Choice with Frames

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Abstract. In order to incorporate recent developments in Bounded Rationality and Behavioral Economics into Choice Theory, we introduce the notion of an *extended choice problem* (A, f) where A is a set of alternatives and f is a *frame*. A frame consists of observable information that appears to be irrelevant to the rational assessment of the alternatives but nonetheless may affect choice. An extended choice function assigns a chosen element to every extended choice problem. We identify conditions under which there exists either a transitive or a transitive and complete binary relation over the alternatives such that an alternative x is chosen in some extended choice problem (A, f) if and only if x is maximal according to the binary relation in the set A. We then investigate several extended choice models in which behavior cannot be described as the maximization of a complete and transitive relation, or alternatively the binary relation provides little information on how the decision maker chooses from extended choice problems. We comment on the possible welfare interpretations of our results along the paper.

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1. Introduction

In economics we usually model a choice situation by describing the set of objects from which the individual makes a choice. Individual behavior is specified by assigning an alternative to every choice situation. In some cases, more than one element is assigned to a choice situation without further specifying the way in which the indeterminacy among the possible choices is resolved.

Mounting evidence from psychology, as well as casual observation and introspection, indicates that in real-life situations behavior often depends on additional observable information, which appears to be irrelevant to the rational assessment of the alternatives. A voter may be influenced by the order in which candidates are listed on a ballot. A consumer may condition an online purchase decision on the alternative designated as the default by the retailer. The choice of a vacation package from a catalogue may depend on whether a casino table appears on the front page. We refer to this additional information as a *frame* and to the dependence of choice on the frame as a *framing effect*.

Following the emergence of behavioral economics, the current trend in economics is to welcome the introduction of frames into economic modeling. In this paper, we propose and analyze a formal framework that incorporates the notion of a frame into the standard model of choice.

Formally, let X be a finite set of alternatives. According to the standard approach, a choice problem is a non-empty subset of X and a choice function attaches to every choice problem $A \subseteq X$ a single element in A. A choice correspondence attaches to every choice problem A a non-empty subset of A.

We enrich the standard model with a set F of frames. For example, F may be various orderings of the set X, a collection of default alternatives or a set of natural numbers interpreted as the number of elements the decision maker can seriously evaluate. An *extended choice problem* is a pair (A, f) where $A \subseteq X$ is a standard choice problem and $f \in F$ is a frame. An *extended choice function* c assigns an element of A to every pair (A, f). Note that Bernheim and Rangel (2006) independently develop a similar framework, which they call choice with ancillary conditions.

Caution should be exercised in applying the model of choice with frames. In many real-life situations the decision maker faces a set of alternatives with additional information that is in fact relevant for the rational assessment of the alternatives and thus should not be regarded as a frame. For example, consider a "matching" choice problem (A, f) where A is a set of males and f is a female for whom we choose a match from A. In this case, the evaluation of a male in A differs as we vary f. Such examples, though they can formally be included in the framework, lie outside the scope of this paper.

The standard argument against extending the description of a choice problem to include frames is that, as economists, we are interested only in behavior. According to this argument, even if the actual choice from a set depends on a frame, it might still be possible to describe behavior as resulting from the maximization of a transitive and possibly complete binary relation. In the first part of the paper, we explore conditions under which the choices of the decision maker can be described in this way even though the frame *does* affect his choices.

Our proofs basically follow the ideas in Sen (1971) adapted to the model of choice with frames. Given an extended choice function, the binary relation that describes choices (when ignoring the frames) must have the property that an alternative a is preferred to an alternative b if a is chosen over b in all extended choice problems $(\{a, b\}, f)$. Bernheim and Rangel (2006) independently reach a similar binary relation and give it a welfare interpretation. Similar constructs also appear in the specific contexts of choice from lists (Rubinstein and Salant, 2006A) and choice with a default alternative (Rubinstein and Salant, 2006B) without any reference to welfare.

The first part of the paper might be thought of as related to the discussion of welfare in the presence of framing effects. The existence of a binary relation whose maximization describes the decision maker's choices often leads the economist to postulate that the relation represents the true underlying preferences of the individual and thus should serve as a welfare criterion. In previous papers we express reservations regarding this approach (see Rubinstein (2005) and Rubinstein and Salant (2007)). However, if one adopts the approach that all welfare considerations should be rooted in behavior, then the conditions we identify for the existence of such a relation can be interpreted as marking the boundaries within which this welfare approach is feasible.

Our results do not imply that studying the details of a frame-sensitive choice procedure that satisfies these conditions is superfluous. On the contrary, analyzing how frames affect choice is crucial to understanding behavior in cases where the binary relation is incomplete or has many indifferences. In addition, these conditions emphasize the wide domain of frame-sensitive choice procedures that cannot be explained by the maximization of a complete and transitive relation.

In the second part of the paper we explore three extended choice models in which behavior cannot be described as the outcome of maximizing a complete and transitive binary relation, or alternatively it can, but the resulting relation has many indifferences. In such cases, the binary relation, if exists, contains little information on how the decision maker actually chooses from extended choice problems. For each model, we conduct the classical choice-theoretic exercise and characterize a family of choice procedures that satisfy interesting behavioral properties. We comment on the possibility of using the langauge of each model in welfare analysis.

The approach presented in this paper provides a possible direction for the development of new and richer models of choice that capture behavioral aspects of decision making discussed in the bounded rationality and behavioral economics literatures. As such, we view this paper as an attempt to bridge between standard choice theory and these literatures.

2. Examples

In this section we discuss several extended choice models that demonstrate the flexibility and richness of the framework.

i. Default alternative

One of the alternatives is designated as the default. The collection of frames is $F = X$. An extended choice problem is a pair (A, x) where $x \in X$ denotes the default alternative which is not necessarily an element of A. As an example of an extended choice function, consider a decision maker who has in mind two functions u and β from X to the real line. Given an extended choice problem (A, x), he chooses x if $x \in A$ and $u(x) + \beta(x) \geq u(a)$ for any other element $a \in A$; otherwise, he chooses the u-maximal element in A. Default bias can be modeled by taking $\beta(x)$ to be positive.

Several previous papers study models of choice with a default alternative. Zhou (1997), Masatlioglu and Ok (2004) and Sagi (2006) study models in which the default serves as a status quo option. Masatlioglu and Ok (2006) investigate a model in which the default alternative serves also as a reference point.

ii. List

The set of alternatives is presented to the decision maker in the form of a list. An extended choice problem is a pair $(A, >)$ where $>$ is an ordering of X. The satisficing procedure (see Simon (1955)) is as an example of an extended choice function: The decision maker has in mind a value function $v : X \to R$ and an aspiration threshold v^*. Given a pair $(A, >)$, he chooses the $>$-first element in A with a value above v^*. If there are none, the $>$-last element in A is chosen. See Rubinstein and Salant (2006A) for a detailed analysis of the model of choice from lists.

The ordering of the elements in the list model is defined over the set X. This means that formally two extended choice problems $(A, >_1)$ and $(A, >_2)$, which order the elements of A identically but differ on $X - A$, may result in different choices. In order to eliminate such peculiar situations, one should restrict attention to extended choice functions that choose the same element from every two extended choice problems that order the elements of A identically. A similar discussion applies to other examples in this section. Regarding examples (iii) and (iv), we will discuss this invariance property in detail in Section 4.

iii. Number of appearances

An alternative may appear multiple times in a "catalogue" which presents the elements of the choice problem. An extended choice problem is a pair (A, i) where i is a function that assigns a natural number to every $x \in X$. An example of an extended choice function is one that assigns to each (A, i) the element $a \in A$ which maximizes $i(x)u(x)$ over all $x \in A$, where u is a utility function.

iv. Time stamps

Some of the alternatives in the choice problem may be available in different timings. For simplicity, assume that there are only two dates, and that an alternative is either available in date 1 or date 2 but not both. An extended choice problem is a pair (A, R) where A is the set of elements available for choice in both dates, $R \subseteq X$ is the set of elements appearing in date 1 and $X - R$ is the set of elements appearing in date 2. As an example of an extended choice function, consider a decision maker who has in mind two functions u and v and a value u^*. Given a problem (A, R), he chooses the u-best element in the first stage if that element passes the threshold u^* or there are no elements in the second stage; otherwise, he chooses the v-best element in the second stage.

v. Limited focus

There is a limit on the number of alternatives that the decision maker can actually consider. An extended choice problem is a pair (A, n) where n is that number. As an example of an extended choice function, consider a decision maker who has in mind two orderings: a "focus" ordering O of X, which determines the alternatives he focuses on and an ordering $\succ$, which represents his preferences. Given an extended choice problem (A, n), the decision maker chooses the $\succ$-best element among the first $\min\{n, |A|\}$ elements in A according to the ordering O.

vi. Deadline

The decision maker is limited by the amount of time he can invest in the decision problem. An extended choice problem is a pair (A, t) where t is interpreted as the deadline for making a choice. An example of an extended choice function is based on two primitives: a value function v and a processing time function d which assigns to each element x the time $d(x)$ it takes the decision maker to understand its meaning. The decision maker chooses from (A, t) the v-maximal alternative among those with $d(x) \leq t$.

3. Choice with frames and the standard choice model

The standard interpretation of a choice correspondence C is that the set $C(A)$ contains all the elements that are chosen from the set A under some circumstances. With this interpretation in mind an extended choice function c induces a choice correspondence C_c, where $C_c(A)$ is the set of elements chosen from the set A for some frame f. That is,

$$C_c(A) = \{x \mid c(A, f) = x \text{ for some } f \in F\}.$$

In this section we analyze the relationship between the model of choice with frames and the standard model of choice correspondences. We discuss three results establishing that, while ignoring the frame, a choice correspondence satisfying certain properties is indistinguishable from an extended choice function satisfying analogous properties.

Our first result is quite trivial. It states that without imposing any structure on any of the two models they are indistinguishable if one ignores the information about the frames.

Proposition 0. For every choice correspondence C, there exists an extended choice function c such that $C = C_c$.

Proof. Let C be a choice correspondence. Let $F = X$ and define

$$c(A, x) = \begin{cases} x & \text{if } \quad x \in C(A) \\ \text{some } y \in C(A) & \text{if } \quad x \notin C(A) \end{cases}.$$

Obviously $C = C_c$. ■

We now investigate cases in which more structure is imposed, and identify circumstances in which an extended choice function can be represented in the correspondence sense as the maximization of a transitive (and possibly complete) binary relation. As mentioned earlier, when such a representation is possible, the binary relation is often interpreted as reflecting the decision maker's well-being. In Rubinstein (2005) and Rubinstein and Salant (2007) we express reservations on this approach. We argue that identifying choices with well-being is a strong assumption made by the modeler, and that the two concepts are in principle independent. In any case, when such a representation does not exist, it is necessary to enrich the model with a well-being criterion as we do in Section 4.

We focus on extended choice functions in which the frame "triggers" the decision maker to use a certain rationale when making a choice. Formally,

Salient Consideration. An extended choice function c is a Salient Consideration function if for every frame $f \in F$ there exists a corresponding ordering $\succ_f$ such that $c(A, f)$ is the $\succ_f$-maximal element in A.

Of the examples discussed in Section 2, the default alternative, the list, the number of appearances, the priority, and the deadline examples are Salient Consideration functions, while the limited focus example is not. In the limited focus model, if $xOyOz$ but $z \succ y \succ x$, then $c(\{x, y, z\}, 2) = y$ while $c(\{y, z\}, 2) = z$.

For any asymmetric and transitive binary relation $\succ$, let $C_\succ$ be a choice correspondence that maximizes $\succ$. That is,

$$C_\succ(A) = \{x \in A \mid \text{ there is no } y \in A \text{ such that } y \succ x\}.$$

Our next result states that the following two explanations of a choice correspondence C are indistinguishable in terms of choice observations when ignoring the frame:

(i) $C = C_\succ$ for some asymmetric and transitive (but not necessarily complete) binary relation $\succ$.

(ii) $C = C_c$ for some Salient Consideration choice function c satisfying property $\gamma-$extended.

Property $\gamma-$extended. If $c(A, f) = x$ and $c(B, g) = x$, then there exists a frame h such that $c(A \cup B, h) = x$.

Property $\gamma-$extended is satisfied by the examples in the default alternative model (let $h = x$), the number of appearances model (one can always find an i such that $i(x)u(x) > u(y)$ for every $y \in A \cup B$), the list model (let h be an ordering in which x appears first if x is satisfactory, otherwise an ordering in which x appears last) and the deadline model (let h be $t(x)$).

Proposition 1. A choice correspondence C satisfies $C = C_{\succ}$ for some asymmetric and transitive binary relation $\succ$ if and only if there is a Salient Consideration choice function c satisfying property $\gamma-$extended such that $C = C_c$.

Proof. Assume that $C = C_{\succ}$ where $\succ$ is an asymmetric and transitive binary relation. For every element $a \in X$, let $\succ_a$ be an extension of $\succ$ to a complete order relation in which only the elements in the set $\{b \mid b \succ a\}$ are ranked above a (the assumption that $\succ$ is transitive and not merely acyclic is important here). Let $F = X$. Define $c(A, a)$ to be the $\succ_a$-maximal element in A. Obviously, the function c is a Salient Consideration function. If $x \in C_{\succ}(A)$, then for no $y \in A$ does $y \succ x$. Therefore, for all $y \in A$, $x \succ_x y$. Hence, $c(A, x) = x$ which implies that $x \in C_c(A)$. Conversely, if $x \in C_c(A)$ then there exists a frame a, such that $c(A, a) = x$. If there were an element $y \in A$ such that $y \succ x$, then x could not be $\succ_a$-maximal in A. Thus, $x \in C_{\succ}(A)$. Therefore, $C_c = C_{\succ}$.

The extended choice function c satisfies Property $\gamma-$extended: if $c(A, a) = x = c(B, b)$, then x is $\succ$-maximal in both A and B which means that x is $\succ$-maximal in the set $A \cup B$. Thus, $x \in C_{\succ}(A \cup B)$ and therefore $x = c(A \cup B, x)$.

In the other direction, assume that $C = C_c$ where c is a Salient Consideration function satisfying Property $\gamma-$extended. The correspondence C satisfies the following standard property:

Property α. If $x \in B \subseteq A$ and $x \in C(A)$ then $x \in C(B)$.

Since c satisfies Property $\gamma-$extended, the correspondence C also satisfies:

Property γ. If $x \in C(A) \cap C(B)$ then $x \in C(A \cup B)$.

It is also easy to verify that C satisfies Property α^+ :

Property α^+. If $C(A)$ is a singleton and $C(A) \in B \subseteq A$ then $C(B) = C(A)$.

The rest follows from Lemma 1.

Lemma 1. A choice correspondence C satisfies Properties α, α^+ and γ if and only if there exists an asymmetric and transitive binary relation $\succ$ such that $C = C_{\succ}$.

Proof: It is easy to verify that $C_{\succ}$ satisfies the three properties. In the other direction, Sen (1971) shows that if a choice correspondence C satisfies properties α and γ, then there exists an asymmetric and acyclic binary relation $\succ$ such that $C = C_{\succ}$. To see that $\succ$ is also transitive assume that $x \succ y$ and $y \succ z$. Then $C(\{x, y\}) = \{x\}$ and $C(\{y, z\}) = \{y\}$. By property α the alternatives y and z do not belong to $C(\{x, y, z\})$ and thus $C(\{x, y, z\}) = \{x\}$. By property α^+ $C(\{x, z\}) = \{x\}$, which implies that $x \succ z$. ■

Note that when we start with an extended choice function c, the binary relation $\succ$ constructed in Proposition 1 must be defined by $x \succ y$ if $C_c(\{x,y\}) = \{x\}$ (just as in Sen (1971)). Thus, $x \succ y$ if and only if $c(\{x,y\},f) = x$ for every frame f. In some contexts, it seems natural to assume that if the decision maker chooses x over y in all frames when only both of them are available, then he truly prefers x to y. A similar relation is discussed in Bernheim and Rangel (2006) who define xP^*y if for every set A such that $x, y \in A$, $c(A,f) \neq y$ for every frame f. Rubinstein and Salant also use a similar relation in the specific contexts of choice from lists (2006A) and choice with a default alternative (2006B).

Obviously, results like Lemma 1 can also be used to check whether other models of extended choice functions can be represented as maximizing an asymmetric and transitive relation. One simply has to verify that the induced correspondence C_c satisfies Properties α, α^+ and γ.

In Proposition 1, the asymmetric and transitive relation is not complete. We now investigate conditions under which an the extended choice model is "behaviorally equivalent" to the maximization of a complete and transitive preference relation.

For any complete and transitive binary relation $\succsim$, let

$$C_{\succsim}(A) = \{x \in A \mid x \succsim y \text{ for every } y \in A\ \}.$$

We need to strengthen Property $\gamma-$extended.

Property γ^+-extended. If $c(A,f) = x$, $c(B,g) = y$ and $y \in A$, then there exists a frame h such that $c(A \cup B, h) = x$.

Note that restricting Property γ^+ to the case in which $y = x$ gives Property $\gamma-$extended. The satisficing procedure in the list model and the number of appearances example satisfy Property γ^+-extended. The default bias example does not necessarily satisfy Property γ^+. Indeed, assume that $X = \{x,y,z\}$ and let $u(x) = 1, u(y) = 2$ and $u(z) = 3$. If $\beta \equiv 1.5$, then $c(\{x,y\},x) = x$, and $c(\{y,z\},y) = y$ but there exists no frame in which x is chosen from $\{x,y,z\}$. In this example, the bonus given to x is not enough to make it more attractive than z even when x is the default alternative.

Our third result states that the following two explanations of a choice correspondence C are indistinguishable when ignoring the frame:

(i) $C = C_{\succsim}$ for some complete and transitive binary relation $\succsim$.

(ii) $C = C_c$ for some Salient Consideration choice function which satisfies property γ^+-extended.

Proposition 2. A choice correspondence C satisfies $C = C_{\succsim}$ for a complete and transitive binary relation $\succsim$ if and only if there is a Salient Consideration choice function c satisfying property γ^+-extended such that $C = C_c$.

Proof. Let $C = C_{\succsim}$ where $\succsim$ is a complete and transitive binary relation over X. Let $\succ$ be its asymmetric component. For every element $a \in X$, let $\succ_a$ be an extension of $\succ$ to a complete order relation in which only the elements in the set $\{b \mid b \succ a\}$ are ranked above a (most importantly, $a \succ_a x$ for every x such that $x \sim a$). Let $F = X$. Define $c(A, a)$ to be the $\succ_a$-maximal element in A. Obviously, the function c is a Salient Consideration function. If $x \in C_{\succsim}(A)$, then for all $y \in A$ we must have $x \succsim y$. Therefore, for all $y \in A$, $x \succ_x y$. Hence, $c(A, x) = x$ which implies that $x \in C_c(A)$. Conversely, if $x \in C_c(A)$ then there exists a frame a, such that $c(A, a) = x$. If there were an element $y \in A$ such that $y \succ x$, then x could not be $\succ_a$-maximal in A. Thus, $x \in C_{\succsim}(A)$.

To see that c satisfies Property γ^+−extended, note that if $c(A, a) = x$ and $c(B, b) = y \in A$, then x is $\succsim$-maximal in A and y is $\succsim$-maximal in B. By $y \in A$ and the fact that $\succsim$ is complete and transitive, x is $\succsim$-maximal in the set $A \cup B$. Thus, $x \in C_{\succsim}(A \cup B)$ and therefore $x = c(A \cup B, x)$.

In the other direction, let $C = C_c$ where c is a Salient Consideration choice function satisfying γ^+-extended. As in Proposition 1, the correspondence C_c satisfies Properties α and α^+. Property γ^+−extended implies the following property of $C_c = C$:

Property γ^+. If $x \in C(A)$, $y \in C(B)$ and $y \in A$, then $x \in C(A \cup B)$.

The rest follows from Lemma 2.

Lemma 2. A choice correspondence C satisfies properties α,α^+ and γ^+ if and only if there exists a complete and transitive binary relation $\succsim$ such that $C = C_{\succsim}$.

Proof. The only if part is immediate. Assume C satisfies Properties α,α^+ and γ^+. Then C also satisfies Property γ, and thus by Lemma 1 there exists an asymmetric and transitive binary relation $\succ$ such that $C = C_{\succ}$. We expand $\succ$ to a complete relation $\succsim$ by defining $x \sim y$ for every two elements x and y such that $x \not\succ y$ and $y \not\succ x$. To see that $\succsim$ is transitive it is enough to show that $\sim$ is transitive. Assume that $x \sim y$ and $y \sim z$. Then, $z \in C(\{y, z\})$ and $y \in C(\{x, y\})$. By Property γ^+, $z \in C(\{x, y, z\})$ and thus by Property α $z \in C(\{x, z\})$. Similarly, $x \in C(\{x, z\})$. Since $C = C_{\succ}$, neither $x \succ z$ nor $z \succ x$, which implies that $x \sim z$. ■

4. Beyond standard maximization

In this section we conduct the traditional choice theoretic exercise in the new framework. We start with an extended choice model and characterize choice procedures that satisfy interesting behavioral properties. We focus on models in which behavior cannot be described as the maximization of a complete and transitive binary relation, or alternatively it can, but the resulting preference relation contains little information on how the decision maker actually chooses from extended choice problems. For each model, we comment on the possibility of using its language in welfare analysis.

4.1. Limited focus

In some cases, there is a limit on the number of alternatives that the decision maker can consider. An extended choice problem is a pair (A, n) where n is the number of alternatives in A that the decision maker actually considers. The frame does not include however any information about the identity of these alternatives. A choice function c that is restricted to the domain of all pairs (A, n) in which n is the cardinality of A expresses choices in which the limit is not binding. If this restricted choice function is consistent with the maximization of a preference relation, it may convey a welfare meaning.

Let c be an extended choice function. We restrict our attention to extended choice problems (A, n) in which $n \leq |A|$.

Definition. Given a pair (A, n), a set $Z \subseteq A$ is called a *focus set* if:

(i) Z contains n elements.

(ii) For every set $B \subseteq A$ such that $Z \cap B \neq \emptyset$, we have $c(B, k) \in Z \cap B$ for $k = |Z \cap B|$, and in addition $c(B, k) = c(A, n)$ when $c(A, n) \in B$.

Thus, a focus set Z is consistent in the following sense. First, $c(A, n) \in Z$ (take $B = A$ in part (ii) of the definition). Second, removing elements from outside the focus set does not change the chosen element (in this case $Z \subseteq B$ and $k = n$). Finally, removing elements from within the focus set together with shrinking the size of the focus set by the same number of elements does not change the chosen element if it is still available; otherwise, an element from the remainder of the original focus set is chosen.

Since the frame in this model has very little structure, we need a strong property in order to ensure that the frame has the limited focus interpretation.

Focus Set property. An extended choice function c satisfies the Focus Set property if for every choice problem (A, n) there exists a set Z which is a focus set with respect to (A, n).

Proposition 3. A choice function c satisfies the Focus Set property if and only if there exists a focus ordering O and a preference ordering $\succ$ of X such that $c(A, n)$ is the $\succ$-maximal element among the n elements in A that are O-maximal.

Thus, the Focus Set property characterizes procedures according to which the decision maker has an ordering O that he applies to determine the focus set. He then rationally chooses from the focus set. This procedure has similarities to the Rational Shortlist Method discussed in Manzini and Mariotti (forthcoming), although the framework and the interpretation are quite different.

Proof. The "if" part is left to the reader.

Assume c satisfies the Focus Set property. For every two elements $a, b \in X$, define aOb if $c(\{a, b\}, 1) = a$ and $a \succ b$ if $c(\{a, b\}, 2) = a$. The rationale for the first definition is that when limited to considering only one option, the decision maker focuses on a rather than b. The rationale for the second definition is that when evaluating both alternatives, the decision maker prefers a to b.

The binary relations O and $\succ$ are complete and asymmetric. To see that O is transitive, assume to the contrary that xOy and yOz but zOx. Consider the set $\{x, y, z\}$ and assume without loss of generality that $c(\{x, y, z\}, 1) = x$. By the definition of a focus set and the Focus Set property, the one-element set $\{x\}$ is the entire focus set with respect to $(\{x, y, z\}, 1)$. Therefore $c(\{x, z\}, 1) = x$, thus contradicting zOx.

To see that $\succ$ is transitive, assume to the contrary that $x \succ y$ and $y \succ z$ but $z \succ x$. Consider the set $\{x, y, z\}$ and assume without loss of generality that $c(\{x, y, z\}, 3) = x$. Then $\{x, y, z\}$ is the focus set with respect to $(\{x, y, z\}, 3)$. Therefore, by the definition of a focus set and the Focus Set property, $c(\{x, z\}, 2) = x$, thus contradicting $z \succ x$.

Let (A, n) be an extended choice problem and Z the guaranteed focus set in A. Then $|Z| = n$.

We first show that Z is the collection of n O-maximal elements in A. Otherwise, there exists an element $x \in Z$ and $y \in A - Z$ such that yOx. By the definition of a focus set, $c(\{x\} \cup \{A - Z\}, 1) = x$ and thus $c(\{x, y\}, 1) = x$ contradicting yOx.

It remains to show that $c(A, n) = x$ is $\succ$-maximal in Z. By the definition of a focus set, $x \in Z$. In addition, $c(\{x, y\}, 2) = x$ for every $y \in Z$ and thus $x \succ y$ for every $y \in Z$. ■

While the limited focus procedures discussed in Proposition 3 do not necessarily induce Salient Consideration functions, they can still be represented in the correspondence sense as maximizing a transitive binary relation. Indeed, any limited focus function induces a correspondence satisfying Properties α, α^+ and γ and thus by Lemma 1 there exists a representation by a transitive relation. Since Property γ^+ may be violated, Lemma 2 implies that the relation is in principle not complete.[1] Nonetheless, it is still possible to identify a complete, asymmetric and transitive relation, namely the preference ordering $\succ$, which is an ingredient of the procedure, that may serve as an intuitive welfare criterion.

4.2. Choice with time stamps

In real life, some of the alternatives in the choice problem may be available in different timings. For simplicity, we assume that there are only two dates. An alternative is either available in date 1 or date 2 but not both. Formally, an extended choice problem is a pair (A, R) where A is the set of elements available for choice in both dates, $R \subseteq X$ is the set of elements appearing in date 1 and $X - R$ is the set of elements appearing in date 2. The following Invariance property states that choice is not sensitive to the time labels of alternatives which are not feasible choices.

Invariance. For any two extended choice problems (A, R) and (A, S) such that $R \cap A = S \cap A$, $c(A, R) = c(A, S)$.

Thus, we can confine our interest to pairs (A, R) in which R is a subset of A with the interpretation that the elements in the set R are available for choice in the first stage and those in $A - R$ are available for choice in the second stage.

[1] Assume that $zOxOy$ and $y \succ z \succ x$ and let c be a limited focus function based on these primitives. Then C_c violates Property γ^+ since $x \in C_c(\{x, y\})$, $y \in C_c(\{y, z\})$ but $x \notin C_c(\{x, y, z\})$.

We now characterize procedures in which the decision maker has a first stage ordering $\succ_1$ and a second stage ordering $\succ_2$. He applies the orderings to an extended choice problem (A, R) as follows. He chooses the $\succ_1$-maximal element from the elements available in the first stage if that element passes some threshold, which the decision maker has in mind, or there are no elements in the second stage. Otherwise, he chooses the $\succ_2$-maximal element from those available in the second stage.

Consistency. An extended choice function c satisfies the Consistency property if for every two choice problems (A, R) and (B, S) such that $c(A, R) = a$, the following holds:

(1) If $a \in R \cap S$ and $R \subset A$ then $c(B, S) \in S$.

(2) If $a, b \in R \cap S$ then $c(B, S) \neq b$.

(3) If $a, b \in (A - R) \cap (B - S)$ then $c(B, S) \neq b$.

Part (1) says that if the chosen element a is considered in the *first stage* ($a \in R$) and the set of alternatives is not exhausted in this stage ($R \subset A$), then an element from the first stage is always chosen when a appears in the first stage. That is, the choice of a reveals the satisfaction of the decision maker in the first stage. Part (2) says that if a is revealed to be better than b in (A, R) and both appear in the first stage, then b cannot be revealed to be better than a when they both appear in the first stage. Part (3) makes the same statement for the case in which both elements appear in the second stage.

Proposition 4. A choice function c satisfies the Consistency property if and only if there exist two orderings $\succ_1$ and $\succ_2$ and a subset $X^* \subseteq X$ satisfying that if $a \in X^*$ and $b \succ_1 a$ then $b \in X^*$, such that:

(i) If $R = A$, then $c(A, R)$ is the $\succ_1$-maximal element in R.

(ii) If $R \subset A$, then $c(A, R)$ is the $\succ_1$-maximal element in R if the set $R \cap X^*$ is not empty; otherwise, $c(A, R)$ is the $\succ_2$-maximal element in $A - R$.

Proof. The "if" part is left to the reader.

Assume that c satisfies the Consistency property. Define $a \succ_1 b$ if $c(\{a, b\}, \{a, b\}) = a$, and $a \succ_2 b$ if $c(\{a, b\}, \emptyset) = a$. These relations are complete and asymmetric. By parts (2) and (3) of the Consistency property they are also transitive.

Define $X^* = \{a \mid \text{there exists } b \text{ s.t. } c(\{a, b\}, a) = a\}$. Thus, an element a is in X^* if a is chosen over some other element when a is considered in the first stage and the other element in the second stage. Assume $a \in X^*$ and $b \succ_1 a$ for some $b \in X$. By part (1) of the Consistency property, $c(A, R) \in R$ for every set A such that $a \in R$. In particular, $c(\{a, b, x\}, \{a, b\}) \in \{a, b\}$ for every x. By part (2) of the Consistency property and the fact that $c(\{a, b\}, \{a, b\}) = b$, $c(\{a, b, x\}, \{a, b\}) = b$. Applying part (1), we get that $c(\{b, x\}, b) = b$ and thus $b \in X^*$.

Let (A, R) be an extended choice problem and denote $a = c(A, R)$.

Assume $R = A$. Then by part (2) of the Consistency property, $c(\{a, b\}, \{a, b\}) = a$ for every $b \in R = A$ and thus $a \succ_1 b$. In particular, a is $\succ_1$-maximal in R.

Assume $R \subset A$. Then, $c(A, R) \in R$ if and only if $R \cap X^* \neq \emptyset$. To verify the "if" part, assume that $x \in R \cap X^*$. Since $x \in X^*$, there exists an element $b \in X$ such that $c(\{x, b\}, x) = x$, and thus by part (1) of the Consistency property $c(A, R) \in R$. To verify the "only if" part assume that $a = c(A, R) \in R$. By part (1), $c(\{a, b\}, a) = a$ for every b in $A - R$ which implies that $a \in X^*$ and thus $R \cap X^* \neq \emptyset$.

Thus, if $R \subset A$ and $R \cap X^* \neq \emptyset$, then $a = c(A, R) \in R$. By part (2) of the Consistency property, $c(\{a, b\}, \{a, b\}) = a$ for every $b \in R$ and thus a is $\succ_1$-maximal in R. If $R \cap X^* = \emptyset$, then $a \in A - R$. By part (3) of the Consistency property $c(\{a, b\}, \{a, b\}) = a$ for every $b \in A - R$ and thus a is $\succ_2$-maximal in $A - R$. ■

Any procedure identified in Proposition 4 is characterized by the relations $\succ_1$ and $\succ_2$ and the set X^*. It can always be represented in the correspondence sense as maximizing a transitive but not necessarily complete binary relation.[2] The binary relation conflicts neither with $\succ_1$, which represents the choices of the restricted function $c(A, X)$, nor with $\succ_2$, which represents the choices of the restricted function $c(A, \emptyset)$. The relation depends on X^*. For example, if $X^* = \emptyset$, then the decision maker is never satisfied in the first stage unless there is really no second stage and thus $C_c(A) = A$. If $X^* = \{x \mid x \succsim_1 x^*\}$ then if $x^* \in A$, $C_c(A)$ contains the $\succ_1$-maximal element in A (since $c(A, A)$ is this element), every element $x \in X^* \cap A$ (since $C(A, \{x\}) = x$) and all elements which are $\succ_2$-maximal in some set B such that $(X^* \cap A) \subseteq B \subseteq A$ (since $c(A, A - B)$ is such an element).

Thus, the binary relation which explains the decision maker's choices is potentially very coarse. But since the decision maker applies two different preference relations $\succ_1$ and $\succ_2$ in the two dates, and there is no a priori reason to assume that one of them better reflects his underlying preferences, there seems to be no other natural welfare criterion.

4.3. Number of appearances

We wish to emphasize the importance of exploring the details of the choice procedure by investigating another extended choice model.

Often an alternative appears more than once in the choice problem without the repetition conveying any significant information about the alternatives. An extended choice problem is a pair (A, i) where i is a function that assigns a natural number $i(x)$ to every $x \in X$. We confine our interest to extended choice functions that satisfy the following invariance property:

Invariance. For any two choice problems (A, i) and (A, i'), if $i(x) = i'(x)$ for every $x \in A$, then $c(A, i) = c(A, i')$.

[2] To see that the binary relation need not be complete, let $z \succ_1 x \succ_1 y$, $y \succ_2 z \succ_2 x$, $X^* = \{z\}$, and c be an extended function based on these primitives. Then C_c violates Property γ^+.

The Invariance property implies that we can restrict attention to the domain of extended problems (A, i) where i assigns the value zero to every element that is not in A and we do so henceforth. We can now interpret the number $i(x)$ as the number of times the alternative x appears in the choice problem (A, i). A choice function c restricted to the domain of all extended choice problems $(A, 1_A)$ where $1_A(x) = 1$ for all $x \in A$ expresses choices in which the repetition frame has no effect. If this restricted choice function is consistent with the maximization of a preference relation, then the preference relation conveys a possible welfare meaning.

The first property of an extended choice function that we investigate states that increasing the number of repetitions of the chosen alternative, decreasing the number of repetitions of an alterative that is not chosen, or removing an alternative that is not chosen and appears only once, does not alter the choice.

Monotonicity (MON). An extended choice function c is monotone if $c(A, i) = x$ implies that $c(A, i') = x$ for every frame i' such that $i'(x) = i(x) + 1$ or $i'(z) = i(z) - 1$ for $z \in A - \{x\}$, and in both cases $i'(y) = i(y)$ for the remaining alternatives. In addition, if $i(z) = 1$ then $c(A - \{z\}, i') = x$, for every i' such that $i'(y) = i(y)$ for every $y \in A - \{z\}$.

The second property states that only the relative rather than the absolute frequencies of the alternatives matter.

Multiplicativity (MUL). An extended choice function c is multiplicative if $c(A, i) = c(A, m \cdot i)$ where $m \cdot i$ is the frame in which each alternative $x \in A$ appears $m \cdot i(x)$ times and m is a natural number.

The third property says that for every two alternatives, if one alternative is repeated enough times, the decision maker chooses that alternative over the other.

Non triviality (NONT). An extended choice function c is non-trivial if for every two elements $x, y \in X$, there exists a natural number M such that $c(\{x, y\}, i) = x$ where $i(x) = M$ and $i(y) = 1$.

The following result is related to the results of Smith (1973), Young (1975) and Myerson (1995) in the context of voting. However, the interpretation of the results in those papers is very different since in the context of voting $i(x)$ conveys important information about the quality of candidate x (e.g., $i(x)$ is a function of x's vote share.)

Proposition 5. A choice function c satisfies MON, MUL and $NONT$ if and only if there exists an array of positive weights $\{\alpha_x\}_{x \in X}$ and a preference relation $\succ_{tie}$ over X such that for every choice problem (A, i),

$$c(A, i) \in \arg\max_{x \in A} \alpha_x i(x)$$

and in the case that there is more than one maximal element, ties are resolved by $\succ_{tie}$.

Dropping any of the three properties invalidates the proposition. Indeed, let $>$ be an ordering of X and consider the following examples that cannot be represented as in Proposition 5:

(i) The decision maker chooses the $>$-best element if all the elements in the choice problem appear the same number of times. Otherwise, he chooses the $>$-worst element. In that case, $NONT$ and MUL are satisfied but MON is not.

(ii) The decision maker chooses the $>$-best element among the ones that appear at least twice. If all elements appear exactly once, he chooses the $>$-best element in the set. In that case, MON and $NONT$ are satisfied but MUL is not.

(iii) The decision maker chooses the $>$-best element in the choice problem. Clearly, MON and MUL are satisfied but $NONT$ is not.

Denote by $(k * x, l * y)$ the extended choice problem $(\{x, y\}, i)$ in which $i(x) = k$, $i(y) = l$ and $i(z) = 0$ for any other element. Define $(k * x, l * y, m * z)$ similarly.

Define the binary relation $\succ$ over the objects $k * x$ where k is a natural number and $x \in X$ as follows:

$$k * x \succ l * y \text{ if } x \neq y \text{ and } c(k * x, l * y) = x, \text{ or if } x = y \text{ and } k > l.$$

The following Lemmas will be useful in proving Proposition 5.

Lemma 1. If c satisfies MON and MUL, then $\succ$ is transitive.

Proof. Assume that $k * x \succ l * y$ and $l * y \succ m * z$. We distinguish between 5 cases:

(i) $x = y = z$. It follows from the definition of $\succ$ that $k > l$ and $l > m$. Thus, $k > m$ and $k * x \succ m * z$.

(ii) $x = y$ and $y \neq z$. Then, $k > l$. MON and $l * y \succ m * z$ then imply that $k * y \succ m * z$.

(iii) $y = z$. Then, $l > m$. MON and $k * x \succ l * y$ imply that $k * x \succ m * z$.

(iv) $x = z$. The negation, $m * z \succ k * x$, would contradict (ii).

(v) x, y and z differ from one another. If $c(k*x, l*y, m*z) = z$, then by MON $c(l*y, m*z) = z$, which is a contradiction. Similarly, $c(k * x, l * y, m * z) \neq y$. Thus, $c(k * x, l * y, m * z) = x$ and therefore by MON it must be that $c(k * x, m * z) = x$, which implies that $k * x \succ m * z$. ■

Lemma 2. Let c be a choice function that satisfies MON and MUL. Then, for every $x, y \in X$ such that $c(1 * x, 1 * y) = x$, there exists a unique number $\alpha_{xy} \in [0, 1]$ such that for any two natural numbers k and l:

(i) If $k/l > \alpha_{xy}$, then $k * x \succ l * y$,

(ii) If $k/l < \alpha_{xy}$, then $k * x \prec l * y$, and

(iii) If $k/l = \alpha_{xy}$, then for every two natural numbers k' and l' such that $k/l = k'/l'$, we always have either $k' * x \succ l' * y$ or $k' * x \prec l' * y$.

Proof. Part (iii) is a direct implication of MUL. To prove parts (i) and (ii), let x and y be two alternatives. Let Λ be the set of all pairs (k,l) such that k and l are natural numbers. Let $\Lambda_x = \{(k,l) \mid k*x \succ l*y\}$ and $\Lambda_y = \{(k,l) \mid l*y \succ k*x\}$. Then, $\{\Lambda_x, \Lambda_y\}$ is a partition of Λ.

It must be that $\inf_{(k,l)\in\Lambda_x} k/l \geq \sup_{(k,l)\in\Lambda_y} k/l$. If not, then there are pairs $(k,l), (k',l') \in \Lambda$ such that $k*x \succ l*y$, $l'*y \succ k'*x$ and $k/l < k'/l'$. By MUL we can assume that $l = l'$, which implies that $k' > k$. By the definition of $\succ$, it follows from $k' > k$ that $k'*x \succ k*x$; however, by the transitivity of $\succ$ we must have the oppositive preference, which is a contradiction.

It is impossible that $\inf_{(k,l)\in\Lambda_x} k/l > \sup_{(k,l)\in\Lambda_y} k/l$ since then we could find some (k^*,l^*) such that $\inf_{(k,l)\in\Lambda_{ix}} k/l > k^*/l^* > \sup_{(k,l)\in\Lambda_y} k/l$ and (k^*,l^*) is not in $\Lambda_i \cup \Lambda_j$. Define $\alpha_{xy} = \inf_{(k,l)\in\Lambda_x} k/l = \sup_{(k,l)\in\Lambda_y} k/l$ to complete the proof. ■

Proof of Proposition 5. We first show that $\alpha_{xy}\alpha_{yz} = \alpha_{xz}$ for all $x,y,z \in X$ such that $E(1*x, 1*y) = x$ and $E(1*y, 1*z) = y$. Assume to the contrary that $\alpha_{xy}\alpha_{yz} < \alpha_{xz}$. Then, there are natural numbers k, l and m such that $\alpha_{xy} < k/l$, $\alpha_{yz} < l/m$ and $k/m < \alpha_{xz}$. Thus, by Lemma 2 we must have $k*x \succ l*y$, $l*y \succ m*z$ and $m*z \succ k*x$, contradicting Lemma 1. Similarly, it is impossible that $\alpha_{xy}\alpha_{yz} > \alpha_{xz}$.

Assume without loss of generality that $X = \{1,..,n\}$ and that $1*1 \succ 1*2 \succ ... \succ 1*n$ and thus $1 \geq \alpha_{i,i+1}$ for all i. Define $\alpha_1 = 1$ and $\alpha_i = \times_{h=1}^{i-1}\alpha_{h,h+1}$. By NONT, we have $\alpha_i > 0$ for all i. For any two elements i and j such that α_{ij} is a rational number, that is, $\alpha_{ij} = k/l$ for two natural numbers k and l, define $i \succ_{tie} j$ if $k*i \succ l*j$. The relation $\succ_{tie}$ is transitive because $\succ$ is transitive.

Let (A,i) be an extended choice problem and assume that $\alpha_x i(x) > \alpha_y i(y)$. Assume without loss of generality that $y > x$. Then, $i(x)/i(y) > \alpha_y/\alpha_x = \times_{h=x}^{y-1}\alpha_{h,h+1} = \alpha_{xy}$. By Lemma 2 $i(x)*x \succ i(y)*y$ which means that $c(A,i) \neq y$. Thus, $c(A,i) \in \arg\max_{x\in A} \alpha_x i(x)$.

To conclude the proof, note that if both y and z are in $\arg\max_{x\in A} \alpha_x i(x)$ and $z > y$, then $\alpha_{yz} = i(y)/i(z)$ and the relation $\succ_{tie}$ is well-defined on y and z. If $y \succ_{tie} z$, then $i(y)*y \succ i(z)*z$ and thus y is not chosen from (A,i). ■

The procedures we characterized in this subsection satisfy the properties discussed in Section 3. In fact, any extended choice function satisfying the conditions of Proposition 5 induces the choice correspondence $C_c(A) \equiv A$ and thus can be trivially "rationalized" using a complete indifference relation. This "preference relation" is obviously less sound as a welfare criterion than the preference relation induced by the array of weights $\{\alpha_x\}_{x\in X}$ that is an ingredient of the choice procedure and represents choices from menus in which every alternative appears exactly once.

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