"Estimating Marginal Returns to Education"
by Carneiro, Heckman and Vytacil (AER, 2011)

Model:

$$Y = \alpha + \beta S + \varepsilon$$

where S is a dummy indicating college attendance (or some other school attainment index).

Two issues:

1. 1. schooling choice is endogenous, likely correlated with individual unobserved characteristics
2. marginal returns to schooling may vary across individuals, with sorting into schooling by those with highest marginal returns

The untreated and treated outcomes (wages) are

$$Y_0 = \mu_0(X) + U_0$$

$$Y_1 = \mu_1(X) + U_1$$

where $\mu_0(X) = E(Y_0 \mid X = x)$ and $\mu_1(X) = E(Y_1 \mid X = x)$. Return to schooling is

$$Y_1 - Y_0 = \beta = \mu_1(X) - \mu_0(X) + U_1 + U_0$$

The average treatment effect of schooling conditional on $X = x$ is

$$\bar{\beta}(x) = E(\beta \mid X = x, S = 1) = \bar{\beta}(x) + E(U_1 - U_0 \mid X = x, S = 1)$$

Schooling decision:

$$Returns \quad : \quad I_s = \mu_s(Z) - V$$

$$Decision \quad : \quad S = 1 \text{ if } I_s \geq 0,\ S = 0 \text{ otherwise}$$

(X, Z) is observed, but not (U_0, U_1, V).which we assume to be independent of (X, Z).

Let $P(z) = Pr(S = 1 \mid Z = z) = F_V(\mu_s(z))$, which is the propensity score. Define $U_s = F_V(V)$, which is uniformly distributed *by construction*. Different values of U_s correspond to different quantiles of V. Then we can write $S = 1$ if $P(Z) \geq U_s$.

The marginal treatment (MTE) is defined as

$$MTE(x, u_s) = E((\beta \mid X = x, U_s = u_s)$$

Tracing out the MTE over u_s shows how the returns to schooling vary with different quantiles of the unobserved component of the index of the desire to enroll in schooling (i.e. *the mean return to schooling of those at the point of indifference between going and not going to school*).

We now show

$$MTE(x,p) = \frac{\partial E(Y \mid X = x, P(Z) = p)}{\partial p}$$

Proof:
Observed earnings are

$$Y = SY_1 + (1-S)Y_0 = \mu_0(X) + [\mu_1(X) - \mu_0(X)]S + U_0 + S(U_1 - U_0)$$

Then

$$E(Y \mid X = x, P(Z) = p) = E(Y_0 \mid X = x, P(Z) = p) + pE(Y_1 - Y_0 \mid X = x, S = 1, P(Z) = p)$$

Using the rule for selecting into schooling, this can be rewritten

$$\begin{aligned} E(Y \quad &\mid \quad X = x, P(Z) = p) \\ &= \quad \mu_0(x) + p[\mu_1(x) - \mu_0(x)] + \int_{-\infty}^{\infty} \int_0^p (u_1 - u_0) f(u_1 - u_0 \mid X = x, U_s = u_s) du_s d(u_1 - u_0) \end{aligned}$$

Recalling that $MTE(x, u_s) = E((\beta \mid X = x, U_s = u_s)$,we can simplify the previous expression as

$$E(Y \mid X = x, P(Z) = p) = \mu_0(x) + \int_0^p MTE(x, u_s) du_s$$

Note: To confirm this, substitute the expression for the MTE into the integral above and remember that $\beta = \mu_1(X) - \mu_0(X) + U_1 + U_0$. Writing out terms will yield the long expression for $E(Y \mid X = x, P(Z) = p)$ given before.

Using

$$E(Y \mid X = x, P(Z) = p) = \mu_0(x) + \int_0^p MTE(x, u_s) du_s$$

we differentiate this expression with respect to p, which yields

$$\frac{\partial E(Y \mid X = x, P(Z) = p)}{\partial p} = MTE(x, p)$$

Interpretation: People with high $P(Z)$ [propensity to get schooling] identify the return for those with a *high value* of U_s values. Those with low values of U_s are already in schooling for that $P(Z)$ so are not affected by the marginal change in $P(Z)$. Thus what we identify are those persons (given by the quantile of the unobserved component of the desire to go to school, U_s) who are induced into schooling by a marginal change in $P(Z)$.

Policy relevant treatment effect (PRTE):

$$PRTE = \frac{E(Y \mid Alternative\ Policy) - E(Y \mid Baseline\ Policy)}{E(S \mid Alternative\ Policy) - E(S \mid Baseline\ Policy)}$$

Interpretation: Normalised effect of a change from a baseline policy to an alternative, and this depend on the alternative being considered. PRTE maps from the proposed policy change (corresponding to some distribution of P*) to the resulting per person change in outcomes. In general, this will differ from the change induced by an instrument (used for IV estimation) unless the instrument and policy change coincide.

Application to Returns to Schooling

Authors use linear in parameters model: $\mu_0(X) = X\delta_0$, $\mu_1(X) = X\delta_1$ and $\mu_s(Z) = Z\gamma$. In this case

$$E(Y \mid X = x, P(Z) = p) = x\delta_0 + px \mid \delta_1 - \delta_0 \mid + K(p)$$

where $K(p) = E(U_1 - U_0 \mid S = 1, P(Z) = p)$ can be estimated non-parameterically as a polynomial.

Test of the null hypothesis there is no selection on gains: This hypothesis says that the MTE is constant in u_s. To do this, we specify $K(p)$ as a polynomial in P (P can be estimated using a Probit or Logit) and then test whether the coefficients on the polynomial terms of order higher than one are jointly equal to zero.

Outcome, regressors and instruments in Table 2, p. 2763

Table 3: First stage estimates of schooling choice

Table 4:

1. Panel A tests and rejects (mostly) that there is no selection on gains.
2. Panel B shows that the $LATE$'s estimated over different ranges of U_s often differ significantly, indicating the potential importance of using PRTE.

Figure 4: MTE estimates

1. Large positive and statistically significant gains from schooling for low values of U_s and no significant returns to schooling for high U_s.
2. Individuals with low values of U_s are those with unobserved characteristics that make them more likely to be in schooling. Thus the MTE estimates show positive selection on gains, and in this (narrow) sense an "efficient" allocation.