof: Again, to simplify the writting we shall drop the time index t. Let us write the income under schemes II and I for a given y:

$$I^{II}(y) = \left[wg(\hat{x}) - R\frac{\hat{x}}{\bar{\varphi}} \right] \varphi_y(\widetilde{A}) \tag{34}$$

$$I^I(y) = wg[x(y)]\varphi_y(\tilde{A}) - Rx(y)$$

To verify (a) of the definition let us **fix** y , hence $x(y)$ is fixed as well. Note that $wg(x(y))\varphi_y(\tilde{A})$, which is random in A, is equally distributed as $[wg(\hat{x}) - R\frac{\hat{x}}{\bar{\varphi}}]\varphi_y(\tilde{A})$. Since $Rx(y)$ is a positive **constant** we derive that it is **more equal** than $wg(x(y))\varphi_y(\tilde{A}) - Rx(y)$ [see, Karni and Zilcha (1995), Lemma 1 on page 283] . This proves part (a) of the definition, hence, $I^{II}(y, A)$ is more equal than $I^I(y, A)$ for each y. ■

Let us state the following Lemma before we proceed,

Lemma 3. *Let z be a real-valued random variable and let $\bar{z} = Ez$. Let $\bar{\varphi}(z)$ and $M(z)$ be increasing non-negative functions of z. Define,*

$$\phi^1(z) = \frac{\bar{\varphi}(z)}{E\{\bar{\varphi}(z)\}} \quad \text{and} \quad \phi^2(z) = \frac{M(z)\bar{\varphi}(z)}{E\{M(z)\bar{\varphi}(z)\}} \tag{35}$$

Then, $\phi^1(z)$ stochastically dominates (SDSD) $\phi^2(z)$.

Define, $\bar{x} = E_y x(y)$. Let us write the mean net income given the signals:

$$I^I(y) = wg[x(y)]\bar{\varphi}(y) - Rx(y)$$

$$I^{II}(y) = \left[wg(\hat{x}) - R\frac{\hat{x}}{\bar{\varphi}}\right]\bar{\varphi}(y)$$

Let us rewrite the (mean) income under funding scheme I as follows:

$$I^I(y) = \left\{wg[x(y)] - R\frac{x(y)}{\bar{\varphi}(y)}\right\}\bar{\varphi}(y) \tag{36}$$

Let us show that the function within the paranthesis above is an increasing function of y. Let us differentiate it with respect to y, we obtain after some manipulations the following expression for the derivative:

$$I^{I'}(y) = \bar{\varphi}(y)\left\{[wg'[x(y)] - \frac{R}{\bar{\varphi}(y)}]x'(y) + wg[x(y)]\frac{\bar{\varphi}'(y)}{\bar{\varphi}(y)}\right\} > 0$$

Due to inequality (16) attained for the funding scheme I case, and $\bar{\varphi}'(y) > 0$ and $x'(y) \geq 0$ by our assumptions.Now, we can apply Lemma 3: the above expression for $I^I(y)$ has a multiplication of $\bar{\varphi}(y)$ by an increasing function of y, while in $I^{II}(y)$, $\bar{\varphi}(y)$ is multiplied by a constant (which does not depend on the signal y); hence, $I^I(y)$ is more unequal than $I^{II}(y)$. ■

We prove now:

Proposition 6. *In equilibrium, income inequality is higher under funding scheme III than under funding scheme II in all dates.*

Proof. Define, $z = E[\varphi_y(\tilde{A})] = \bar{\varphi}(y)$. As we have seen in scheme III the optimal investment in education $x_t^*(z)$ depends on z. Now, the **mean** net income for each given z under scheme III is given by:

$$I_t^a(z) = \left\{w_t g[x_t^*(z)] - R_t\frac{x_t^*(z)}{z}\right\} z$$

We claim that Lemma 3 can be applied here to show that this income distribution is dominated (second degree stochastic dominance, hence condition (1b) of the definition holds). By :

$$I_t^b(z) = \left\{w_t g[x_t^*(\bar{z})] - R_t\frac{x_t^*(\bar{z})}{\bar{z}}\right\} z$$

where $\bar{z} = E_y z$. To verify that Lemma 3 is applicable we need to show that the following function: $M(z) = w_t g[x_t^*(z)] - R_t\frac{x_t^*(z)}{z}$ is increasing in z. Write,

$$\begin{aligned} M'(z) &= w_t g'[x_t^*(z)]x_t^{*\prime}(z) - R_t\frac{z x_t^{*\prime}(z) - x_t^*(z)}{z^2} \\ &= \frac{1}{z}\{[w_t g'[x_t^*(z)]z - R_t\}x_t^{*\prime}(z) + \frac{R_t x_t^*(z)}{z^2} > 0 \end{aligned}$$

Since the first term on the RHS is 0 under the optimality condition under the funding scheme III. But I_t^b is *equally distributed* as the mean of I_t^{II} : $I_t^{IIa}(z) = \left[w_t g(\hat{x}) - R_t \frac{\hat{x}}{\bar{\varphi}}\right] z$. Thus, $I_t^a(z)$ is more unequal than $I_t^{IIa}(z)$. The first part of the definition, part (1a), trivially holds; therefore, the income distribution $I_t^{II}(z)$ is more equal than $I_t^{III}(z)$ for all dates t. ■

■

Let us compare now income inequality under the funding schemes I and III. We claim that, in general, we cannot derive a Lorentz-dominance when we consider the (mean) *income* distributions under these two regimes. Consider the mean-income distribution, given the signals y , for these two cases, denoted by $I^{Ia}(y)$ and $I^{IIIa}(y)$ (again, to simplify the notations we drop the time index t). We claim that Lorentz-dominance can go either way, but we shall demonstrate only one case. Assume that $g(x) = x^\alpha$, where $0 < \alpha < 1$. We have shown already that $x_t(y) < x_t^*(y)$ holds for all y; let us assume that for certain choice of utility functions we have

$$x_t(y) = \rho x_t^*(y), \quad \text{where} \quad 0 < \rho < 1.$$

Since $x_t^*(y)$ is utility-independent it means that we should choose utility functions that guarantee that ρ is signal-independent. We can write in such a case the average income for regime I as :

$$I^{Ia}(y) = \rho^\alpha \left\{ wg(x^*(y))\bar{\varphi}(y) - Rx^*(y)\rho^{1-\alpha} \right\} \tag{37}$$

Since $\rho^{1-\alpha} < 1$ we shall compare the inequality of the income distribution $I^{Ia}(y)$ to the distribution of :

$$I^{IIIa}(y) = wg(x^*(y))\overline{\varphi}(y) - Rx^*(y)$$

Lemma 4. *Consider the positive random variables $W(y)$ and $Z(y)$. Assume that*

$Z(y)$ is either more unequal than $W(y)$ or equally distributed as $W(y)$. Let $\psi > 0$ be a constant, then, the random variable $Z(y)$ is more unequal than $W(y) + \psi Z(y)$.

Using the FOCs we can write, in this case: $\overline{\varphi}(y) = m[x^*(y)]^{1-\alpha}$. Thus, we can represent $I^{IIIa}(y)$ by:

$$I^{IIIa}(y) = w[(x^*(y)]^{\alpha}\overline{\varphi}(y) - Rx^*(y) = wm[x^*(y)] - Rx^*(y)$$

Which implies that $I^{IIIa}(y)$ and $x^*(y)$ are equally distributed. Now, write: $I^{Ia*}(y) = \rho^{\alpha}\{I^{III}(y) + Rx^*(y)[1 - \rho^{1-\alpha}]\}$. Since, $I^{IIIa}(y)$ and $x^*(y)$ are equally distributed, by Lemma 4 we obtain that, in this case, $I^{Ia}(y)$ is more equal than $I^{IIIa}(y)$. Now let us state,

Conclusion: Consider the mean-income distributions characterized by the signals y. Under certain conditions, the funding scheme III results in income distribution with *higher* inequality than the income distributions under funding scheme I.

6. Conclusion

The role of government in providing education is clear, but in most modern coutries it extends beyond compulsory schooling. The economic implications of higher education provision are compelling and, hence, we observe significant government intervention in the various higher education systems. It is our goal, in this paper, to compare various possible manners of governmental intervention in making resources available to students. Our conclusions are that a program that allows students to invest in higher education freely, while making the paybacks of these loan dependent on their **future income** 'dominates' the case where student loans are provided at the existing credit markets condition. There are three dimensions to our 'dominance criterion'; namely, the funding schemes that include risk sharing are 'better' in the following sense: (a) Efficiency: under funding schemes that include some risk sharing human capital accumulation is higher in equilibrium (hence, economic growth), (b) Welfare: the funding schemes with risk-sharing provide higher ex-ante expected lifetome utilities, (c) Income inequality: in most cases these risk sharing arrangements have **lower** income inequality in equilibrium.

Since the two funding schemes that include income-linked payback of education loans require intervention (see the cases of Australia and other countries discussed in Lleras (2004)), the role of government is restricted to creating the environment that makes such a mechanism operate. Basically, it requires minimal financing from the government (for example, in case of personal bankruptcy or fraud). Similar intervention is needed when the government guarantees availability of students loans through the existing credit markets. The economic benefits obviously outweight the public costs in this case and our analysis shows that the income-linked payback system is the 'better' system. Thus, in countries with higher education system that imposes heavy financial burden, such as sizeable tuition fees and other related direct costs, should adopt some variant of the 'Australian model'. This type of mechanism is, currently, being examined in Israel. In the US loans for students are provided under conditions similar toour funding scheme I, which is inferior in many senses to the income-linked rate of return case.

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8. Appendix

Proof of Lemma 1: A direct calculaion shows that :

$$\phi''(\varphi) = -\{[g''\phi'\varphi g'' - g'[g'''\phi'\varphi + g''\}\Gamma, \quad \text{where } \Gamma > 0.$$

Now, we derive that: $\quad \text{sign } \phi'' = -\text{sign} \left\{ 2 + \frac{g'''}{g''} \left[-\frac{g'}{g''} \right] \right\}$, which easily establishes the Lemma.

Proof of Lemma 2: Assume by negation that for some two signals y' and y'' we have: $y'' = y' + \epsilon$, $\epsilon > 0$ small, and $x_t(y') > x_t(y'')$. Due to the CRRA utility functions we obtain that $c_t^1(y) = \zeta s_t(y)$ for all y, for some positive constant ζ. For all realizations of A for which,

$$w_t \varphi(A) g_1(x_t(y')) - R_t > 0 \tag{38}$$

we have: $c_t^1(y'') + s_t(y'') < c_t^1(y') + s_t(y')$ [see equation (8)]. Thus, in this case, due to the monotonicity (decreasing) of u_1' , we have $u_1'(c_t^1(y') < u_1'(c_t^1(y''))$. Now, for all realizations of A for which,

$$w_t\varphi(A)g_1(x_t(y')) - R_t < 0 \tag{39}$$

we have: $c_t^1(y'') + s_t(y'') > c_t^1(y') + s_t(y')$. Hence in this case we have : $u_1'(c_t^1(y') > u_1'(c_t^1(y''))$. Thus, we obtain that for all value of A we have:

$$[w_t\varphi(A)g_1(x_t(y')) - R_t]u_1'(c_t^1(y')) \leq [w_t\varphi(A)g_1(x_t(y'')) - R_t]u_1'(c_t^1(y'')) \tag{40}$$

From the first order conditions [equation (11)] it can be shown that,

$$c_t^1 \uparrow \iff s_t \uparrow \quad \text{as } A \text{ and } y \text{ vary} \tag{41}$$

$$E\{[w_t\varphi(A)g_1(x_t(y'')) - R_t]u_1'(c_t^1(y'')) \mid y'\} > 0$$

Now let us use the MLRP to complete our argument, namely, to derive a contradiction to our assumption. It is easy to verify that since $\gamma_1 \leq 1$ the function $w_t\varphi(A)g_1(x_t(y))u_1'(c_t^1(y))$ is nondecreasing in A for each value of y (note that it holds even though $c_t^1(y)$ is an increasing function of A). Therefore, by the MLRP we obtain that,

$$E\{[w_t\varphi(A)g_1(x_t(y'')) - R_t]u_1'(c_t^1(y'')) \mid y''\} > 0$$

Which contradicts the first order conditions. This proves that we must have $x_t(y') \leq x_t(y'')$.

Proof of Lemma 3: Obviously, $\phi^1(z)$ and $\phi^2(z)$ have the same mean. The proof is , therfore, complete if we can show that there exists some $\widehat{z}$ such that:

$$\phi^2(z) \leq \ [\geq] \ \phi^1(z) \quad \text{for} \quad z \leq \ [\geq] \ \widehat{z} \tag{42}$$

Define,

$$\phi(z) = \phi^2(z) - \phi^1(z) = \bar{\varphi}(z)\{\frac{M(z)}{E\{M(z)\bar{\varphi}(z)\}} - \frac{1}{E\{\bar{\varphi}(z)\}}\}$$

Since the term in brackets is increasing in z there exists $\widehat{z}$ such that $\phi(z) \leq 0$ for $z \leq \widehat{z}$ and $\phi(z) \geq 0$ for $z \geq \widehat{z}$. By the definition of $\phi(z)$, this property is equivalent to the inequalities in (42). ■

Proof of Lemma 4: This Lemma is a generalization of Lemma 2 in Karni and Zilcha (1995). Assume that the income $W(y)$ is more equal than $Z(y)$. Let us show that any non-trivial convex combination of the incomes $W(y)$ and $Z(y)$ is more equal than $Z(y)$. This follows from definition: Assume that $0 < \theta < 1$, then for any strictly concave function u we have:

$$E\{u[\theta W(y) + (1-\theta)Z(y)]\} \geq \theta Eu(W(y)) + (1-\theta)Eu(W(y)) \geq Eu(Z(y)) \tag{43}$$

since $Eu(W(y)) \geq Eu(W(y))$ holds for any such u. Now, to show the claim let us rewrite,

$$(1+\psi)\left[\frac{1}{1+\psi}W(y) + \frac{\psi}{1+\psi}Z(y)\right] = W(y) + \psi Z(y) \tag{44}$$

Since the LHS is more equal than $Z(y)$ this demonstrates the claim. The rest of the proof is similar to that of Lemma 2 in Karni and Zilcha (1995). ■

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